The Rational Postulates for *DEL* in the Context of Real Administering of Knowledge

by

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Introduction

Determination of administering knowledge by a rational agent acting in compliance with certain protocols within a real system of interaction, a system of communicating, requires postulating rationality of acting by the agent, as well as postulating restriction of this rationality appropriately to the real actions.

A description of administering knowledge by the rational agent in compliance with *DEL* protocols was presented in [1] – [8].

The notion of bounded rationality was introduced by H.A. Simon in the 20th century [10], who proposed to distinguish: (1) a set of agents, (2) a set of behaviour alternatives, (3) a set of outcomes of choice among the behaviour alternatives, and (4) a set of order of preferences for making choices of behaviours. According to him, an agent who is invested with "perfect rationality" possesses a full knowledge of distinguished sets, whereas an agent with bounded rationality, in contrast, might not know all alternatives, nor need it know the exact outcome of each; what is more, such an agent might lack a complete preference ordering which is indispensable to obtain the outcomes.

We assume that establishing a proper *DEL* protocol for the agent with bounded rationality leads to linking the real system of interactions with relevant types of communicating. Thanks to fixing the type of communicating, it becomes possible to assign a suitable class of Kripke models for *DEL* to this type. In a real system of interactions, a set of rational agents is limited to a set of subjects of such actions of communicating as: production, rendering available and possession or allocating the objects distinguished by agents. The objects are products of the action of communicating. The products are divided into resources, goods, services and values arising in consequence of actions realized within the real system of interactions. The order of preferences for making choices of actions necessary to obtain certain products expected by agents as a result of a given action, is determined by real conditions that establish the beginning and the end of this action.

In this paper we will present rational postulates which allow executing a certain typology of real systems of interactions. They are divided into the following four groups:

- postulates concernig information networks (**P1 – P3**),
- postulates of the real interactivity system (**P4 – P6**),
- postulates of administering knowledge (**P7 – P11**),
- postulates of approximated semantics for logics *Real-DEL* (**P12**).

The postulates will be introduced in the successive Sections of the paper.

1. Postulates concerning information networks

P1. ***Information about an object O*** (*in short:* ***information***) *is a set of data about the object* ***O****, or more precisely – a set of data* ***identifying*** *the object* ***O*** *or any object being part of the object* ***O****.*
Pieces of information are ***indiscernible*** *when they identify the same objects. Identification of an object* ***O*** *groups information about the object* ***O****, thus it groups indiscernible pieces of information.*

P2. ***Reference of information about objects*** *is an ordered set of information about objects. The first piece of information in the given reference identifies the object which the last piece of information is about in this reference.*

P3. ***Information transmission and processing.*** *References on elements determining the same object* ***transmit information*** *on this object. The first element of this reference is a piece of* ***input*** *information, while the last one –* ***output*** *information. References not only transmit information,* ***process information****: the first piece of information – the* ***input*** *one – into the last piece of reference information – the* ***output*** *one. Information transmission is a particular case of information processing. We call the object which assigns ordered systems of objects to references* ***information channel****. The first object of the system determined by the information channel is the* ***input of the channel****, while the last object of this system – the* ***output of the channel****. The information channel processes information if each n-th piece of information of reference determines the n-th object of the system of objects ordered by this channel system of objects. We call the collection of information channels an* ***information network****. The inputs and outputs of information channels will be called the inputs and outputs of the information network. The Internet is a model example of an information network.*

2. Postulates of the real interactivity system

We will understand the **real interactivity system** as a system of communicating, whose model example is the Internet. In such a system, processing information means producing **resources** of knowledge and respective rendering them available, which leads to possessing or allocating of the knowledge, for instance, producing, rendering available, possessing or allocating of files which include some data or serve the purpose of processing these data. Production and making available of the resources of knowledge, according to common needs of users of the system, is a certain **good** provided by informatics for the users. Production and rendering available of the resources of knowledge, as requested by the users in order to satisfy individual needs, is – for the users – a certain **service** provided by informatics. The equivalent usefulness of resources, goods and services establishes their **value** for users of the communication system. Possession or allocation of accessibility to the goods and services, as well as to the value is – at the same time – a process of producing new information resources.

P4. ***System of communicating*** *is a system of human activity and – at the same time – an information network defined for sets of objects that are subjects or objects of production, rendering available and possession or allocation of resources, goods, services and values being effects of people's informatics-related activity within the system. Still, each input and output of this information network is a subject of production, rendering available, possession or allocation.* ***Knowledge*** *is information processed in a certain system of communicating. A set of data on the subject, relating to the kind of knowledge that the subject possesses, is*

understood to be information about the subject. ***Communicating*** *is processing information within the system of communicating. Pairs of such attributes of input/output, subjects' activity at the inputs and outputs of the communicating system as production, rendering available, possession or allocation allow distinguishing the basic types of communicating. We accept that the informatics-related activity of those communicating with one another, which is determined by the above-listed attributes points – with the dominance of this activity – to only one type of their activity. We accept that communicating is as follows:*

Interactive (*with index 1*) *– when, at the input, there dominates production of knowledge of the net user, while – at the output – this knowledge is rendered available to the user, e.g. ordering to have money transferred to the bank account, in consequence of which the knowledge about the operation made is made available on the account, or the other way round: when at the input one net user renders available knowledge to another user at the output in order to process it, e.g. logging on the bank account and calculating – with the use of the calculator accessible there – the interests rate of the credits granted,*

Verbal (*with index 2*) *– when, at the input, there dominates possession of knowledge, while – at the output – allocation of the knowledge, or the other way round – one of the users possesses* (*e.g. on a website*) *of another user, or the other way round – on the website of the first net user there is allocated knowledge which the other user possesses in his computer, this knowledge is automatically acquired from the computer of the other user; let us note that this kind of communicating can occur without referring to the meaning of sentences which represent the processed knowledge* (*content of the information*)*, therefore this communicating can be called verbal,*

Public (*with index 3*) *– when at the input and at the output there dominates allocation of knowledge, e.g. readers of a published title, by means of questionnaires meant to examine what kind of knowledge they allocate, cause the editors of the title – after getting acquainted with the questionnaires – to allocate and present this knowledge in the title they edit; it also happens that titles – through presentation of the allocated knowledge – influence the type of knowledge their readers will allocate,*

Private (*with index 4*) *– when at the input and at the output there dominates possession of knowledge, which most often takes place while transferring personal data, e.g. the data are passed when the provider of a service must possess the data which the receiver of the service does; in a similar way a person's identity card is displayed to a police officer,*

Static (*with index 5*) *– when at the input there dominates rendering knowledge available and at the output – allocation of knowledge, e.g. an Internet website displays a road map and the Internet user – on the basis of the map – allocates knowledge about roads to reach Copenhagen; or the other way round – when at the input there dominates allocation of knowledge, while at the output – rendering it available, e.g. the Internet user renders knowledge allocated by an Internet forum on the very forum itself; in the process of communicating no new data are produced* (*the data are only made available and are allocated*)*,*

Dynamic (*with index 6*) *– when at the input there dominates production of knowledge, while at the output – possession of knowledge, e.g. one of the communicating subjects produces new data in order to change the resources of knowledge of the other subject; or the other way round – at the input there dominates possession of knowledge, while at the output – production of knowledge, e.g. the subject, at the output, makes use of knowledge of the other subject in order to make alterations,*

Decision-making (*with index 7*) – *when at the input and at the output there dominates production of knowledge – the first subject of communication changes the data in the way such that the other of the subjects could implement the changes to make his own alterations; it can also be otherwise – the other subject will be able to make appropriate alterations of data obtained in the process of communicating then and only then when the first subject makes relevant changes of the data; thus, the changes being made depend on decisions on making the changes undertaken by the subjects,*

Discursive (*with index 8*) – *when at the input and at the output there dominates rendering knowledge available, which most often takes place in a discourse, i.e. when two subjects communicating with each other process knowledge in order to mutually make it available,*

Intelligent (*with index 9*) – *when at the input there dominates production of knowledge, while at the output – allocation of knowledge and such production of knowledge that by the first subject that the knowledge could be allocated by the other subject,*
or the other way round – when at the input there dominates such allocation of knowledge by the first subject that the other subject could produce something out of it at the output;
both of the described actions can be considered a manifestation of intelligence,

Behavioural (*with index 10*) – *when at the input there dominates rendering knowledge available and at the output – possession of knowledge, e.g. if the first of the subjects holds a lower social rank than the other subject* (*is dependent on the other one*), *then the first of the subjects must make the knowledge available to the other in order that the latter would expand his knowledge,*
or the other way round – when at the input there dominates possession of knowledge, while at the output – rendering knowledge available, e.g. if the first subject has a higher social rank than the other (*the other subject is dependent on the first*), *then the first subject must possess knowledge which can be made rendered available to the other one in order that the rank of the former could be established.*

We accept that the above-mentioned types of communicating are disjoint in the aspect of subjects' activity: if, between two subjects, there occurs communicating of one of the types, then the other types of communicating do not occur.

Table 1. Types of communicating determined by input/output attributes

input\output	***Production***	***Rendering avail.***	***Possession***	***Allocation***
Production	***Decision-making 7***	***Interactive 1***	***Dynamic 6***	***Intelligent 9***
Rendering avail.	***Interactive 1***	***Discursive 8***	***Behavioural 10***	***Static 5***
Possession	***Dynamic 6***	***Behavioural 10***	***Private 4***	***Verbal 2***
Allocation	***Intelligent 9***	***Static 5***	***Verbal 2***	***Public 3***

The opposition of the types is represented by means of the following juxtapositions of textures of opposing patterns (*opposing colours*): (,), (,), (,), (,), (,).

P5. ***Epistemic agent*** (*in short:* ***agent***) *is an object at the input or output of a system of communicating.*

P6. *The following* **aspects of knowledge** *are distinguished*:

Common-sense knowledge – applied knowledge and habitual knowledge, which – for agent *a* is distinguished by the operator of assertiveness (A_a): ***agent a thinks that …***,

Emotive knowledge – knowledge related to feelings distinguished for agent *a* by the operator of feeling (F_a): ***agent a feels that …***,

Sensual knowledge – sensual knowledge distinguished for agent *a* by the operator of perception (P_a): ***agent a perceives that …***,

Empirical knowledge – knowledge attained in the way of experiencing, verifying, testing, distinguished for agent *a* by the operator of experience (E_a): ***agent a experiences that…***

Rational knowledge – knowledge attained through thinking and reasoning distinguished for agent *a* by the operator of understanding (K_a): ***agent a knows that …***.

The rational knowledge consists of the above-listed aspects of knowledge, as well as types of knowledge defined through relations between the above aspects of knowledge:

I know that φ if

(***alternative of the aspects of knowledge***)
I think that φ or my feeling is that φ or I perceive that φ or I experience that φ;

(***principle of subordination***)
when I think on the basis of experience or feel on the basis of perceiving;

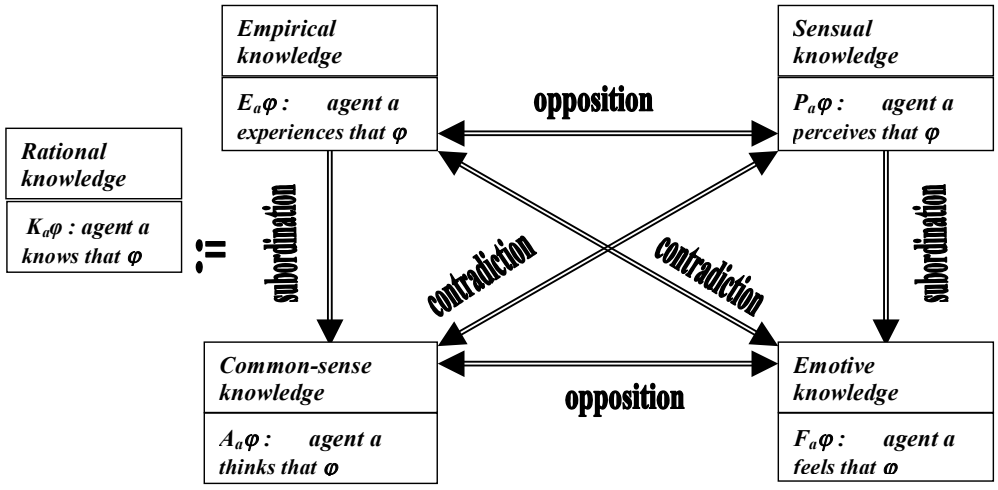
(***principle of oppositions***)
if I think, I do not feel,
if I feel, I do not think,
if I experience, then I do not perceive,
if I perceive, then I do not experience;

(***principle of contradiction***)
I do not think iff I perceive, I do not experience iff I feel.

The above-listed aspects of knowledge and the types of communicating defined earlier allow determining bounded activities of agents in the communication practice. These limitations may be established by carrying appropriate observations of communicating agents and by applying relevant research methods such as: questionnaire-based surveying, testing, computer simulations, etc. Results of research yield also a reliable picture of agents' interactions, leading to revealing the real interaction system.

The basic epistemic operators applied in the real system of interaction satisfy the following logical square given in Diagram 1:

Diagram 1. Square of epistemic operators for different aspects of knowledge



Juxtaposing the fundamental epistemic operators with types of communication with indexes 1-10, we obtain the following matrix of epistemic operators:

Matrix of epistemic operators

type\aspect	*1. A_a*	*2. F_a*	*3. P_a*	*4. E_a*	*5. K_a*
1. Interactive	A_a^1	F_a^1	P_a^1	E_a^1	K_a^1
2. Verbal	A_a^2	F_a^2	P_a^2	E_a^2	K_a^2
3. Public	A_a^3	F_a^3	P_a^3	E_a^3	K_a^3
4. Private	A_a^4	F_a^4	P_a^4	E_a^4	K_a^4
5. stanic	A_a^5	F_a^5	P_a^5	E_a^5	K_a^5
6. Dynamic	A_a^6	F_a^6	P_a^6	E_a^6	K_a^6
7. Decision-mak.	A_a^7	F_a^7	P_a^7	E_a^7	K_a^7
8. Diskursive	A_a^8	F_a^8	P_a^8	E_a^8	K_a^8
9. Inteligent	A_a^9	F_a^9	P_a^9	E_a^9	K_a^9
10. Behavioural	A_a^{10}	F_a^{10}	P_a^{10}	E_a^{10}	K_a^{10}

For any set of agents, set of types of communicating and sets of aspects of knowledge processed in the communicating process there exists relevant logic *DEL* with epistemic operators determined by types of communicating and aspects of knowledge (as in the matrix of epistemic operators). These logics can be called *Real-DEL*.

Proposed axioms for logics *Real-DEL*

Subordination

$E_a^i\varphi \Rightarrow A_a^i\varphi$

$P_a^i\varphi \Rightarrow F_a^i\varphi$

Contradiction

$E_a^i\varphi \Leftrightarrow \neg F_a^i\varphi$

$A_a^i\varphi \Leftrightarrow \neg P_a^i\varphi$

Opposition

$E_a^i\varphi \Rightarrow \neg P_a^i\varphi$
$P_a^i\varphi \Rightarrow \neg E_a^i\varphi$
$A_a^i\varphi \Rightarrow \neg F_a^i\varphi$
$F_a^i\varphi \Rightarrow \neg A_a^i\varphi$

Alternative of the aspects of knowledge

$$A_a^i\varphi \vee P_a^i\varphi \vee E_a^i\varphi \vee F_a^i\varphi \Rightarrow K_a^i\varphi$$

P7. ***Administering knowledge*** *is processing knowledge within information channels in which communicating occurs. It follows from the definition of the information channel and determining the agent that the input and the output of the information channel is a certain agent. Information channels which compose administering the knowledge are* ***dispositions of knowledge****. The fact that the agent knows something, encodes, decodes and represents knowledge, acquires knowledge, announces knowledge, is convinced* (*believes in something*)*, is interpreted as making use of suitable dispositions of knowledge by the agent: possessing knowledge, encoding, decoding, etc. We call the whole of administering the knowledge the* ***state of administering knowledge*** (*in short:* ***state***).

P8. *In order to administer knowledge, a group of agents who realize a certain type of communicating accept an appropriate* ***protocol of processing knowledge*** *that implements this type of communicating.*

3. Administering resources of knowledge

The presented rational postulates for *DEL* allow establishing sets S of all states of administering knowledge within the selected real system of interaction. Let $\boldsymbol{P}$ be a set of atomic propositions expressing knowledge, and $\boldsymbol{A}$ be a set of agents. Relations of using – by agents – information channels, are then determined by the mapping R_A: $\boldsymbol{A} \rightarrow \wp(S\times S)$, and also the mapping $V^{\boldsymbol{P}}$: $\boldsymbol{P} \rightarrow \wp(S)$ is known as it determines a set of states, in which for the given atomic proposition there occurs communicating that processes this atomic proposition. *Structure* $M = \langle S, R_A, V^{\boldsymbol{P}} \rangle$ *is then a Kripke model for DEL* (cf. [5]).

Let us note that determining the real system of interaction is executed in a certain relational data basis. The above-mentioned postulates allow identifying attributes of this data basis and values of these attributes. This aspect of the research offers the possibility, in the case of vagueness in determining results of communicating, of applying the method of rough sets in Pawlak's sense [9] to describe this communicating. Administering resources of knowledge in social and economic systems of managing knowledge can be described in this sense as relational data bases, and then – by means of these bases – certain classes of Kripke models can be fixed for *DEL*. A result of such research can be fixing of this type of *DEL* for the given system of managing knowledge. The rational actions proposed here, which lead to fixing certain classes of models for *DEL* can be made precise by accepting the following postulates:

P9. *Protocols of processing knowledge must be established for each type of communicating so that the agents communicating* (*within this type*) *could administer, in certain states, a set of atomic sentences that are true only within this type of communicating: with the established semantics of DEL other sentences can also be processed within this type of communicating and acknowledged or not to be true.*

P10. *The set S of states of administering knowledge is a sum of disjoint sets* $S_1, S_2, \ldots, S_{10}$, *and* S_i *– in compliance with* **P9** *– corresponds to the type of communication with index i given in* **P4**.

P11. *In the language of DEL there are distinguished epistemic operators: assertive* $\boldsymbol{A_a^i}$, *of feeling* $\boldsymbol{F_a^i}$, *perception* $\boldsymbol{P_a^i}$, *experiencing* $\boldsymbol{E_a^i}$, *understanding* $\boldsymbol{K_a^i}$, *where each operator, respectively* (*as in* **P9**), *distinguishes atomic sentences in the i-th type of communicating.*

4. Approximate semantics for *Real-DEL*

The truthfulness of the formula of *DEL* language in model $M = <S, R_A, V^P>$ can be defined in an equivalent way to the standard definition through an extension of valuation function V^P: $\boldsymbol{P} \rightarrow \wp(S)$ to function V: $\boldsymbol{FORM} \rightarrow \wp(S)$, where ***FORM*** is a set of properly built *DEL* formulas so that for any formula $\varphi \in \boldsymbol{FORM}$

$$M,s \models \varphi \quad \text{iff} \quad s \in V(\varphi).$$

Accepting postulates **P9** and **P10** one can ask the question in what way sets $V(\varphi)$ of *states of administering knowledge* depend on sets $S_1, S_2, \ldots, S_{10}$, that is what is the relationship between types of communicating and truthfulness of formulas. An answer to this question can be obtained by using the method of rough sets in Pawlak's sense [9]:

P12. *Assessing set* $X = V(\varphi)$ ***from the bottom*** (**as a lower approximation**) *by means of the set*

$$A^-(X) = \bigcup \{S_i : S_i \subseteq X, i = 1,2,\ldots,10\},$$

and also ***from the top*** (**as an upper approximation**) *by means of the set*

$$A^+(X) = \bigcup \{S_i : S_i \cap X \neq \emptyset, i = 1,2,\ldots,10\}$$

we can determine the relation of types of communicating and truthfulness of formulas in the following way:

Truthfulness of the two formulas φ, ψ *does not depend on a choice of type of communication, when*

$$A^-(V(\varphi)) = A^-(V(\psi)),$$

$$A^+(V(\varphi)) = A^+(V(\psi)).$$

Logical values of formulas φ and ψ are *indiscernible* (equivalent) in all types of communicating, symbolically: $V(\varphi) \approx V(\psi)$ if their lower approximations and their upper approximations are the same.

Postulate **P12** admits existence of equivalence classes $[V(\varphi)]_{\approx}$ of sets $X \subseteq S$ of states of administrating knowledge such that

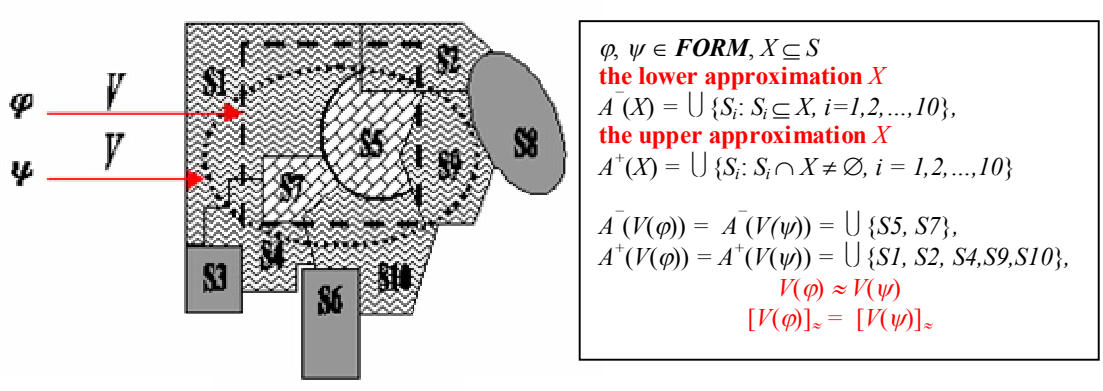
$$A^-(V(\varphi)) = A^-(X),$$

$$A^+(V(\varphi)) = A^+(X)$$

for any formula $\varphi \in \boldsymbol{FORM}$.

The above-described method of approximating logical values of formulas φ and ψ. is illustrated by Diagram 2 below.

Diagram 2.



The square and the ellipse represent sets of states: the logical values of formulas φ and ψ, respectively; the wavy part of the diagram corresponds to the upper approximation, while the checked part – to the lower approximation of these values. Inside the box on the right, there are definitions of the lower approximation and the upper approximation of sets of states; an equivalence relation ≈ defined on sets of states is also determined. The last equation expresses the identity of equivalence classes for equivalent sets of states (values of logical formulas φ and ψ). In 1982, Zdzisław Pawlak called equivalence classes defined in an analogous manner – rough sets.

Accepting postulates **P1–P12**, the mapping defined by the following formula:

$$[V]: \boldsymbol{FORM} \rightarrow \{[V(\varphi)]_{\approx} : \varphi \in \boldsymbol{FORM}\} ,$$

can be called the ***approximation valuation***, and the structure $[M] = \langle S, R_A, V^P, [V] \rangle$ can be called the ***approximation Kripke model***.

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