1990 JAPAN – U.S.A. SYMPOSIUM ON FLEXIBLE AUTOMATION
– A PACIFIC RIM CONFERENCE –
ISCIE, KYOTO, JAPAN

Motion/Force Redundancy of Manipulators

Oussama Khatib

Robotics Laboratory
Computer Science Department
Stanford University

Abstract

Redundancy is a source of freedom in task execution. Positioning and orienting the end-effector of a redundant manipulator can be obtained with an infinity of postures of the mechanical structure. Another important aspect of redundancy is concerned with forces. End-effector forces are affected by the joint torques delivered by the redundant number of actuators. Determining how generalized joint torques are reflected at the end-effector is crucial in tasks that involve active force control. This is the central issue addressed in this paper. The limitations of kinematic and static analyses are shown and a dynamic treatment of the problem is performed. The dynamically consistent relationship between joint torques and end-effector forces is established and a general strategy for the control of redundant manipulators is discussed.

1 Introduction

Manipulator redundancy has received an increased attention in recent years. Most of the work in this area, however, has focused on what can be called *the motion redundancy problem*. For some specified motion of the end-effector, the problem is to find the appropriate motions of manipulator's joints. This problem has been generally addressed by solving the linearized kinematic model using generalized inverses and pseudo-inverses of the Jacobian matrix (Whitney 1972; Liegois 1977; Fournier 1980; Hanafusa, Yoshikawa, and Nakamura 1983). More recently, an interesting approach aimed at finding inverse kinematic functions has been investigated (Wampler 1987).

Another important problem in redundancy is *the force redundancy problem*. One aspect of this problem is found in the control of multi-manipulator and multi-fingered hand systems. This is the problem associated with finding the internal forces acting on the grasped object (Salisbury and Craig 1982; Kerr and Roth 1986; Nakamura, Nagai, and Yoshikawa 1987).

The force redundancy problem, however, is also encountered in the control of redundant manipulators for tasks that involve active force control. Exerting forces on the environment requires accurate control of the end-effector forces generated by the redundant number of actuators. In many constrained motion operations, these forces are to be maintained while the end-effector contact point is moving along a surface or while the posture of the manipulator is changing. The applied joint torques used in the control of the manipulator for the execution of these additional tasks will clearly have an impact on the resulting forces at the end-effector. Determining how joint torques are reflected at the end-effector of a redundant manipulator, the central issue in this paper, is therefore essential for the development of a control strategy that allows to achieve accuracy and performance in compliant motion operations.

2 Kinematic Relationships

One of the basic relationships in manipulator kinematics is the linearized kinematic model which expresses the relationships between elementary displacements, $\delta \mathbf{q}$, of the joint coordinates, $\mathbf{q}$, and the corresponding elementary displacements, $\delta \mathbf{x}$, of the operational coordinates, $\mathbf{x}$, which describe the end-effector's position and orientation. This model is

$$\delta \mathbf{x} = J(\mathbf{q})\delta \mathbf{q}; \tag{1}$$

where $J(\mathbf{q})$ is the Jacobian matrix. For an n-degree-of-freedom manipulator whose end-effector is operating in an m-dimensional space, the operational space, the Jacobian, $J(\mathbf{q})$, is an $n \times m$ matrix.

Using this kinematic model, Whitney (1972) proposed the *resolved motion-rate control* approach for the coordination of manipulator joint motions. The resolved motion-rate control uses the inverse of the Jacobian matrix. For a non-redundant manipulator, i.e, $n = m$, the solution is simply

$$\delta \mathbf{q} = J^{-1}(\mathbf{q})\delta \mathbf{x}. \tag{2}$$

For a given trajectory of the end-effector, motion control is achieved by continuously controlling the manipulator from its current configuration $\mathbf{q}$ to the configuration $\mathbf{q} + \delta\mathbf{q}$, where $\delta\mathbf{q}$ is evaluated in accordance with $\delta\mathbf{x}$ using equation (2).

Redundant Manipulators

The position and orientation of the end-effector of a redundant mechanism can be obtained with an infinite number of postures of its links. Generalized inverses and pseudo-inverses have been used to solve the inverse kinematic problem. Using a generalized inverse $J^{\#}(\mathbf{q})$ of the Jacobian matrix, the general solution is

$$\delta \mathbf{q} = J^{\#}(\mathbf{q})\delta \mathbf{x} + [I - J^{\#}(\mathbf{q})J(\mathbf{q})]\delta \mathbf{q}_0; \tag{3}$$

where I is the identity matrix of appropriate dimensions and $\delta\mathbf{q}_0$ denotes an arbitrary vector. The matrix $[I - J^{\#}(\mathbf{q})J(\mathbf{q})]$ defines the mapping to the null space associated with $J^{\#}(\mathbf{q})$, and vectors of the form $[I - J^{\#}(\mathbf{q})J(\mathbf{q})]\delta\mathbf{q}_0$ correspond to zero-variation of the position and orientation of the end-effector. The additional freedom of motion associated with the null space is generally used to minimize some criteria or to achieve an additional task.

3 Torque/Force Relationships

The basic relationship between end-effector forces, $\mathbf{F}$, and joint torques, Γ, is

$$\Gamma = J^T(\mathbf{q})\mathbf{F}; \tag{4}$$

This relation is obtained using the identity between the virtual works associated with torques and forces in the virtual displacements $\delta\mathbf{q}$ and $\delta\mathbf{x}$.

Redundant Manipulators

At a given configuration of a redundant mechanism, we have seen that there is an infinity of elementary joint displacements that can take place without altering the configuration of the end-effector. These displacements take place in the null space associated with the Jacobian matrix.

Let us consider the case of a three-degree-of-freedom manipulator whose task is to exert a force $\mathbf{F}$ on the environment at some point, $\mathbf{x}$, as illustrated in Figure 1. The required joint torques are still given by the relationship of equation (4). The application of the joint torques Γ will achieve the desired vector of force $\mathbf{F}$ at the end-effector. Submitted to the environment reaction forces $-\mathbf{F}$ and to the applied joint torques Γ, the mechanism will remain in static equilibrium.

This manipulator is redundant with respect to the task of applying $\mathbf{F}$ at the end-effector at a point $\mathbf{x}$. If the manipulator's posture was to change or if the contact point was

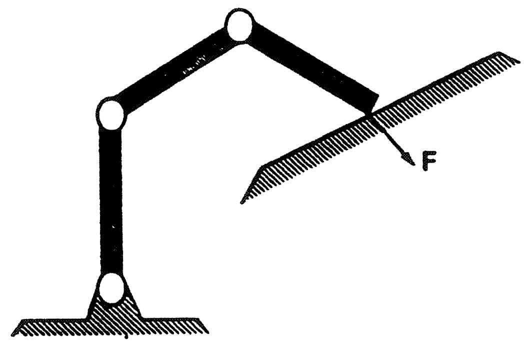

Figure 1: A Force Control Task

to evolve (sliding along the surface), while maintaining the desired forces, additional control will be needed. Whatever control methodology is used, the additional control will result by the application of some additional torques, Γ_{add}, at the manipulator joints. The total joint torques become

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + \Gamma_{add}. \tag{5}$$

Determining how the additional joint torques, Γ_{add} affect the resulting forces at the end-effector is crucial for the design of a control strategy that allows the execution of both tasks. This issue is part of the more general problem of finding how applied generalized joint torques are reflected at both the redundant structure and the end-effector.

The relationship (4) has been established by expressing the identity between the virtual works done by the generalized joint torques and the end-effector forces:

$$\Gamma^T \delta\mathbf{q} \equiv \mathbf{F}^T \delta\mathbf{x}. \tag{6}$$

In stating this identity the underlying assumption is that the mechanism is held at static equilibrium. This assumption will not be any more verified, for instance, for the task of changing the posture of the manipulator, while the exerted force at the end-effector is to be maintained.

For such a task, indeed, the applied joint torques will not be any more totally supported by the reaction forces of the environment, that is the total virtual work will be greater than $\mathbf{F}^T\delta\mathbf{x}$. For this task, the total virtual work involves, in addition, the work of Γ_{add} in the displacements associated with the changes in the manipulator's posture.

Maintaining the position and orientation of the end-effector implies that the trajectory along which the manipulator will be taken from its initial configuration to the desired posture must lay in the null space associated with the Jacobian matrix. Let us assume, for instance, that the actual displacement of the manipulator is done following the relationship (3). The total virtual displacement will be

$$\delta \mathbf{q} = J^{\#}(\mathbf{q})\delta \mathbf{x} + [I - J^{\#}(\mathbf{q})J(\mathbf{q})]\delta \mathbf{q}_0.$$

This shows that, in addition to $\delta \mathbf{x}$, displacements $[I - J^{\#}(\mathbf{q})J(\mathbf{q})]\delta \mathbf{q}_0$ in the null space must be accounted for. The total virtual work

$$\delta W = \Gamma^T \delta \mathbf{q};$$

done by the generalized joint forces Γ in the virtual displacement $\delta \mathbf{q}$ becomes

$$\delta W = \delta W_1 + \delta W_2; \tag{7}$$

with

$$\delta W_1 = [J^{\#^T}(\mathbf{q})\Gamma]^T \delta \mathbf{x}; \tag{8}$$

and

$$\delta W_2 = \{[I - J^{\#}(\mathbf{q})J(\mathbf{q})]^T \Gamma\}^T \delta \mathbf{q}_0. \tag{9}$$

δW_1 corresponds to the virtual work done in the virtual displacements $\delta \mathbf{x}$ and δW_2 corresponds to the virtual work done by the joint torque vector $[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma$ in the virtual displacement $\delta \mathbf{q}_0$.

In order for the additional torques Γ_{add} to not produce any work along $\delta \mathbf{x}$, the force $J^{\#^T}(\mathbf{q})\Gamma$ in (8) must be equal to $\mathbf{F}$. Using equation (5), this yields

$$J^{\#^T}(\mathbf{q})[J^T(\mathbf{q})\mathbf{F} + \Gamma_{add}] = \mathbf{F};$$

which reduces to

$$J^{\#^T}(\mathbf{q})\Gamma_{add} = 0.$$

This equation implies that the additional joint torques must lay in the null space associated with the matrix $J^{\#^T}(\mathbf{q})$, this is

$$\Gamma_{add} = [I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma_0. \tag{10}$$

And δW_2 becomes $\Gamma_{add}^T \delta \mathbf{q}_0$.

In these conditions, the total joint torques Γ of (5) can be written in the form

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + [I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma_0; \tag{11}$$

4 Dynamic Considerations

The relationship of joint torque/effector force (11) has been established with specific assumptions about the relationship of joint position/effector position (3). The consistency between forces and motions is governed by the dynamic equations of the system and these equations must be taken into account in order to enforce that consistency.

The relationship (11) can be interpreted as a decomposition of the joint vector Γ following some generalized inverse, $J^{\#}(\mathbf{q})$, this is

$$\Gamma = J^T(\mathbf{q})[J^{\#^T}(\mathbf{q})\Gamma] + [I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma. \tag{12}$$

In this decomposition, $(J^{\#^T}(\mathbf{q})\Gamma)$ corresponds to the end-effector forces and $[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma$ to torques acting in the null space.

This decomposition is arbitrary since it depends on the generalized inverse being used (any matrix $J^{\#}$ such that $J = JJ^{\#}J$). Clearly different selections of generalized inverses correspond to different joint torque vectors, $[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma_0$, and ultimately to different responses of the manipulator.

The Gravity Example

Considering, for instance, the gravity vector $\mathbf{g}(\mathbf{q})$ associated with the three-degree-of-freedom manipulator, the relationship (12) leads to the decomposition into: $J^T(\mathbf{q})\mathbf{p}(\mathbf{q})$ and $[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\mathbf{g}(\mathbf{q})$, where

$$\mathbf{p}(\mathbf{q}) = J^{\#^T}(\mathbf{q})\mathbf{g}(\mathbf{q}).$$

Two cases are to be considered. First, for configurations $\mathbf{q}$ such that

$$[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\mathbf{g}(\mathbf{q}) = 0;$$

the manipulator can be clearly maintained in static equilibrium by the application of end-effector forces $\mathbf{p}(\mathbf{q})$. This is, for instance, the case for the configuration shown in figure 2.a, where the gravity vector $\mathbf{g}(\mathbf{q})$ is orthogonal to all displacements in the null space. At this configuration, the gravity forces can be supported by a force, $\mathbf{p}(\mathbf{q})$, acting at the end-effector. In contrast, if we started from the configuration shown in figure 2.b, the gravity, $\mathbf{g}(\mathbf{q})$, will have a component in the null space.

$$[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\mathbf{g}(\mathbf{q}) \neq 0.$$

Let us select the pseudo inverse of the Jacobian, $J^+(\mathbf{q})$, for the evaluation of the gravity forces $\mathbf{p}$. The response of the manipulator to the application of $\mathbf{p}(\mathbf{q})$, as obtained from a dynamic simulation of the system, is shown in Figure 3.b. The manipulator initial configuration is shown in Figure 3.a. The resulting behavior clearly shows that the forces being reflected at the end-effector to be different from $\mathbf{p}(\mathbf{q})$ and illustrates the need for a dynamic treatment of this problem.

Dynamics of Redundant Manipulators

The joint space equations of motion of a manipulator are

$$A(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \Gamma; \tag{13}$$

where $\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, and Γ, represent the Coriolis and centrifugal, gravity, and generalized forces in joint space.

The basic question to answer here is how the manipulator dynamic forces are reflected at the end-effector. This is part of the study of operational space dynamics for redundant manipulators.

First, let us examine the effects of the acceleration forces. Starting from static equilibrium, equations (1) and (13) show that the acceleration at the end-effector resulting from the application of a joint torque vector Γ is given by $J(\mathbf{q})A^{-1}(\mathbf{q})\Gamma$ (at zero-velocity $\ddot{\mathbf{x}} = J(\mathbf{q})\ddot{\mathbf{q}} = J(\mathbf{q})A^{-1}(\mathbf{q})\Gamma$).

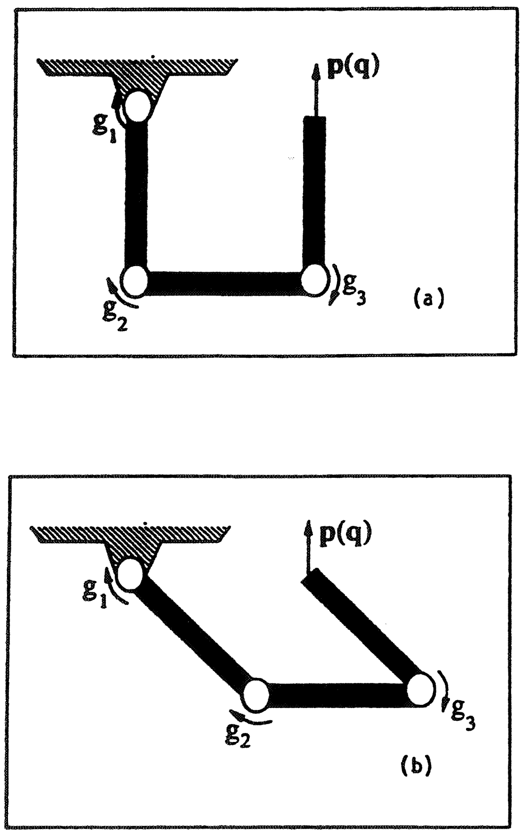

Figure 2: Gravity Components in the Null Space

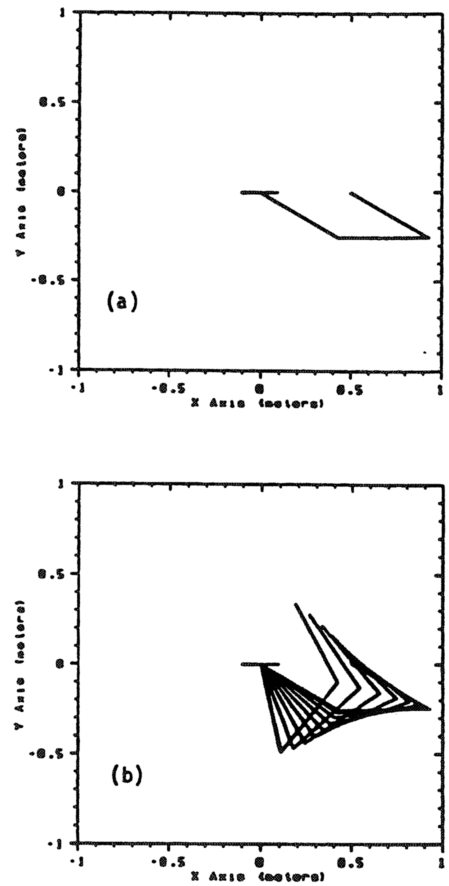

Figure 3: Gravity Compensations (Pseudo-Inverse)

Dynamically Consistent Null Space

In order for the null space joint torques, $J(q)A^{-1}(q)[I - J^T(q)J^{\#^T}(q)]\Gamma$, to not produce any acceleration at the end-effector, it is necessary to have

$$J^T(q)A^{-1}(q)[I - J^T(q)J^{\#^T}(q)]\Gamma = 0. \quad (14)$$

The null space associated with a generalized inverse satisfying the above constraint is said to be dynamically consistent.

<u>Theorem 1</u>: *(Dynamic Consistency)*

A generalized inverse that is consistent with the dynamic constraint of equation (14), $\overline{J}(q)$, is unique and given by

$$\overline{J}(q) = A^{-1}(q)J^T(q)\Lambda(q). \quad (15)$$

where

$$\Lambda(q) = [J(q)A^{-1}(q)J^T(q)]^{-1}; \quad (16)$$

is defined as the operational space kinetic energy matrix. $\overline{J}(q)$ in equation 15 is actually a generalized inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy.

End-Effector Dynamics

The operational space study shows in fact that all dynamic forces (inertial and gravitational) are reflected at the end-effector by the matrix $\overline{J}^T(q)$. The end-effector dynamic behavior resulting from the application to the manipulator (13) of the joint torque vector

$$\Gamma = J^T(q)F + [I - J^T(q)\overline{J}^T(q)]\Gamma_0;$$

is described by

$$\overline{J}^T(q)[A(q)\ddot{q} + b(q,\dot{q})g(q)] = F;$$

Table 1: Position/Force Duality

	Position	Force
Non-Redundant Manipulator Systems	$\delta \mathbf{q} = J^{-1}(\mathbf{q})\ \delta \mathbf{x}$	$\Gamma = J^T(\mathbf{q})\ \mathbf{F}$
Redundant Manipulator Systems	$\delta \mathbf{q} = \bar{J}(\mathbf{q})\ \delta \mathbf{x} + [I - \bar{J}(\mathbf{q})\ J(\mathbf{q})]\ \delta \mathbf{q}_0$	$\Gamma = J^T(\mathbf{q})\ \mathbf{F} + [I - J^T(\mathbf{q})\ \bar{J}^T(\mathbf{q})]\ \Gamma_0$

The dynamics of the end-effector of a redundant manipulator can be written in the form (Khatib 1987)

$$\Lambda(\mathbf{q})\ddot{\mathbf{x}} + \mu(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{p}(\mathbf{q}) = \mathbf{F}; \qquad (17)$$

where $\mu(\mathbf{q}, \dot{\mathbf{q}})$, representing the centrifugal and Coriolis forces acting at the end-effector, is related to the joint space dynamics by

$$\mu(\mathbf{q}, \dot{\mathbf{q}}) = \bar{J}^T(\mathbf{q})\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) - \Lambda(\mathbf{q})\dot{J}(\mathbf{q})\dot{\mathbf{q}};$$

and where $\mathbf{p}(\mathbf{q})$, representing the gravity forces acting at the end-effector, is

$$\mathbf{p}(\mathbf{q}) = \bar{J}^T(\mathbf{q})\mathbf{g}(\mathbf{q}).$$

Generalized Torque/Force Relationship

In consistency with the manipulator dynamics, the torque/-force relationship becomes

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + [I - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_0; \qquad (18)$$

With this relationship, the manipulator control can be decomposed into two dynamically decoupled control vectors:

- End-effector control using $\mathbf{F}$.
- Internal motion control using the joint torques Γ_0.

The force/position duality for non-redundant manipulators is extended with the relationship (18) to the case of redundant manipulators as summarized in Table 1.

The Gravity Example (Dynamic Compensation)

Let us consider the example of the three-degree-of-freedom manipulator in the configuration shown in Figure 4.a. The goal is to maintain the end-effector at static equilibrium by the only application of operational forces $\mathbf{F}$. All motions of the redundant manipulator should be constrained to the null space. Starting from rest, the operational forces, $\mathbf{F}$, should be designed to compensate for the gravity, centrifugal, and Coriolis forces reflected at the end-effector. The response of the manipulator to the application at the end-effector of the compensation force $\mu(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{p}(\mathbf{q})$, as obtained by a dynamic simulation, is shown in Figure 4.b. Figure 5 shows the manipulator response, when starting from a different configuration. In these two examples, the end-effector is maintained at static equilibrium by the only application of end-effector forces (which do not involve any position error feedback control). Compared to the result obtained with the transpose of the Jacobian pseudo-inverse (see Figure 3), these two examples illustrate the significance of dynamic considerations in the control of redundant manipulators.

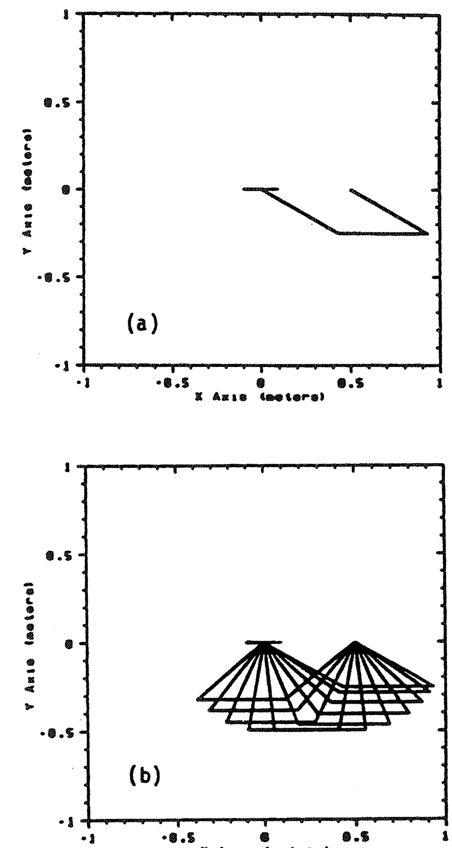

Figure 4: Dynamic Compensation (Configuration 1)

Internal Motion Control

The generalized torque/force relationship (18) provides an effective means for the design of a dynamically decoupled control of internal motions. An additional task to be carried out using the manipulator internal motions can be realized by constructing a potential function, $V_0(\mathbf{q})$, whose minimum corresponds to the desired task. By selecting Γ_0 as the gradient of this function

$$\Gamma_0 = -\nabla V_0;$$

one obtains the needed attraction (Khatib, 1986) to the desired task. The interference of the additional torques on the end-effector is simply eliminated by projecting this gradient in the dynamically consistent null space. This is

$$\Gamma_n = -[I - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\nabla V_0. \qquad (19)$$

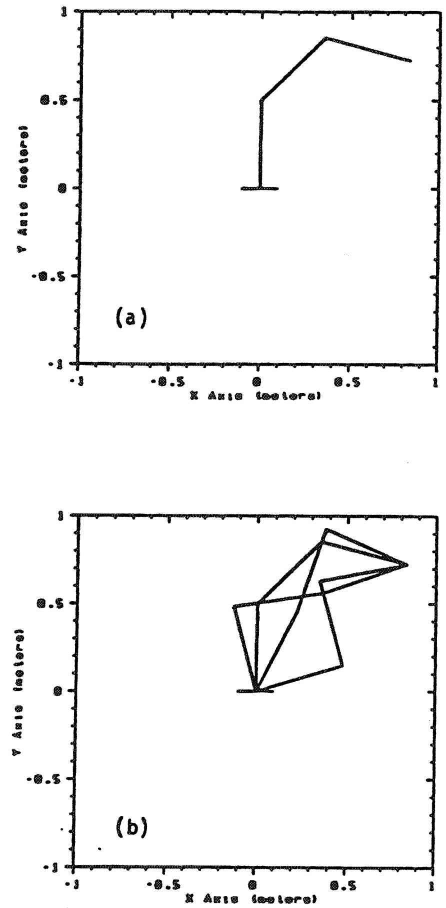

Figure 5: Dynamic Compensation (Configuration 2)

The avoidance of obstacles and kinematic singularities can be achieved by integrating an "artificial potential field" function into V_0. However, the avoidance will be only realized within the freedom of internal motions, due to the projection of the gradient into the null space. Asymptotic stabilization of the redundant mechanism requires additional dissipative joint forces, which should also be selected from the dynamically consistent null space.

5 Conclusion

The general relationship between joint torques and end-effector forces for redundant manipulator mechanisms has been established. Consistent with the dynamics of the redundant manipulator and the effector, this relationship provides a decomposition of joint torques into two dynamically decoupled control vectors:

- Joint torques corresponding to forces acting at the end-effector;
- Joint torques only affecting the internal motions.

With this dynamically decoupled decomposition, the end-effector is then controlled by the action of forces based on its dynamic model, while tasks involving internal motions are controlled by joint torques which do not alter the effector dynamic behavior.
The generalized joint-torque/effector-force relationship provides an important tool for motion and active force control for redundant manipulator systems. This relationship is also important for dealing with the dynamic forces resulting at the manipulated object in multi-manipulator and multi-fingered hand systems.

Acknowledgments

The financial support of SIMA and DARPA (DAAA21-89-C0002) are acknowledged.

References

Fournier, A. 1980 (April), "Génération de Movements en Robotique. Application des Inverses Généralisées et des Pseudo Inverses," *Thèse d'Etat*, Mention Science, Université des Sciences et Techniques du Languedoc, Montpellier, France.

Hanafusa, H., Yoshikawa, T., and Nakamura, Y., 1981, "Analysis and Control of Articulated Robot Arms with Redundancy," *8th IFAC World Congress*, vol. XIV, pp. 38-83.

Kerr, J. and Roth, B. 1986, "Analysis of Multifingered Hands," *Int. J. of Robotics Research*, vol. 4-4, pp. 4-17.

Khatib, O. 1986, "Real-Time Obstacle Avoidance for Manipulators and Mobile Robots," Int. J. of Robotic Research, vol. 5, no. 1, pp. 90-98.

Khatib, O. 1987, "A Unified Approach to Motion and Force Control of Robot Manipulators: The Operational Space Formulation," *IEEE J. on Robotics and Automation*, vol. 3, no. 1.

Liegois, A. 1977, "Automatic Supervisory Control of the Configuration and Behavior of Multibody Mechanisms," *IEEE Trans. on Systems, Man and Cybernetics*, vol. 7, no. 12.

Nakamura, Y., Nagai, K., and Yoshikawa, T. 1987, "Mechanics of Coordinative Manipulation by Multiple Robotic Mechanisms," *Proc. IEEE Conf. of Robotics and Automation*, Raleigh, North Carolina, pp. 991-998.

Salisbury, J.K., and Craig, J.J., 1982, "Articulated Hands: Force Control and Kinematic Issues," *Int. J. of Robotics Research*, vol. 1-1, pp. 4-17.

Wampler, C. 1987, "Inverse Kinematic Functions for Redundant Manipulators," *Proc. IEEE Conf. of Robotics and Automation*, Raleigh, North Carolina, pp. 610-617.

Whitney, D.E. 1972, "The Mathematics of Coordinated Control of Prosthetic Arms and Manipulators," Journal of Dynamic Systems, Measurement, and Control, pp. 303-309.