Combinatorial Auction Glossary

additive set function   A set function f is *additive* if and only if $f(S \cup T) = f(S) + f(T)$ for all disjoint S and T.

activity   A measure of the quantity of bidding in a round by a bidder in a simultaneous ascending auction. Activity includes both standing high bids and new bids in the round. In spectrum auctions, activity is measured in units of bandwidth times the number of people covered by the license (e.g., MHz-pop). In electricity auctions, the quantity is in energy units (e.g., MWh).

activity rule   A restriction on allowable bids intended to maintain the pace and encourage price discovery in simultaneous ascending auctions. This is done by preventing the ''snake in the grass'' strategy, in which a bidder maintains a low level of activity early in the auction and then greatly expands his demand late in the auction. The activity rule forces a bidder to maintain a minimum level of activity to preserve current eligibility. Typically, the activity requirement increases in stages. For example, the activity requirement might be 60 percent in stage 1, 80 percent in stage 2, and 100 percent in stage 3 (the final stage). With a 60 percent activity requirement, each bidder must be active on a quantity of items, equal to at least 60 percent of the bidder's current eligibility. If activity falls below the 60 percent level, then the bidder's current eligibility is reduced to its current activity divided by 60 percent. With a 100 percent activity requirement, the bidder must be active on 100 percent of its current eligibility or its eligibility drops to its current activity. The lower activity requirement early in the auction gives the bidder greater flexibility in shifting among license aggregations early on when there is the most uncertainty about what will be obtainable. Typically, bidders are given some small number of waivers of the activity requirement.

affiliated   The random variables $Z = \{S, T_1, \ldots, T_n\}$ are *affiliated* if the joint density $f(z)$ is such that for all $z, z' \in Z$, $f(z \vee z') f(z \wedge z') \geq f(z) f(z')$, where $z \vee z' = \max\{z, z'\}$ and $z \wedge z' = \min\{z, z'\}$.

affiliated values   Model of bidder values in which bidder i's value $v_i(s, t_1, \ldots, t_n)$ depends on the state of world s and the realization of each bidder's private information $(t_1, \ldots, t_n)$, where the random variables determining the state of the world and each bidder's private information are *affiliated*. The affiliated values model is a general model, allowing both *private values* uncertainty and *common values* uncertainty. Affiliation implies that if one bidder has a high estimate of value it is more likely that the other's estimate of value is high. See *interdependent values*.

agent   A party or computer program that acts on behalf of another. A player in a game. See *bidding agent*.

agents are substitutes See *bidders are substitutes*.

allocation An assignment of items to bidders as a result of the auction.

allowable bids Bids that bidders are allowed to submit. Auction rules may prevent bids on certain combinations, bids that are too low or too high, or that claim too many goods.

anonymous pricing Pricing that does not depend on bidder identity (in contrast with *non-anonymous pricing*).

approximation A solution to an optimization problem that is not necessarily optimal but that has a performance guarantee. See *heuristic*.

ascending auction An iterative auction in which bids on any biddable combination can only increase during the course of the auction.

auction design The process of defining the products being auctioned, as well as the auction rules and procedures.

auction designer One designing an auction.

auctioneer One conducting the auction.

Bayesian Nash equilibrium The extension of Nash equilibrium to games with incomplete information. Each player plays a best response to the strategies of the other players. Best responses are evaluated after a player learns his private information, but before he learns the private information of the other players. Hence, the player's strategy maximizes his expected utility given his private information, the joint distribution of others' private information, and the strategies of the other players. Private information is drawn from a common joint distribution and beliefs about strategies are consistent. See *Nash equilibrium* and *ex post equilibrium*.

bid An expression of preference by the bidder, indicating an interest in buying or selling particular items under certain conditions. Typically, bids are binding commitments, with the exception of *indicative bids*.

bid increment In an ascending auction, the minimum amount that a standing high bid must be raised for the bid to be valid. Typically, bid increments are set at the discretion of the auctioneer.

bid preparation The act of preparing a bid for an auction.

bid retraction In a discrete round auction, removing a bid before the bid submission period for the round has concluded. The bid becomes null and void, equivalent to as if it was never made. See *bid withdrawal*, which is fundamentally different.

bid withdrawal The situation under which a high bidder can withdraw his bid subject to a penalty. In such a case, the auctioneer is then listed as the high bidder and the minimum bid is then set at a lower level. Bid withdrawals typically are allowed in simultaneous ascending auctions without package bidding. A bid withdrawal occurs after the bid submission period of the round the bid was placed.

biddable combination A combination on which it is possible to place an allowable bid.

bidder A company or person participating in an auction.

bidder submodular A *coalitional value function w* where bidders are more valuable when added to smaller coalitions: for all $l \in L \setminus \{0\}$ (0 denotes the seller) and all coalitions S and S' satisfying $0 \in S \subset S'$, $w(S \cup \{l\}) - w(S) \geq w(S' \cup \{l\}) - w(S')$. Bidder submodular implies *bidders are substitutes* (or *agents are substitutes*). Bidder submodular requires the bidders are substitutes condition for every subset of the grand coalition (the set L of all bidders and the seller).

bidders are substitutes A coalitional value function w where the incremental value of a subset of bidders to the grand coalition is at least as great as the sum of the incremental contributions of each of its members: for all $K \subset I$ (the set of all bidders), $w(I) - w(I \setminus K) \geq \sum_{i \in K} w(I) - w(I \setminus \{i\})$. The Vickrey payoff is in the core if and only if bidders are substitutes. The stronger condition, bidder submodular, requires bidders are substitutes for every subset of the grand coalition. Hence, it assures the Vickrey payoff is in the core of the game with the reduced set of bidders.

bidding agent A party or computer program that submits bids on behalf of a bidder according to a predetermined procedure. See *proxy*.

bidding language The set of all allowable bids.

bid-taker See *auctioneer*.

bipartite graph An undirected graph whose nodes can be partitioned into two sets such that each edge connects nodes from different sets.

Boolean combination An expression using the primitive Boolean connectives AND, OR, and NOT, and possibly other logical connectives defined by these such as XOR (exclusive OR).

budget constraint An upper bound on the amount of money a bidder can spend in the auction.

bundle See *package*.

bundle prices Prices defined on bundles.

clock auction A type of *dynamic auction* used for identical items or divisible goods, in which the auctioneer announces prices and bidders respond with quantities desired at the announced prices. Bidding is continuous in a theoretical clock auction, although practical implementations have a discrete clock and discrete rounds.

coalitional value function A mapping from a set of players S to a real number $w(S)$, equal to the maximum value created by the set of players working together. In auctions, this is the total value from trade among the players S. See *cooperative game*.

collusion Cooperative behavior among bidders in an auction, which may be explicit (as in a bidding ring) or tacit (as in a shared understanding derived from past or current bidding behavior).

combination See *package*.

combinatorial bid A Boolean combination of package bids.

complement The set of all items that do not belong to a given combination.

complements Items such that the value of the items combined is greater than the sum of individual values. See *superadditive*.

combinatorial auction (CA) An auction that allows combinatorial bids, or, as a special case, package bids. See *package bid* and *combinatorial bid*.

common values   Model of bidder values in which packages of items have the same value to all bidders. Typically, these values are unknown, and bidders only have estimates of the common value $v(S, s, t_1, \ldots, t_n)$, where S is the package of items, s is the state of the world (reflecting common uncertainty), and t_i is bidder i's private information. See *interdependent values* and *private values*.

communication complexity   The minimum number of bits whose transmission is required in order to compute a given function.

computational complexity   The intrinsic minimum amount of resources—such as time or storage capacity—needed to solve instances of a computational problem, measured as a function of the number of bits needed to represent an instance (input size).

computationally tractable   A computational problem is *computationally tractable* if it is possible to provide an algorithm that solves each instance within a computation time that is bounded by a polynomial function of the size of the instance.

computationally intractable   A computational problem is *computationally intractable* if it is not possible to provide an algorithm that has a computation time bounded by a polynomial function of the size of the instance.

conditional bid   A bid that is valid only if a specified set of requirements are met, such as package, minimum volume, delivery, or payment requirements. See *multiattribute bid*.

continuous auction   An auction that is not subdivided into rounds, allowing bid submission at any time. Often, trades occur on a continuous basis, as well as bids.

cooperative game   A game in which players decide what group of players to work with (a coalition) and how to split up the maximum value created by their working together. The value by their working together is defined by a *coalitional value function*. Players can make binding commitments.

core   A concept for cooperative games. For the case of transferable utility (utility is linear in money), the core is the set of value allocations among coalition members such that each member does at least as well within the coalition as in any subcoalition. For the case of non-transferable utility, the *NTU-core* is the set of feasible unblocked allocations. An allocation ω is *blocked* if there exists some coalition S and allocation ω' feasible for coalition S such that all the members of S strictly prefer ω' to ω.

correspondence   A one-to-many mapping: every point in the domain is mapped to possibly many points in the range. In contrast, a function is a one-to-one mapping: every point in the domain is mapped to a single point in the range.

cost minimization   A common objective in auctions to buy, such as procurement auctions, in which the auctioneer minimizes the cost of acquiring the items. See *revenue maximization*.

decision problem   A computational problem consisting of a class of instances and a definition of a property of these instances. The problem to be solved is to decide whether a given instance from the class has this property or not. For example, given a set of combinatorial bids and a target revenue w, is there an allocation of items to bidders with a total revenue of at least w.

direct revelation mechanism   For games with private information, a mapping from player reports of private information into utility outcomes. For any game with private information, a direct revelation mechanism is the composition of the strategy functions (mapping private information into actions) and the utility functions (mapping actions and private information into payoffs). See *mechanism* and *incentive compatibility*.

discriminatory price   An auction where two bidders winning identical item combinations may nonetheless pay different prices for those bundles.

discriminatory auction   In economics, a format for auctioning multiple identical items (or divisible goods) where bidders simultaneously submit demand curves. Each bidder wins the quantity he demanded at the clearing price, paying the amount that he bid for each unit won. See *pay-as-bid auction*. Computer scientists sometimes refer to any auction with a discriminatory price as a discriminatory auction.

dominant strategy   A strategy that, for every possible realization of types of players, does at least as well as any other, regardless of what strategies are played by the other players. The strategy is a *strictly dominant strategy* if it does for every possible realization of types of players strictly better than all other strategies, regardless of the strategies chosen by others.

dominant strategy equilibrium   A refinement of *Nash equilibrium* (or *Bayesian Nash equilibrium* for games of incomplete information). Each player's strategy is a best response, regardless of the strategies of the other players. Behavior in dominant strategy equilibria is robust to uncertainty about what strategies the other players adopt and to uncertainty over the other players' private information. With private values, bidding one's value is a dominant strategy equilibrium in the *Vickrey auction*.

dual   Every *linear program* has a corresponding linear program called the *dual*, with the property that its objective is always a bound on the original linear program, called the *primal*. When a point is feasible to both the primal and the dual linear programming problems, the point is optimal to both problems.

Dutch auction   As used in auction theory, a format for auctioning a single item also known as a *descending clock auction*. The auctioneer starts at a high price and announces successively lower prices. The first bidder to bid wins the item, and pays the current price at the time of the bid. Unfortunately, the same term is sometimes used in the finance literature to refer to the *uniform-price auction* for multiple items.

dynamic auction   Any auction format that involves multiple opportunities to bid and where some information about the bidding is revealed to the bidders during the course of the auction. An English auction is the most common form of dynamic auction. Dynamic auctions can either have discrete rounds (as in an *iterative auction* or *multiround auction*) or continuous bidding, as in a *clock auction*.

(Pareto) efficiency   An assignment of resources such that it is impossible to make someone better off without making anyone else worse off. Equivalently, when the utility possibility set is convex, an assignment of resources that maximizes a weighted sum of players' utilities for some positive welfare weights. For games with incomplete information, there are several definitions of efficiency

depending on the time at which efficiency is evaluated (*ex ante*—before players learn their private information, *interim*—after each player learns his own private information, but before he learns the others' private information, and *ex post*—after all information has been revealed), and whether incentive constraints are recognized. The most common form of efficiency applied to auction games is *ex post efficiency*, which examines efficiency after all private information is known and ignoring incentive constraints. For games with transferable utility (utility that is linear in money), this is the same as maximizing the ex post gains from trade; that is, assigning the items to those that value them the most.

electronic auction   An *electronic market* in which the market is conducted by auction.

electronic market   A market that is designed for or requires use of electronic communication and computation.

eligibility   The maximum quantity of items that a bidder may bid on. In a dynamic auction, eligibility may decline as a result of a bidder's failure to satisfy an activity rule. Eligibility is initially set based on a bidder's financial qualifications, its incumbent position, a bid deposit, or a letter of credit. See *activity rule*.

ending rule   The rule in iterative auctions defining the point at which the auction ends. A common ending rule is the point at which no bidder is willing to submit another bid.

equilibrium bidding   A set of bidding strategies, one for each bidder, such that each bidder's strategy is a best response to the other bidders' strategies. Typically, bidders have private information, in which case the best responses are calculated in expected terms. See *Bayesian Nash equilibrium*.

ex post equilibrium   A refinement of Bayesian Nash equilibrium for games with incomplete information, in which each player's equilibrium strategy remains an equilibrium even after learning the realization of each players' private information. An ex post equilibrium is robust to the distribution of private information; that is, ex post equilibrium strategies remain best responses regardless of the distribution of private information. Hence, the ex post outcome is invariant to the distribution of private information. Ex post equilibrium is a weakening of *dominant strategy equilibrium*. It typically arises in auction settings with interdependent values, because dominant strategy equilibria typically do not exist in these settings, but ex post equilibria may exist.

exposure problem   The problem of winning some—but not all—of a complementary collection of items in an auction without package bids. The bidder is ''exposed'' to a possible loss if his bids include synergistic gains that might not be achieved.

English auction   A format for auctioning a single item. Bidders submit successively higher bids for the item, until no bidder is willing to bid higher. The final bidder wins the item, and pays the amount of his final bid.

failure-freeness   The property that the auction procedure never fails to allocate and price items and combinations according to predetermined auction rules.

first-price (sealed bid) auction   A format for auctioning a single item. Bidders simultaneously submit sealed bids for the item. The highest bidder wins the item, and pays the amount of his bid.

free disposal   A valuation function v satisfies free disposal if $v(S \cup T) \geq v(S)$ for all combinations S and T. In particular, disposing an item from a combination cannot increase the combination value.

gap   The difference between the minimum bid on a combination that would be needed to make the bid a winning one under the assumption all other bids remain unchanged, and the current highest bid on that combination. The gap is zero if the current highest bid is a provisional winner; otherwise the gap is positive.

gross substitutes   See *substitutes*.

heterogeneous items   Items that are not identical. See *homogeneous items*.

heuristic   An algorithm that is believed to be valuable but is not guaranteed to be optimal and is lacking in performance guarantees. An algorithm can be heuristic with respect to some properties (for example, running time) but not others (for example, correctness or optimality of the output). See *approximation*.

homogeneous items   Identical items. See *heterogeneous items*.

incentive compatibility   A direct revelation mechanism is *incentive compatible* if truthfully reporting private information is a Bayesian Nash equilibrium in the revelation game; that is, it is a best response truthfully to report private information assuming all other players report truthfully. A direct revelation mechanism is *incentive compatible in dominant strategies* if truthfully reporting private information is a dominant strategy equilibrium in the revelation game; that is, it is a best response, regardless of the reports of the other players. Every mechanism with a *Bayesian Nash equilibrium* has a corresponding direct mechanism that is incentive compatible. See *direct revelation mechanism*.

incumbent bidder   A bidder who already has existing operations in the market.

independent private values   A *private values* model for bidder values in which the bidders' values are independent random variables.

indicative bids   A nonbinding indication of interest. This is used in certain auctions to identify a short list of bidders that are allowed to bid in the subsequent auction with binding bids.

integer program   An optimization problem, typically with a linear objective and linear constraints, where one or more of the choice variables must take on integer values. See *linear program*.

integer programming   A collection of techniques from mathematics, operations research, and computer science for solving integer programs. The problem of finding a solution to an arbitrary integer program is *NP-hard*. See *linear programming*.

item   Basic unit of good or service offered at the auction. Equivalent terms include *product*, *property*, and *object*. See *unit*.

item prices   See *linear prices*.

iterative auction   A *dynamic auction* with discrete rounds.

interdependent values   Model of bidder values with a general valuation function in which each bidder's value of a package depends on his private information as well as the private information

of the other bidders. Typically, these values are unknown, and each bidder i only has estimates of the value $v_i(S, s, t_1, \ldots, t_n)$, where S is the package of items, s is the state of the world (reflecting common uncertainty), and t_i is bidder i's private information. See *common values* and *private values*.

jump bid   A bid in an ascending auction that is higher than the minimum bid required in the round. Jump bids are sometimes used to signal a strong interest in particular items. In auctions with package bids, jump bids can be used on large packages to exploit the *threshold problem*.

lattice   A lattice $(L, \geq)$ is a pair consisting of a nonempty set L and a partial order $\geq$ on L in which any two elements $a, b \in L$ have 1) a least common upper bound (join) $a \vee b$, and 2) a largest common lower bound (meet) $a \wedge b$.

linear pricing   Each item has a price that is the same for each bidder. The price for any package is the sum over all items in the package of the price of each item times the quantity of the item in the package.

linear program   An optimization problem with a linear objective and linear constraints, in which any real valued solutions are allowed. See *integer program*.

linear programming   A collection of techniques from mathematics, operations research, and computer science for solving linear programs. Linear programming is computationally tractable. See *integer programming*.

linear programming relaxation   A linear program obtained from a given integer program by removing the constraints that certain choice variables take on integer values.

linear valuation   A valuation function that is additive.

market   An opportunity for sellers and buyers of items to engage in trade.

market design   Defining the products to be traded, as well as the rules and procedures of trade, with the goal of maximizing an objective, often efficiency or revenues.

mechanism   A description of the possible actions for each agent, and a mapping from the agents' actions into an outcome. See *protocol* and *direct revelation mechanism*.

minimum bid   The smallest acceptable bid. In ascending auctions, this is one bid increment more than the prior bid. See *bid increment*.

monotone likelihood ratio property   A probability density function f satisfies the monotone likelihood ratio property if the ratio $f(v|t)/f(v|s)$ is weakly increasing in v for all $t > s$. Typically, $f(v|s)$ is the probability density of a bidder's value v conditional on the signal s (an estimate of value). Intuitively, the likelihood of high values increases with the estimate of value.

multiattribute bid   A bid in an auction, especially procurement auctions, where the bidder specifies several predefined attributes of an item, such as price, payment terms, delivery schedule, quality, and material.

multiple-round auction   A *dynamic auction* where bidders have a chance to modify their bids in a sequence of rounds.

multi-item auction An auction for many items. Also called a multiple-object auction or multi-object auction. See *multiunit auction* for the special case where the items are identical.

multiunit auction An auction for many identical items. Bidders express the quantity desired of each type of item.

Nash equilibrium A set of strategies, one for each player of the game, such that each player's strategy is a best response to the others' strategies.

net substitutes See *substitutes*.

non-anonymous pricing A pricing rule in which two bidders may win the same set of items and yet pay different prices. See *anonymous pricing*.

nonlinear pricing The price of a package does not equal the dot product of item prices times item quantities. Volume discounts are an example of nonlinear pricing. *Vickrey pricing* is an example of both nonlinear and *non-anonymous* pricing. See *linear pricing*.

NP (nondeterministic polynomial time) A complexity class of *decision problems* for which there exists a polynomial time algorithm that can check the validity of the property in question, if it is given as input next to the instance a certificate for the validity (regardless of how hard it is to compute such certificates). Equivalently, it is the class of problems computable by a nondeterministic Turing machine in polynomial time. It is generally believed, but not known, that the class NP contains problems that are computationally intractable. See *NP-complete*.

NP-complete Those problems in NP that are the hardest problems in NP in the following sense: any polynomial time algorithm that solves an NP-complete problem could be used to solve any other problem in NP. For this reason, it is generally believed, but not known, that NP-complete problems are computationally intractable.

NP-hard The complexity class of computational problems that are intrinsically as hard as any in NP. By definition, NP-complete problems are NP-hard, but also problems not contained in NP can be NP-hard.

NP-complete (resp. hard) optimization problem An optimization problem whose corresponding *decision problem* is NP-complete (resp. NP-hard). The decision version of an optimization problem "find for a given instance an optimal solution" takes the form "Is there a solution for this instance of quality at least …"

object See *item*.

online auction An auction conducted with electronic means, typically over the Internet.

quasilinear utility Utility that is linear in money and the good: $u(S, p) = v(S) - p$, where $v(S)$ is the bidder's value for the package S and p is his payment.

package A set collection of items. Also called *bundle* or *combination*.

package auction An auction for multiple items, allowing package bids. A bid is interpretable as an all-or-nothing offer for the specified package at the specified price. See *combinatorial auction*.

package bid A price for a specified collection of items. In the indivisible, single-unit case, the collection is simply the set of items. In the indivisible, multiunit case, the collection is the number of units of each good. In the divisible-good case, the collection is the fraction of each good.

pay-as-bid auction   A format for auctioning multiple identical items (or divisible goods) where bidders simultaneously submit demand curves. Each bidder wins the quantity demanded at the clearing price, paying the amount that he bid for each unit won. See *discriminatory auction*.

permitted bids   See *allowable bids*.

polynomial time algorithm   An algorithm that has an execution time that is no more than a polynomial function of the size of the input.

price discovery   A feature of dynamic auctions in which tentative price information is reported to bidders, giving bidders the opportunity to adjust subsequent bids based on the price information.

pricing rule   An auction rule defining prices for all allowable combinations at any given point of the auction procedure.

privacy preservation   A property that precludes bidders from making their valuations contingent on the valuations of other bidders.

private values   Model of bidder values in which each bidder's value does not depend on the private information of the other bidders. Bidder i's value for the package S is given by $v_i(S)$. See *common values* and *interdependent values*.

procurement auction   An auction to buy services or goods. Also called a *reverse auction*.

product   See *item*.

property   See *item*.

protocol   A specification for sequential contingent actions (such as communication or computation) by one or more agents. See *mechanism*.

provisional winner   In an interactive auction, a bidder that would be a winner if the auction were to end at that point. See *tentative winner*.

provisional winning bid   In an iterative auction, a bid that would be a winning bid if the auction were to end at that point.

proxy   An agent process that automatically submits bids for a bidder according to a predetermined procedure.

proxy auction   An auction in which bids are submitted by proxy. Most typically, real bidders submit preferences, and then the proxy agents iteratively submit bids using straightforward bidding. The use of a proxy speeds up the iterative auction and constrains how bidder preferences are translated into bids. See *straightforward bidding*.

reserve price   The minimum price at which the seller will sell an item. The amount is sometimes known to the bidders and is sometimes not known, in which case it is called a "secret reserve." In an auction to buy, the reserve price is the maximum price at which the buyer will buy an item.

reservation price   A bidder's valuation or willingness to pay. Also called *valuation* or *value*.

revelation principle   A principle of mechanism design, which states that under certain conditions, there is no loss in restricting attention to direct revelation mechanisms in which the players simply report their private information to the mechanism, and in equilibrium do so truthfully.

revenue maximization   A common objective in auctions to sell, in which the auctioneer maximizes the revenue obtained for the items. See *cost minimization*.

reverse auction   An auction to buy goods or services with one buyer and many competing sellers. This is ''reverse'' of the standard auction to sell goods or services with one seller and many competing buyers. See *procurement auction*.

scalable   A property that the auction procedure would still be practically solvable if the number of items auctioned were increased significantly.

sealed-bid auction   A single round auction where bids are simultaneously submitted by the bidders.

second-price (sealed-bid) auction   A format for auctioning a single item. Bidders simultaneously submit sealed bids for the item. The highest bidder wins the item, and pays the amount bid by the second-highest bidder. See *Vickrey auction*.

set packing problem   See weighted set packing problem.

shortfall   See *gap*.

simultaneous ascending auction (SAA)   A format for auctioning multiple items, commonly used for auctioning spectrum licenses. The auction is a natural generalization of the English auction, especially useful when selling many related items. The items are auctioned simultaneously in a sequence of rounds. In each round, each bidder can submit bids on any of the items, raising the standing high bids by at least the bid increment. The auction does not end until no bidder is willing to bid higher on any item. Typically, bidding is constrained by activity rules that force bidders to maintain a level of activity throughout the auction that is consistent with their desired winnings. An equivalent term is *simultaneous multiple round* (SMR) *auction*.

single-round auction   An auction in which bidders have a single opportunity for bid submission. See *sealed-bid auction*.

sincere bidding   See *straightforward bidding*.

straightforward bidding   A bidding strategy in dynamic auctions in which, in each round, the bidder bids the minimum bid on the package of items that maximizes his net value given the current item and package prices. Such a strategy is optimal if this is the last bid placed by any bidder. However, in other cases straightforward bidding typically is not optimal. This is sometimes called *sincere bidding*.

subadditivity   A set function f is locally subadditive at the disjoint sets S and T if $f(S \cup T) \leq f(S) + f(T)$. A set function f is subadditive if it is locally subadditive for all disjoint S and T. See *superadditivity*.

submodular   A set function $f : 2^m \rightarrow \Re$ is submodular if for all sets S and T, $f(S \cap T) + f(S \cup T) \leq f(S) + f(T)$. A lattice function $f : L \rightarrow \Re$ is submodular if for all $a, b \in L$, $f(a \wedge b) + f(a \vee b) \leq f(a) + f(b)$. See *lattice* and *bidder submodular*.

substitutes   Goods are substitutes when increasing the price of one does not reduce demand for the other. The modified terms ''gross substitutes'' and ''net substitutes'' are often used to distinguish between substitutes for uncompensated and compensated demand, respectively. With quasilinear utilities, there is no distinction between ''gross'' and ''net.''

superadditivity   A set function f is locally superadditive at the disjoint sets S and T if $f(S \cup T) \geq f(S) + f(T)$. A set function f is superadditive if it is locally superadditive for all disjoint S and T. See *subadditivity*.

synergy   When a player's valuation function is superadditive on a specified subset of items, the player is said to have a synergy for those items. See *complements*.

tentative winner   In an interactive auction, a bidder that would be the winner if the auction were to end at that point. See *provisional winner*.

tie   A situation in which there are distinct alternative solutions to the winner determination problem.

tie-breaking   A procedure for determining which of alternative sets of bids that solve the winner determination problem should be preferred.

threshold problem   Allowing package bids may favor bidders seeking larger packages, because small bidders do not have the incentive or capability to top the tentative winning bids of the large bidder.

tractable   See *computationally tractable*.

transparency   The property that, in any instance of the auction, the participants or the public can verify that the auction rules have been properly followed; for example, a losing bidder can see why he lost.

uniform-price auction   A format for auctioning multiple identical items. Bidders simultaneously submit demand curves. Each bidder wins the quantity demanded at the clearing price, and pays the clearing price for each item won.

unit   A measure of quantity in the identical good case or the divisible good case, as in multiunit auction. (However, in electricity auctions the term ''unit'' refers to a physical generator, as in ''unit commitment.'') See *item*.

valuation   See *reservation price*.

value   See *reservation price*.

valuation function   A function describing a bidder's valuation for all biddable packages.

VCG mechanism   See *Vickrey-Clarke-Groves mechanism*.

Vickrey allocation   An allocation defined by the VCG mechanism.

Vickrey auction   An auction format for multiple identical items. Bidders simultaneously submit demand curves. Each bidder wins the quantity demanded at the clearing price, and pays the opportunity cost of its winnings. For a single-item auction, the Vickrey auction is a *second-price auction*. When the approach is applied to the auction of non-identical items, the Vickrey auction is often referred to as the *generalized Vickrey auction* or the *Vickrey-Clarke-Groves mechanism*.

Vickrey-Clarke-Groves (VCG) mechanism   A format for auctioning multiple items. Bidders simultaneously submit sealed bids giving their value for each possible package. The auctioneer then determines an efficient assignment of the items based on the bids. Payments are determined so as to allow each bidder a payoff equal to the incremental surplus that he brings to the auction. See *Vickrey auction*.

Vickrey outcome   An allocation and pricing defined by the VCG mechanism.

Vickrey pricing   Pricing defined by the VCG mechanism.

Walrasian pricing process   A dynamic pricing process in which market participants express quantities in response to announced prices. Prices are increased in response to excess demand and decreased in response to excess supply. The process continues until supply equals demand for all items.

weighted set packing problem   The problem of finding a disjoint collection of weighted subsets of a larger set with maximal total weight. Weighted set packing is a classical *NP-hard* problem, and is closely related to the *winner determination problem*.

winner's curse   The insight that winning an item in an auction is bad news about the item's value ($E(v_i|i \text{ wins}) < E(v_i)$), where v_i is bidder i's uncertain value), because winning implies that no other bidder was willing to bid as much for the item. Hence, it is likely that the winner's estimate of value is an overestimate. Because a bidder's bid is only relevant in the event that the bidder wins, the bidder should condition the bid on the negative information winning conveys about value. Bidders that fail to condition their bids on the bad news winning conveys suffer from the winner's curse in the sense that they often pay more for an item than it is worth.

winner determination problem (WDP)   The computational problem of determining the winning bids in a combinatorial auction by labeling bids as either winning or losing so as to maximize the sum of the accepted bids under the constraint that each item can be allocated to at most one bidder. This is equivalent to the *weighted set packing problem*, a computationally intractable problem.

winning bid   The bid that is selected by the winner determination problem. A bidder placing a winning bid is assigned all items defining the combination on which the bid was placed.