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# A Unified Framework for Whole-Body Humanoid Robot Control with Multiple Constraints and Contacts

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**Summary.** Physical interactivity is a major challenge in humanoid robotics. To allow robots to operate in human environments there is a pressing need for the development of control architectures that provide the advanced capabilities and interactive skills needed to effectively interact with the environment and/or the human partner while performing useful manipulation and locomotion tasks. Such architectures must address the robot whole-body control problem in its most general form: task and whole body motion coordination with active force control at contacts, under various constraints, self collision, and dynamic obstacles. In this paper we present a framework that addresses in a unified fashion the whole-body control problem in the context of multi-point multi-link contacts, constraints, and obstacles. The effectiveness of this novel formulation is illustrated through extensive robot dynamic simulations conducted in SAI, and the experimental validation of the framework is currently underway on the ASIMO platform.

## 1 Introduction

Robotics is rapidly expanding into human environments and vigorously engaged in its new emerging challenges. Interacting, exploring, and working with humans, the new generation of robots will increasingly touch people and their lives. The successful introduction of robots in human environments will rely on the development of competent and practical systems that are dependable, safe, and easy to use. Physical interactivity is a key characteristic for providing these robots with the ability to perform and interact with the surrounding world. While much progress has been made in robot locomotion and free-space motion behaviors, the physical interaction ability has remained very limited. Over the past ten years, our effort in humanoid robotics was aimed at addressing the various aspects of humanoid robot control (motion, contacts, constraints, and obstacles) in an integrated coherent fashion. An important milestone in this direction was reached in 2004 [1], where full integration of free-space task and posture control was established. This result was based on the concept of task-consistent posture Jacobian and the development of models of the robot dynamic behavior in the posture space. These models were the basis for the development of the whole-body free-space motion control structure, which we have demonstrated on various platforms. In subsequent development, our effort has also addressed constraints [2] and multiple contacts [3]. This paper finally brings the integration of these components in a unified fashion. The new framework addresses the essential capabilities for task-oriented whole-body control in

the context of integrated manipulation and locomotion under constraints, multiple contacts, and multiple moving obstacles.

In this paper. We will first summarize our previous results on the dynamic decomposition of task and posture behaviors. We will then propose a novel representation for under-actuated systems in contact with the environment, which will serve as the basis for the development of the unified whole-body control framework.

Several research groups in academic and private institutions have developed and implemented whole-body control methods for humanoid systems, with leading research by Honda Motor Corporation [4] and the National Institute of Advanced Industrial Science and Technology in Japan [5]. Their platforms, based on inverse kinematic control techniques, are designed for position actuated robots. In contrast, the unified control framework discussed here relies on torque control capabilities which are typically not available in most humanoid robots. Addressing this limitation in actuation, we have developed the Torque to Position Transformation Methodology [6] that allows to implement torque control commands on position controlled robots.

## 2 Whole-Body Control Framework

To create complex behaviors, humanoids need to simultaneously govern various aspects of their internal and external motion. For instance, locomotion, manipulation, posture, and contact stability are some important tasks that need to be carefully controlled to synthesize effective behaviors (see Figure 1).

As part of a methodological framework, we will develop here kinematic, dynamic, and control representations for multi-task control of humanoid systems in contact with their environment.

### 2.1 Task and Posture Control Decomposition

The task of a human-like robot generally involves descriptions of various parts of the multi-body mechanism, each represented by an operational point  $x_{t(i)}$ . The full task is represented as an  $m \times 1$  vector,  $x_t$ , formed by vertically concatenating the coordinates of all operational points. The Jacobian associated with this task is denoted as  $J_t$ . The derivation of the operational space formulation begins with the joint space dynamics of the robot [7]

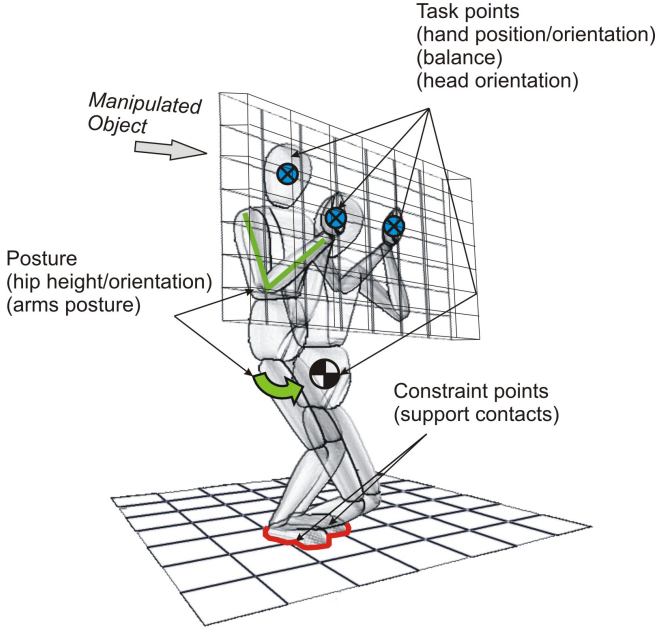
$$A\ddot{q} + b + g = \Gamma \quad (1)$$

where  $q$  is the the vector of  $n$  generalized coordinates of the articulated system,  $A$  is the  $n \times n$  kinetic energy matrix,  $b$  is the vector of centrifugal and Coriolis generalized forces,  $g$  is the vector of gravity forces, and  $\Gamma$  is the vector of generalized control forces.

Task dynamic behavior is obtained by projecting (1) into the space associated with the task, which can be done with the following operation

$$\bar{J}_t^T [A\ddot{q} + b + g = \Gamma] \implies \Lambda_t \ddot{x}_t + \mu_t + p_t = \bar{J}_t^T \Gamma. \quad (2)$$

Here,  $\bar{J}_t^T$  is the dynamically-consistent generalized inverse of the  $J_t$  [7],  $\Lambda_t$  is the  $m \times m$  kinetic energy matrix associated with the task and  $\mu_t$  and  $p_t$  are the associated centrifugal/Coriolis and gravity force vectors.



**Fig. 1. Task decomposition:** We depict here a decomposition into low-level tasks of a whole-body multi-contact behavior. Each low-level task needs to be instantiated and controlled individually as part of a whole-body behavior.

The above equations of motion provide the structure for the decomposition of behaviors into separate torque vectors: the torque that corresponds to desired task behavior and the torque that only affects postural behavior in the tasks's null space, i.e.

$$\Gamma = \Gamma_{\text{task}} + \Gamma_{\text{posture}}. \quad (3)$$

The following task control torque yields linear control of task forces and accelerations [8]

$$\Gamma_{\text{task}} = J_t^T F_t, \quad (4)$$

providing a flexible platform to implement motion or force control strategies for various task points in the robot's body. The second vector,  $\Gamma_{\text{posture}}$ , provides the means for posture control. The general form of these term is [8]

$$\Gamma_{\text{posture}} = N_t^T \Gamma_p \quad (5)$$

where  $\Gamma_p$  is a control vector assigned to control desired posture behavior and  $N_t^T$  is the dynamically-consistent null-space matrix associated with  $J_t$  [8].

Given a desired posture coordinate representation  $x_p$  with Jacobian  $J_p$ , in [1] we introduced a novel representation called task-consistent posture Jacobian defined by the product

$$J_{p|t} \triangleq J_p N_t. \quad (6)$$

Therefore the equation

$$\dot{x}_{p|t} = J_{p|t}\dot{q} \quad (7)$$

defines a vector of postural velocities that comply with task constraints, i.e. that do not involve motion of task coordinates [9]. Here, the subscript  $p|t$  denotes that postural behavior that is consistent with task behavior. With this projections in mind, postural dynamics can be derived by multiplying (1) by the dynamically-consistent generalized inverse of  $J_{p|t}$  [1], leading to the following equation of motion in postural space

$$\bar{J}_{p|t}^T[A\ddot{q} + b + g = \Gamma_{\text{task}} + \Gamma_{\text{posture}}] \implies \Lambda_{p|t}\ddot{x}_{p|t} + \mu_{p|t} + p_{p|t} = F_{p|t} + \bar{J}_{p|t}^T\Gamma_{\text{task}}, \quad (8)$$

where  $\Lambda_{p|t}$  is the inertia matrix in posture space,  $\mu_{p|t}$  and  $p_{p|t}$  are the associated Coriolis/centrifugal and gravity force vectors, and

$$F_{p|t} \triangleq \bar{J}_{p|t}^T\Gamma_{\text{posture}} \quad (9)$$

is a vector of desired control forces that only affect postural motion. In [9] we explored different methods to control postural behavior by means of the above equations.

## 2.2 Handling of Internal and External Constraints

Humanoids are aimed at executing realtime manipulation and locomotion tasks in complex environments, possibly with a high degree of autonomy. Operating in these environments entails responding to dynamic events such as moving obstacles and contact events without interrupting the global task.

Realtime response to motion constraints has been extensively addressed as a secondary process. In contrast, our approach consists on handling motion constraints as priority processes and executing operational tasks in the null space of constrained tasks [9].

To illustrate our approach, let us consider the control example shown in Figure 2, where the robot's end-effector is commanded to move towards a target point. When no constraints are active, the robot's right hand behavior is controlled using the decompositions presented in the previous section, i.e.

$$\Gamma = J_t^T F_t + J_{p|t}^T F_{p|t}. \quad (10)$$

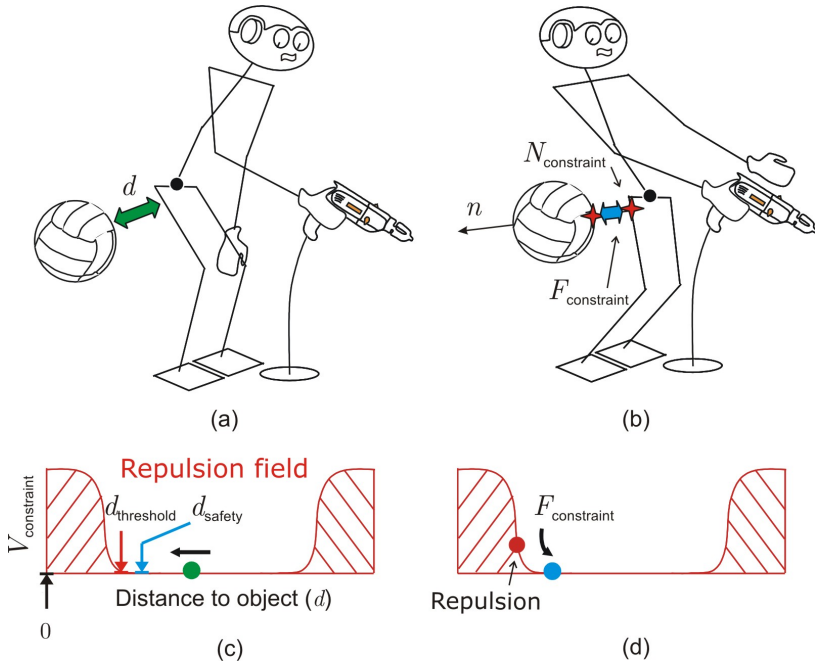
When the obstacle enters an activation zone, we project the task and posture control structures in a constraint-consistent motion manifold, decoupling the task from the constraint (see [9]). At the same time, an artificial attraction potential is implement to keep a minimum safety distance between robot and obstacle. The simultaneous control of constraints and operational tasks is expressed as [9]

$$\Gamma = J_c^T F_c + J_{t|c}^T F_{t|c} + J_{p|t|c}^T F_{p|t|c} \quad (11)$$

where  $J_c$  and  $F_c$  are the Jacobian and control forces associated with the constrained parts of the robot (e.g. the closest point on the robot to upcoming obstacles in Figure 2), and

$$J_{t|c} \triangleq J_t N_c, \quad (12)$$

$$J_{p|t|c} \triangleq J_p N_t N_c \quad (13)$$



**Fig. 2. Illustration of obstacle avoidance:** When an incoming obstacle approaches the robot’s body a repulsion field is applied to the closest point on the robot’s body. As a result, a safety distance can be enforced to avoid the obstacle.

are constrained projections of task and postural Jacobians. Here the subscript  $t|c$  indicates that the task point is consistent with the acting constraints and the subscript  $p|t|c$  indicates that postural motion is consistent with both task motion control and physical constraints [9].

### 2.3 Unified Whole-Body Control Structure

To support the synthesis of whole-body behaviors, we define here unified control structures meant to serve as the main primitives of motion for interfacing with high-level execution commands.

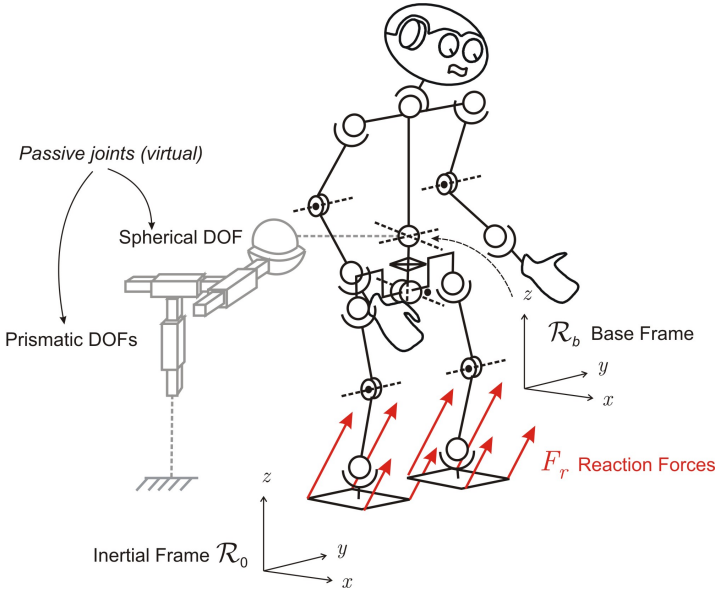
In view of Equation (11) a global control primitive is sought to abstract the motion behavior of the entire robot. This primitive is expressed in the following form similar in structure to the original operational space formulation [8]

$$\Gamma = J_{\otimes}^T F_{\otimes} \tag{14}$$

where

$$J_{\otimes} \triangleq \begin{bmatrix} J_c \\ J_{t|c} \\ J_{p|t|c} \end{bmatrix}, \quad F_{\otimes} \triangleq \begin{bmatrix} F_c \\ F_{t|c} \\ F_{p|t|c} \end{bmatrix}. \tag{15}$$

Here  $J_{\otimes}$  is a unified Jacobian that relates the behavior of all aspects of motion with respect to the robot’s generalized coordinates and  $F_{\otimes}$  is the associated unified control force vector.



**Fig. 3. Kinematic representation of a humanoid robot:** The free moving base is represented as a virtual spherical joint in series with three prismatic virtual joints. Reaction forces appear at the contact points due to gravity forces pushing the body against the ground as well as due to COG accelerations.

### 2.4 Whole-Body Dynamics and Control Under Supporting Contacts

To create kinematic and dynamic representations under supporting constraints we represent multi-legged robots as free floating systems in contact with the ground and analyze the impact of the associated constraints and the resulting reaction forces. Reaction forces appear on the supporting surfaces due to gravity forces and center of gravity (COG) accelerations (see Figure 3). These reaction forces or contact constraints provide the means for stability, locomotion, and postural stance. Using Lagrangian formalism and expressing the system’s kinetic energy in terms of the individual link kinetic and potential energies we can derive the following equation of motion describing robot dynamics under supporting contacts

$$A\ddot{q} + b + g + J_s^T F_s = \Gamma. \tag{16}$$

Here the term  $J_s^T F_s$  corresponds to the projection of reaction forces acting on the feet into forces acting in passive and actuated DOFs and  $J_s$  corresponds to the Jacobian associated with all supporting links. Supporting contacts at the feet, and in general in

any other place in the robot's body provide the support to realize advanced locomotion and manipulation behaviors. Therefore, they affect the robot's motion at the kinematic, dynamic, and control levels.

With the premise that stable balance is maintained and that internal forces are controlled to keep the feet flat against the ground, no relative movement occurs between contact points and the supporting ground. Therefore, relative velocities and accelerations at the contact points are equal to zero, i.e.

$$\dot{x}_s = 0, \quad \ddot{x}_s = 0. \quad (17)$$

By right-multiplying (16) by the term  $J_s A^{-1}$  and considering the equality  $\ddot{x}_s = J_s \ddot{q} + \dot{J}_s \dot{q}$ , we solve for  $F_s$  using the above constraints leading to the following estimation of supporting forces [10]

$$F_s = \bar{J}_s^T \Gamma - \mu_s - p_s. \quad (18)$$

which leads to the following more elaborate expression of (16) [10]

$$A\ddot{q} + b + g - J_s^T \mu_s - \bar{J}_s^T p_s = N_s^T \Gamma. \quad (19)$$

Here

$$N_s \triangleq I - \bar{J}_s J_s \quad (20)$$

is the dynamically-consistent null-space matrix associated with  $J_s$ .

The following constrained expression determines the mapping between arbitrary base and joint velocities to task velocities [9]

$$J_{\otimes|s} \triangleq J_{\otimes} N_s. \quad (21)$$

Here we use the subscript  $\otimes|s$  to indicate that the unified Jacobian  $J_{\otimes}$  is projected in the space consistent with all supporting constraints.

The tasks' equation of motion can be obtained by left multiplying (19) by the transpose of the dynamically consistent generalized inverse of the constrained Jacobian,  $\bar{J}_{\otimes|s}^T$  yielding the following task space equation of motion

$$\Lambda_{\otimes|s} \dot{\vartheta}_{\otimes} + \mu_{\otimes|s} + p_{\otimes|s} = \bar{J}_{\otimes|s}^T N_s^T \Gamma \quad (22)$$

Because we represent humanoids as holonomic systems with  $n$  actuated articulations and 6 passive DOFs describing the position and orientation of its base, the vector of generalized torques  $\Gamma$  contains 6 zeros in its upper part and  $n$  actuated values in its lower part corresponding to control values. This is reflected in the following expression

$$\Gamma = U^T \Gamma_a, \quad (23)$$

where  $U \triangleq [0_{n \times 6} I_{n \times n}]$  is a selection matrix that projects actuated torques,  $\Gamma_a$  into the above underactuated equation of motion. To enforce linear control of whole-body accelerations and forces, we impose the following equality on the RHS of Equation (22)

$$\bar{J}_{\otimes|s}^T N_s^T U^T \Gamma_a = F_{\otimes|s}, \quad (24)$$

where  $F_{\otimes|s}$  is an arbitrary vector of whole-body control forces. Although there are several solutions for the above equation, our choice is the following

$$\Gamma_a = \overline{[UN_s]}^T J_{\otimes|s}^T F_{\otimes|s}, \tag{25}$$

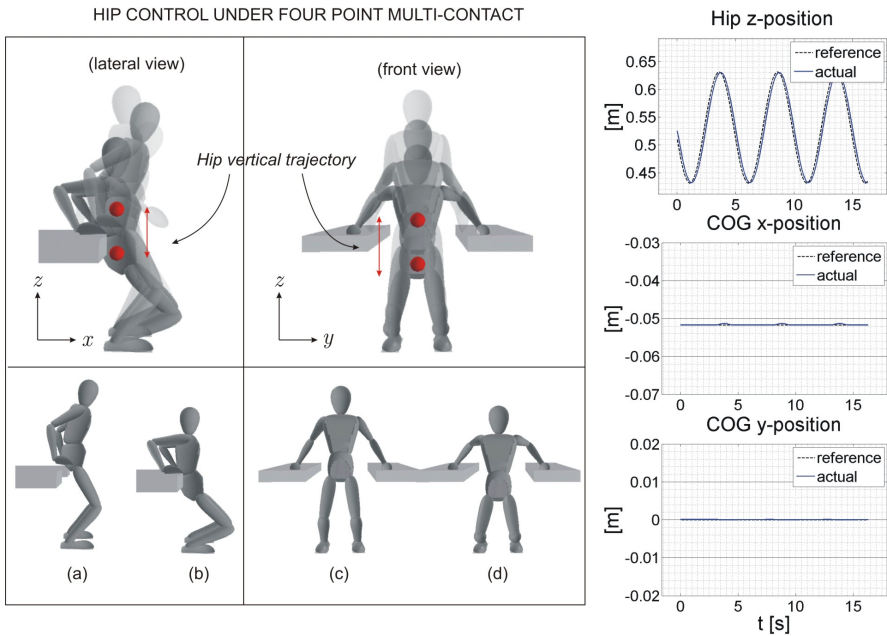
where the term  $\overline{[UN_s]}$  is a dynamically-consistent generalized inverse of  $UN_s$  as described in [9]. The above solution reflects the kinematic dependency between task and joint velocities (also shown in [9]), i.e.

$$\dot{x}_{\otimes|s} = UN_s J_{\otimes|s} \dot{q}_a, \tag{26}$$

where  $\dot{q}_a$  is the vector of actuated joint velocities.

### 3 Results

We conduct an experiment on a simulated model of a humanoid shown in Figure 4. This model measures 1.70 m in height and weights 66kg. Its body is similar in proportions to the body of an average healthy human. Its articulated body consists of 29



**Fig. 4. Hip control under four point multi-contact:** In this experiment, a virtual point located between the robot’s hips is commanded to track a vertical sinusoidal trajectory while maintaining stable control of all contact points on its hands and feet. The two top sequences correspond to the same experiment seen from lateral and front view perspectives. The second row of images (a), (b), (c), and (d) correspond to snapshots of the resulting behavior. The data graph correspond to hip and center of gravity trajectories.



joints, with 6 joints for each leg, 7 for each arm (3 shoulder, 1 elbow, 3 wrist), 2 for waist movement, 1 for chest rotations, and 2 for head pitch and yaw movements. The masses and inertias of each link have been calculated to approximate those of real humans. A dynamic simulation environment and a contact solver based on propagation of forces and impacts are used to conduct the experiment [11]. The whole-body controller described in Equations (23) and (25) is implemented. For this experiment, the tasks being controlled are the center of gravity, the hip height, the hip saggital position, and a posture that resembles that of a human standing up (see [9] for details on multi-task and posture control). Initially, the robot stands up with its hands away from the side strips. When the multi-contact behavior is commanded, the hands reach towards the contact goals. When the hands make contact with the strips, a multi-contact behavior with four contact points is initiated. This behavior involves projecting task controllers in the contact-consistent null-space matrix as shown in Equation (25), while commanding the hip vertical position to track the sinusoidal trajectory shown in Figure 4. The simulated experiment reveals excellent tracking of hip trajectories and COG positions. Despite the fast commanded movements, the maximum error on both the COG and hip trajectories is around 3 mm.

## 4 Conclusion

In this paper, we have presented a unified whole-body control framework that integrates manipulation, locomotion, and diverse dynamic constraints such as multi-contact interactions, obstacle avoidance, and joint limits. The proposed methodology provides the basic structures to synthesize whole-body behaviors in realtime and allows human-like robots to fully interact with their dynamic environments. This framework is currently being implemented in the Honda humanoid robot Asimo, providing a platform to explore advanced manipulation and locomotion behaviors.

Beyond robotics, this framework is being applied in related fields: synthesis of human motion (biomechanics), optimization and design of spaces where humans operate (ergonomics), and synthesis of realistic interactions in computer-simulated environments (interactive worlds).

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