FSR'99 International Conference on Field and Service Robotics, Pittsburgh, PA, Aug. 1999

Development of a Holonomic Mobile Robot for Mobile Manipulation Tasks

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Abstract

Mobile manipulator systems hold promise in many industrial and service applications including assembly, inspection, and work in hazardous environments. The integration of a manipulator and a mobile robot base places special demands on the vehicle's drive system. For smooth accurate motion and coordination with the manipulator a holonomic, vibration-free wheel system which can be dynamically controlled is preferred. The work presented here is part of the commercial efforts of Nomadic Technologies Inc. and continuing research at the Stanford University Computer Science Robotics Laboratory focused on dextrous mobile manipulation.

Figure 1: Nomadic XR4000 and PUMA 560

1 Introduction

A holonomic system is one in which the number of degrees of freedom are equal to the number of coordinates needed to specify the configuration of the system. In the field of mobile robots, the term holonomic mobile robot is applied to the abstraction called the robot, or base, without regard to the rigid bodies which make up the actual mechanism. Thus, any mobile robot with three degrees of freedom of motion in the plane has become known as a holonomic mobile robot. Holonomic mobile robots are desirable because they do not have kinematic motion constraints, which makes path planning and control much simpler.

Many different mechanisms have been created to achieve holonomic motion. These include various arrangements of universal or omni wheels, double universal wheels, Swedish or Mecanum wheels, chains of spherical or cylindrical wheels, orthogonal wheels, and ball wheels [1, 2, 3, 4, 5, 6, 7, 8]. All of these mechanisms, except for some types with ball wheels, have discontinuous wheel contact points which are a great source of vibration; primarily because of the changing support provided; and often additionally because of the discontinuous changes in wheel velocity needed to maintain smooth base motion. These mechanisms tend to have poor ground clearance due to the use of small peripheral rollers and/or the arrangement of the mechanism leaves some of the support structure very close to the ground.

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The design and actuation of these mechanisms has been driven by kinematic concerns for minimum actuation and minimal sensing to make to the implementations of odometry and control mathematically exact. Yet, many of these designs have multiple rollers with the contact points of the wheel on the ground moving from one row to the other while it is assumed to remain stationary in the middle of each wheel. This emphasis on minimal design has led to many three wheeled designs which are more likely to tip over, or at least lift a wheel, as performance and payload is increased. Also, the minimal use of actuators often led to complex mechanical transmissions to distribute the power to the driving elements. The designs discussed are mechanically complex; often with many moving parts, some active, some passive.

Just as a kinematic approach was used in the design of these holonomic mechanisms, the control of these mechanisms was looked at from a purely kinematic perspective. Many of the designs incorporate passive rollers without sensing of their motions, so that the dynamics of these elements cannot be accounted for. Without dynamic control, it is difficult to perform coordinated motion of a mobile base and dynamically controlled manipulator.

We present here another type of holonomic vehicle mechanism which we will refer to as a *powered caster vehicle* or PCV. It was conceptually described by Muir and Neuman [9] as early as 1986 as an “omnidirectional wheeled mobile robot” having “non-redundant conventional wheels”. (A “powered office chair” is maybe a simpler conceptual description.) They dismissed pursuing the idea since it had the potential for actuator conflict. Others have also chosen to not implement such a design because of the difficulty of the control [8]. We will present the design of a working PCV mechanism and the framework for dynamic control.

The use of a dynamically-controlled, holonomic mobile robot in a mobile manipulation system is particularly desirable because it provides easier planning and navigation for gross motion, along with the ability to fully use the null space motions of the system to improve the workspace and overall dynamic endpoint properties.

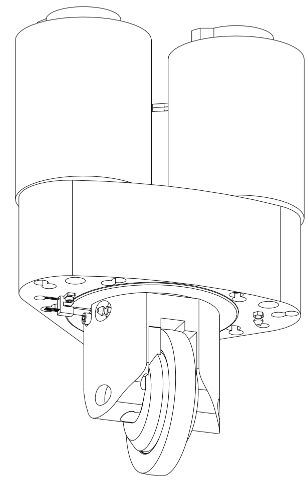

Figure 2: Powered Caster Module

2 Design

The PCV is good mechanism to provide holonomic mobility for many reasons. The contact points between the wheels and the ground move in a continuous manner and thus do not induce vibrations from shifting support points or discontinuous wheel velocities. The location of each contact point is well known so that control is more exact. Each wheel mechanism contains a single nonholonomic wheel which is large enough for good ground clearance [10]. One final point which has not been addressed previously is that this is the only holonomic mechanism which can be designed to effectively use currently available pneumatic tires. Because there are no passive and more importantly no unmeasured bodies in a powered caster design the dynamics of the system can be accurately modeled.

A PCV is composed of $n \geq 2$ powered caster modules as illustrated in Figure 2. The modules could vary in size and power from module to module, but without loss of generality, we will assume that all the modules are identical. The PCV design is defined by the strictly positive geometric parameters: wheel radius(r), caster offset(b), and wheel module placement(h, β) (see Figure 3). Along with the mass and inertia of each component in the design, parameters which affect the system dynamics include the gear ratios and motor sizes. Values for the geometric parameters must be selected so that the area swept out by each wheel does not intersect any other. The wheels should have a large enough radius to surmount anticipated obstacles. The dynamic tradeoffs involve the geometry as well as the motors and gearing. Careful selection must be made to result in a mechanism which has good acceleration while maintaining the ability to reach the desired top speed. At the same time, by choosing components so that motor and gearbox speeds are kept low, mechanical noise due to high component speeds can be minimized.

The PCV mechanism shown in Figure 1, a Nomadic Technologies XR4000 mobile robot, was designed to be a high performance holonomic vehicle for mobile robotics and mobile manipulation. It has four 11 cm diameter wheels with 2 cm caster offset. It can accelerate at 2 m/s^2 on most surfaces and has a top speed of 1.25 m/s. The controller of the XR4000 used herein was modified at Stanford University by replacing the standard PWM motor amplifiers with current controlled motor amplifiers.

3 Dynamic Modeling

Typically, the dynamic equations of motion for a parallel system with nonholonomic constraints such as a PCV are formed in one of two ways: the unconstrained dynamics of the whole system can be derived and the the constraints are applied to reduce the number of degrees of freedom [11]; or the system is cut up into pieces, the dynamics of these subsystems are found, and the loop closure equations are used to eliminate the extra degrees of freedom. For our four-wheeled XR4000 robot, using the first method, we will obtain 11 equations for the unconstrained system and 8 constraint equations for a total of 19 equations. The second method will yield 12 equations for the unconstrained subsystems and 9 constraint equations for a total of 21 equations. These systems of equations must then be reduced to 3 equations. Ideally, both these methods would yield the same minimal set of dynamic equations, but in practice it is difficult to reduce the proliferation of terms that are introduced in a large number of equations.

To get a more efficient form of the dynamic equations of motion we will use a method which uses compatible 3 DOF systems. We can model the PCV as a collection of cooperating manipulators such as shown in Figure 3. The dynamic equations

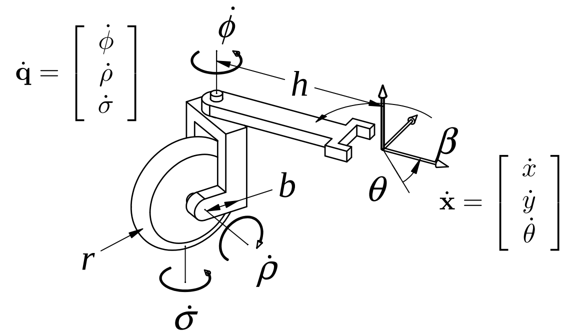

Figure 3: Powered Caster "Manipulator"

of motion for this three DOF serial manipulator can be written [12],

$$A(\mathbf{q})\ \ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{\Gamma} \tag{1}$$

where $\mathbf{q}$ and its derivatives are the joint positions, velocities, and accelerations, A is the symmetric mass matrix, and $\mathbf{b}$ is the vector of centripetal and coriolis coupling terms. We assume that the PCV is on level ground and have dropped the effects of gravity.

Because of the parallel nature of the final mechanism we choose to write the relationship between joint speeds and local Cartesian speeds, $\dot{\mathbf{x}}$, as

$$\dot{\mathbf{q}} = G\dot{\mathbf{x}} \tag{2}$$

$$G = \begin{bmatrix} -\mathrm{s}\phi/b & \mathrm{c}\phi/b & h[\mathrm{c}\beta\mathrm{c}\phi + \mathrm{s}\beta\mathrm{s}\phi]/b - 1 \\ \mathrm{c}\phi/r & \mathrm{s}\phi/r & h[\mathrm{c}\beta\mathrm{s}\phi - \mathrm{s}\beta\mathrm{c}\phi]/r \\ -\mathrm{s}\phi/b & \mathrm{c}\phi/b & h[\mathrm{c}\beta\mathrm{c}\phi + \mathrm{s}\beta\mathrm{s}\phi]/b \end{bmatrix}$$

As shown in Figure 3, $\dot{\phi}$ is the steering rate, $\dot{\rho}$ is the angular speed of rolling, and $\dot{\sigma}$ is the angular twist rate at the wheel contact. For compactness we use s$\cdot$ and c$\cdot$ as shorthand for $\sin(\cdot)$ and $\cos(\cdot)$. It is interesting to note that the first two rows of G express the nonholonomic constraints due to ideal rolling while the third row is a holonomic constraint: $\theta = \sigma - \phi$.

Using the joint space dynamics from eqn. 1 and the Jacobian in eqn. 2, we can express the operational space dynamics [13] of the i^{th} manipulator as

$$\Lambda_i(\mathbf{q}_i)\ddot{\mathbf{x}} + \boldsymbol{\mu}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \dot{\mathbf{x}}) = \mathbf{F}_i \tag{3}$$

with

$$\Lambda_i = G_i^T A_i G_i$$

$$\boldsymbol{\mu}_i = G_i^T \left(A_i \dot{G}_i \dot{\mathbf{x}} + \mathbf{b}_i \right)$$

where Λ is the operational space mass matrix, $\boldsymbol{\mu}$ is the operational space vector of centripetal and coriolis terms, and $\mathbf{F}$. is the force/torque vector at the origin of the end effector coordinate system. Since our manipulator is simple and not redundant we compute G directly, thus avoiding an inversion operation which is traditionally required. Also note that as expressed here $\boldsymbol{\mu}_i$ is a function of $\mathbf{q}_i, \dot{\mathbf{q}}_i$ and $\dot{\mathbf{x}}$. This representation allows us to use exact local information, such as the rolling speed of the wheel, which is measured directly and to use the best estimates of the base speeds which we develop in section 4.

If we choose the end effector frames of the various manipulators such that they are coincident while the wheel modules are correctly positioned with respect to one another, then, using the augmented object model of Khatib [14], we can write the overall operational space dynamics of the mobile base.

$$\Lambda\ \ddot{\mathbf{x}} + \boldsymbol{\mu} = \mathbf{F} \tag{4}$$

with

$$\Lambda = \sum_i^n \Lambda_i \quad ; \quad \boldsymbol{\mu} = \sum_i^n \boldsymbol{\mu}_i \quad ; \quad \mathbf{F} = \sum_i^n \mathbf{F}_i$$

Here, Λ, $\boldsymbol{\mu}$, and $\mathbf{F}$ have the same meanings as before but now represent the properties of the entire robot.

With this algorithm we have determined the operational space dynamic equations of motion directly. For our four-wheeled XR4000 robot we generate 12 equations, 3 for each i in eqn. 3, which are then added in groups of four to give the required 3 operational space equations. Using the symbolic dynamic equation generator AUTOLEV to create Λ and $\boldsymbol{\mu}$, the number of multiplies and additions are reduced from 8180 and 2244, to 2174 and 567.

4 Dynamically Decoupled Control

One of the more effective techniques for controlling a coupled non-linear system such as the PCV is the *nonlinear dynamic decoupling approach* [13]. Nonlinear dynamic decoupling in operational space is obtained by the selection of the following control structure:

$$\mathbf{F} = \Lambda\ \mathbf{F}^* + \boldsymbol{\mu} \tag{5}$$

where $\mathbf{F}$ is the operational space force which is to be applied to the PCV and $\mathbf{F}^*$ is the control force for our linearized unit mass system. As an example we can choose to implement a simple P-D controller

$$\mathbf{F}^* = -K_p(\mathbf{x} - \mathbf{x}_d) - K_v(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + \mathrm{I}_3\ddot{\mathbf{x}}_\mathrm{d} \tag{6}$$

with K_p, K_v the position and velocity gains and $\mathbf{x}_d$ and its derivatives the desired position, velocity and acceleration.

This approach requires that we know the operational space velocities, $\dot{\mathbf{x}}$, of the PCV and the actuated joint torques, $\boldsymbol{\Gamma}'$, necessary to produce the commanded operational space force, $\mathbf{F}$. The XR4000 powered casters (see Figure 2) have an encoder on each motor. The encoders together with knowledge of the gearbox kinematics allow us to calculate the positions and velocities of the steering and rolling joints of each module. We can write the relationships between the observed joint speeds and the operational speeds of the i^{th} wheel as the *wheel constraint matrix*, C_i, which contains the two nonholonomic constraints from "manipulator" model in eqn. 2. We will use $\dot{\mathbf{q}}_i' = [\,\dot{\phi}_i\ \dot{\rho}_i\,]^T$ to designate the observed joint speeds of the i^{th} wheel.

$$\dot{\mathbf{q}}_i' = C_i\ \dot{\mathbf{x}} \tag{7}$$

$$C_i = \begin{bmatrix} -\mathrm{s}\phi_i/b & \mathrm{c}\phi_i/b & h_i[\mathrm{c}\beta_i\mathrm{c}\phi_i + \mathrm{s}\beta_i\mathrm{s}\phi_i]/b - 1 \\ \mathrm{c}\phi_i/r & \mathrm{s}\phi/r & h_i[\mathrm{c}\beta_i\mathrm{s}\phi_i - \mathrm{s}\beta_i\mathrm{c}\phi_i]/r \end{bmatrix}$$

The overall motion of the joints in the robot can be described by gathering the wheel constraint matrices into the *constraint matrix*, C.

$$\dot{\mathbf{q}}' = C\ \dot{\mathbf{x}} \tag{8}$$

$$\dot{\mathbf{q}}' = \begin{bmatrix} \dot{\mathbf{q}}_1' \\ \vdots \\ \dot{\mathbf{q}}_n' \end{bmatrix} \quad ; \quad C = \begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix}$$

The dual of this relationship describes the operational space force produced by the torques at the actuated joints.

$$\mathbf{F} = C^T\ \boldsymbol{\Gamma}' \tag{9}$$

To find the operational space velocities and actuated joint torques we need to find the reciprocal relationships to eqns. 8,9. Since the constraint matrix is not square with size $2n \,\mathrm{x}\, 3$, it is not invertible. For our overconstrained system with more measurements than states, we can use a generalized inverse of the constraint matrix, $C^{\#}$, to give results we desire. For an ideal robot with no measurement error, using any arbitrary left inverse will yield the same results. However, when there is unmodeled slippage at the wheel contacts along with the ever present measurement errors of real hardware, the particular choice of generalized inverse will yield different results.

One common choice of generalized inverse is the Moore-Penrose pseudo-inverse [9]. This leads to an $\dot{\mathbf{x}}$ which minimizes, in a least-squares manner, the joint velocity differences between the measured system and the consistent set of joint velocities associated with that robot velocity. The physical meaning of fitting a solution to the joint velocities is elusive.

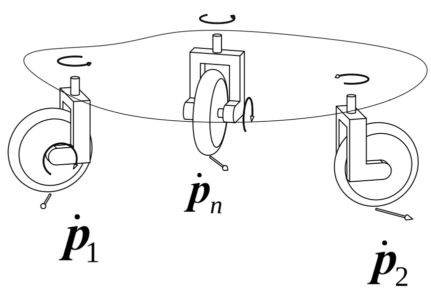

Figure 4: Contact point velocities

A more physically meaningful solution can be found by looking at the set of wheel velocities at the contact points, $\dot{\mathbf{p}}$, as shown in Figure 4. The sensed velocities of these points can be found from the measured joint speeds with the mapping below where C_q is square, full rank, block diagonal, and invertible.

$$\dot{\mathbf{q}} = C_q \ \dot{\mathbf{p}} \tag{10}$$

When the robot obeys the ideal rolling assumptions there exists a robot velocity where the sensed contact speeds are identical to the consistent set of contact speeds, $\dot{\hat{\mathbf{p}}}$, found with the kinematic relationship

$$\dot{\hat{\mathbf{p}}} = C_p \ \dot{\mathbf{x}} \tag{11}$$

However, as is to be expected, when there is some slippage and measurement noise, $\dot{\mathbf{p}} \neq \dot{\hat{\mathbf{p}}}$. By using the Moore-Penrose pseudo-inverse of the non-square matrix C_p we will minimize the total perceived slip by minimizing the differences between $\dot{\mathbf{p}}$ and $\dot{\hat{\mathbf{p}}}$. Our estimate of the robot velocity assuming that slip is minimized uses a generalized inverse of the constraint matrix and is

$$\dot{\mathbf{x}} = C^{\#} \ \dot{\mathbf{q}}' \tag{12}$$

with

$$C^{\#} = C_p^{+} C_q^{-1} \tag{13}$$

We have tested the odometry of our XR4000 moving randomly for one minute in a 1.5m x 2.5m area and then returning to its starting position. When using the generalized inverse from eqn. 13 the dead-reckoning error was less than half as large as when the pseudo-inverse of the constraint matrix was used.

The dual of this result is just as ideal. There are many ways to distribute the effort among the joints to achieve a desired operational space force. By distributing the joint torques using the transpose of the generalized inverse in eqn. 13

$$\mathbf{\Gamma}' = C^{\#^T} \ \mathbf{F} \tag{14}$$

we minimize, in a least squares way, the contact forces developed by the wheels. The consequence is that the tractive effort is spread as evenly as possible among the wheels and the tendency for any one wheel to loose traction is minimized.

Other useful, physically meaningful generalized inverses can be found. For instance there exists a different generalized inverse to distribute the torques in a way that minimizes the actuator power.

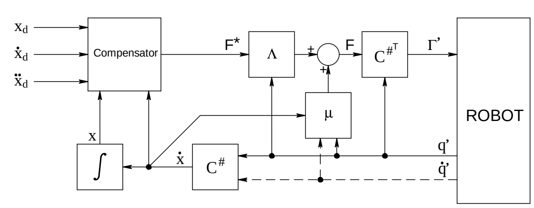

Figure 5: Controller Schematic

The controller outlined in Figure 5 was implemented on a modified Nomadic Technologies XR4000 mobile robot. The real-time operating system QNX was used to run the controller containing the full dynamics of PCV at 1000 Hz. on an onboard 450 MHz. Pentium II.

5 Conclusions

We have presented the design of a new wheeled holonomic mobile robot, the *powered caster vehicle*, or PCV, which is being produced as the XR4000 mobile robot by Nomadic Technologies. The design of the powered casters provides smooth accurate motion with the ability to traverse the hazards of a typical office environment. The design can be used with two or more wheels and as implemented with four wheels provides a stable platform for mobile manipulation. A modular, efficient dynamic model was derived by using the augmented object model originally developed for the study of cooperating fixed-base manipulators. The framework for dynamically decoupled control of the PCV, an over-actuated parallel system, was developed using physically meaningful generalized inverses of the kinematic constraint matrix. We look forward to further results demonstrating the coordination of the dynamically controlled mobile base with the dynamically controlled PUMA manipulator it carries.

Acknowledgements

We gratefully acknowledge Nomadic Technologies Inc., where the development of the powered caster mechanism took place; for the resources devoted to this project, and to the work of all the individuals there, especially Anthony del Balso, Rich Legrand, Jim Slater and John Slater who were instrumental in the creation of the XR4000 mobile robot.

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