Acey Deucey

- Have a standard deck of 52 cards
  - Ranks of cards: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
  - Three cards drawn (without replacement)
  - What is probability that rank of third card drawn is between the ranks of the first two cards, exclusive?
    - E.g., if ranks of first two cards drawn are 4 and 9, then want probability that third card is a 5, 6, 7 or 8
- Solution set-up
  - Let X = difference between rank of 1st and 2nd card
  - P(X = 0) = 3/51
    - After picking first card, there are 3 others with same rank
    - This is not really relevant. Just a warm-up to get you thinking!

Acey Deucey Solution

- Solution
  - P(X = *i*) = $(13-i)\cdot\frac{2}{13}\cdot\frac{4}{51}$, where $1 \le i \le 12$
    - (13 – *i*) ways to choose two ranks that differ by *i*
    - First card has 2/13 chance of being one of those 2 ranks
    - Second card is one of 4 cards (out of 51) that differ in rank by *i*
  - Want: $\sum_{i=1}^{12} P(X=i)P(\text{3rd card between first two} \mid X=i)$

    $= \sum_{i=1}^{12} \frac{8(13-i)}{(13)(51)} P(\text{3rd card between first two} \mid X=i)$
    - Of remaining 50 cards, there are 4 cards of each (*i* – 1) ranks

    $= \sum_{i=1}^{12} \frac{8(13-i)}{(13)(51)} \cdot \frac{4(i-1)}{50}$

Birthdays Tres Compadres

- Have a group of 100 people
  - Let X = number of days of year that are birthdays of *exactly* 3 people in group
- What is E[X]?
  - First, compute probability *p* that a particular day is the birthday of exactly 3 people in the group
    - Let A_i = number of people that have birthday on day *i*
    - A_i ~ Bin(100, 1/365)
    - $p = P(A_i = 3) = \binom{100}{3}\left(\frac{1}{365}\right)^3\left(\frac{364}{365}\right)^{97}$
  - Let X_i = 1 if A_i = 3, and 0 otherwise
  - $E[X] = E[\sum_{i=1}^{365} X_i] = \sum_{i=1}^{365} E[X_i] = \sum_{i=1}^{365} P(A_i = 3) = \sum_{i=1}^{365} p = 365p$

More Birthdays, More Fun

- Have a group of 100 people
  - Let Y = number of distinct birthdays
  - What is E[Y]?
- Solution
  - Let Y_i = 1 if day *i* is the birthday of at least 1 person, and 0 otherwise
  - $E[Y_i] = P(Y_i) = 1 - P(Y_i^c) = 1-\left(\frac{364}{365}\right)^{100}$
  - $E[Y] = E[\sum_{i=1}^{365} Y_i] = \sum_{i=1}^{365} E[Y_i] = 365\left[1-\left(\frac{364}{365}\right)^{100}\right]$

MOM Loves the Geometric

- Consider I.I.D. random variables $X_1, X_2, \ldots, X_n$
  - X_i ~ Geo(*p*)
- Estimate *p* using Method of Moments
- Solution
  - Recall, for X_i ~ Geo(*p*), we know $E[X_i] = 1/p$
  - Rewrite as $p = 1/E[X_i]$
  - Using Method of Moments:

$$p = \frac{1}{E[X_i]} \approx \frac{1}{\hat{m}_1} = \frac{1}{\bar{X}} = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} X_i} = \hat{p}$$