



# Multi-Contact Compliant Motion Control for Robotic Manipulators

Jaeheung Park\*,  
Rui Cortesao\*\*,  
Oussama Khatib\*

*System Setup*

*Motivation*

*Multi-Contact Formulation*

*Control*

*Results*

*Movie*

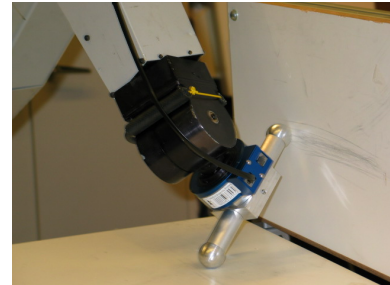
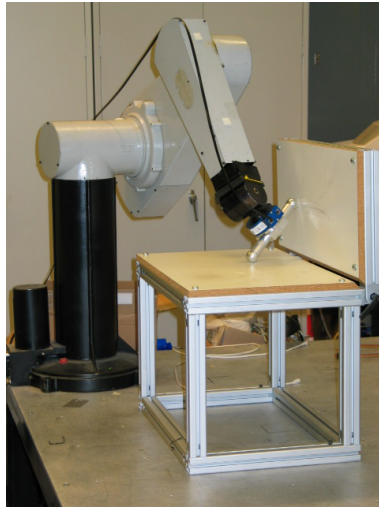
*Conclusion*

*Future Works*

\*Stanford AI Lab, Stanford University

\*\*University of Coimbra, ISR

# 1. System Setup



- PUMA560
- Two contacts with vertical board and horizontal table

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## 2. Motivation

### Multi-Contact Formulation

- Our previous work<sup>1</sup> developed a general multi-contact model, which cannot be described by the Raibert-Craig model.<sup>2</sup>
- Extend the framework by modeling the stiffness of the environment.

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## 2. Motivation

### Multi-Contact Formulation

- Our previous work<sup>1</sup> developed a general multi-contact model, which cannot be described by the Raibert-Craig model.<sup>2</sup>
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### Force Control

A modified Kalman estimation(AOB) is well suited for our system.

- Uncertain input torque - additional input error state.
- Varying measurement noise - on-line variance calculation.

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<sup>1</sup>Roy Featherstone, Stef Sonck Tiebaut, and Oussama Khatib. A general contact model for dynamically decoupled force/motion control, 1999.

<sup>2</sup>Raibert, M. H., and Craig, J. J. Hybrid Position/Force Control of Manipulators, ASME Jnl. Dynamic Systems, Measurement & Control, 1981

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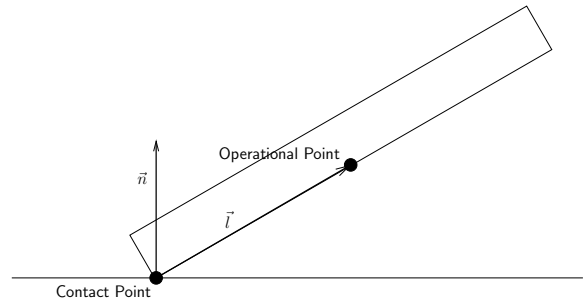
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### 3. Multi-Contact Formulation

#### Multi-Contact model

$$f_c = N\alpha$$
$$N = \begin{bmatrix} \vec{n} \\ \vec{n} \times \vec{l} \end{bmatrix}$$

$\alpha$  : magnitude of contact force



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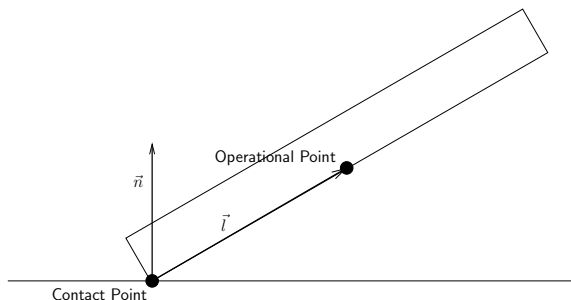


### 3. Multi-Contact Formulation

#### Multi-Contact model

$$f_c = N\alpha$$
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#### Projection Matrices

$$f_c = \Omega_f f$$
$$\vartheta_t = \Omega_m \vartheta$$

- $\vartheta$  velocity of the operational point
- $f$  force at the operational point
- $N$  spans contact normal space

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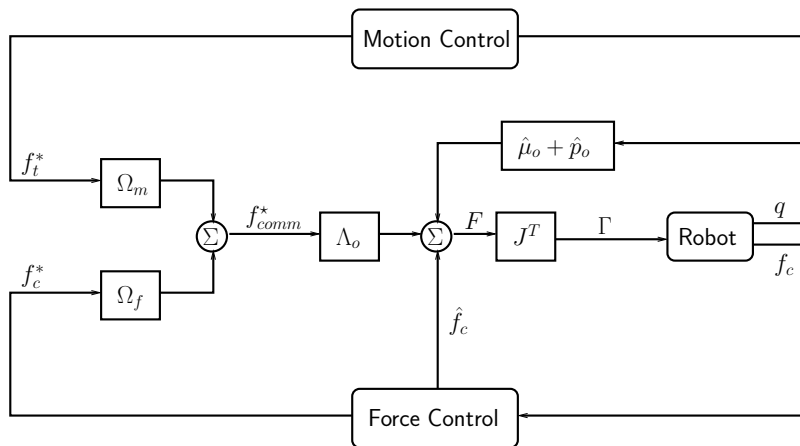
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## 4. Control



Equations of Motion with Operational Space Formulation

$$\Lambda_o(x)\dot{\vartheta} + \mu_o(x, \vartheta) + p_o(x) + f_c = F,$$

$$F = f_{com}^* + \hat{\mu}_o(x, \vartheta) + \hat{p}_o(x) + \hat{f}_c$$

$$f_{com}^* = \Lambda_o \Omega_m f_t^* + \Lambda_o \Omega_f f_c^*.$$

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## Force control

With equations of motion in *Contact Normal Space*

$$\dot{v}_c = \Omega_f f_c^*$$



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## Force control

With equations of motion in *Contact Normal Space*

$$\dot{\vartheta}_c = \Omega_f f_c^*$$

and a spring model

$$\dot{f}_{c,i} = k_{s,i} \vartheta_{c,i},$$



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## Force control

With equations of motion in *Contact Normal Space*

$$\dot{\vartheta}_c = \Omega_f f_c^*$$

and a spring model

$$\dot{f}_{c,i} = k_{s,i} \vartheta_{c,i},$$

The system transfer function can be derived as

$$G(s) = \frac{k_{s,i} e^{-sT_d}}{s(s + K_2)}.$$

$T_d$  system input delay

$K_2$  additional damping

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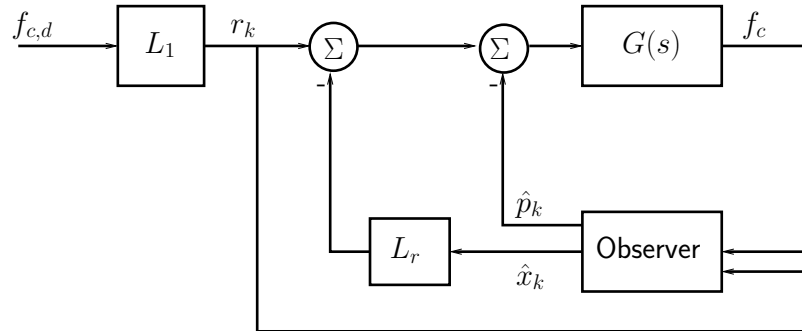
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## Force control Design



- $L_r$  a full state feedback gain obtained by Pole Placement Method
- $L_1$  a scaling factor to compute reference input
- $f_c$  contact force
- $f_{c,d}$  desired contact force
- $r_k$  reference input
- $\hat{x}_k$  state estimate
- $\hat{p}_k$  input error estimate

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## Noise Variance( $R_k$ ) Estimation

- The discrete time first order high-pass filter

$$\alpha_f(z) = G_f(z)\alpha(z),$$

$G_f(z)$  the filter with a zero at 3[Hz] and a pole at 60[Hz]  
 $\alpha(z)$  the measured contact force for each contact force space



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## Noise Variance( $R_k$ ) Estimation



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$G_f(z)$  the filter with a zero at 3[Hz] and a pole at 60[Hz]

$\alpha(z)$  the measured contact force for each contact force space

- The estimation of the measurement noise,  $\hat{R}(t_i)$

$$\hat{R}(t_i) = \frac{1}{N} \sum_{j=i-N+1}^i \{[\alpha_f(t_j) - \bar{\alpha}_f][\alpha_f(t_j) - \bar{\alpha}_f]^T\},$$

where  $\bar{\alpha}_f$  is the mean of the filtered force over a time window.

- 50 samples have been used in the experiments.

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## 5. Results



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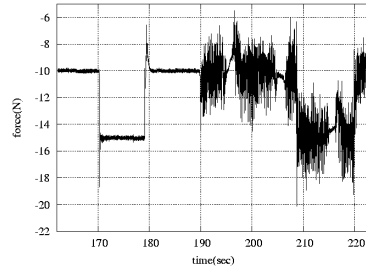
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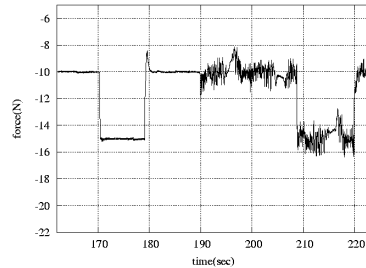
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Experiment for Analysis

## Measured and Estimated forces in contact with the table.



(a) Measured force of the first contact.  $z$  direction.



(b) Estimated force of the first contact.  $z$  direction.

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## Measured and Estimated forces in contact with the vertical board.



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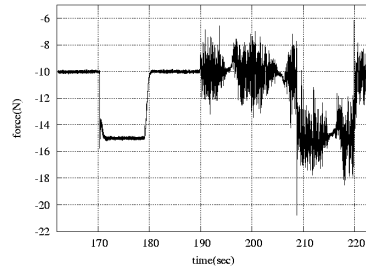
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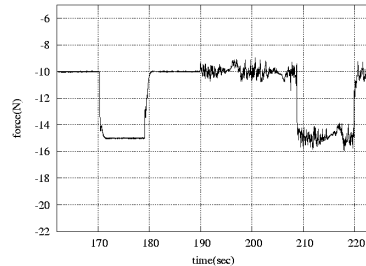
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(a) Measured force of the second contact.  $y$  direction.



(b) Estimated force of the second contact.  $y$  direction.



## Noise Variance Estimations.



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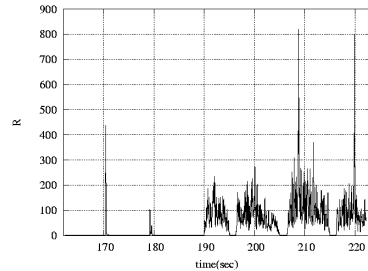
Control

**Results**

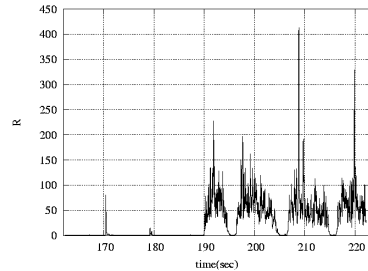
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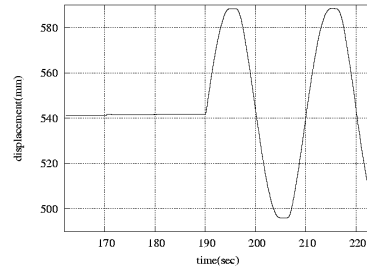


(a) Noise Covariance Estimation for the first contact force.



(b) Noise Covariance Estimation for the second contact force.

## Wrist translational motion in $x$ direction.



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## 6. Movie



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Linear/Angular Motion with Contacts ( 90/120 degree )



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Without/With online calculation of Noise Variance,  $R_k$



## 7. Conclusion

### Multi-Contact Formulation

- Extend our previous work(multi-contact motion/force control for rigid contact) to deal with compliant contact.
- This new formulation sets up dynamic equation for contact force control.

### Force Control

- Apply a modified Kalman filter estimator(AOBs).
- On-line noise Estimation.

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## 8. Future Work

- More experiments with different stiffness environment.
- Implement on-line stiffness estimation strategy.
- Multi-contact with multi-link.

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