Previewed Signal Decoupling Problem in Discrete-Time Systems

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Abstract

This paper deals with the decoupling problem of previewed signals. The discrete time case is considered. In this domain previewing a signal by p steps means that the sample at step $k+p$ of the signal to be decoupled is available at instant k. New necessary and sufficient conditions are proposed for the problem with stability to be solved. These conditions are constructive, easily checkable and appear to be computable in a more convenient way if compared to previous results in literature.

The framework throughout is the geometric approach to the control theory of linear systems. The main tool is the lattice of self-bounded controlled invariants that leads to the minimization of the resolving subspace which corresponds to the infimum of the lattice.

1 Introduction

The disturbance decoupling problem (DDP) has been one of first applications considered in the geometric approach framework. In its first formulation [3, 11], disturbance signals were assumed to be unknown and unaccessible. However, in some cases the signals to be decoupled may be measurable, like for instance in noninteracting control. These cases are referred to as *measured signal decoupling problems* (MSDP) and have been studied by Bhattacharyya through both static and dynamic feedback [5]. The structural conditions for solvability of MSDP are less restrictive than those for DDP, while stabilizability conditions are similar.

In this paper the decoupling problem is approached in a more general setting. Signals known in advance, or previewed by a given amount of time, are considered and the decoupling problem will be referred to as *previewed signal decoupling problem* (PSDP).

Preaction and preview have been recently studied in tracking problems. The use of a supervising unit feeding the compensator that solves the problem of perfect tracking with preaction in the non minimum-phase case was developed in the SISO case with standard transfer functions [6] and in the MIMO case with geometric techniques [8].

The previewed signal decoupling problem in continuous time was first studied by Willems in [10] where a necessary and sufficient condition solving the PSDP with pole placement was proposed. This solution was based on the so called proportional-integral-derivative control laws consisting of a feedback of the state system and of a linear combination of the signal (to be decoupled) and its time derivatives. This approach, being in continuous time, involves distributions and hence is not practically implementable. In [7], Imai and Shinozuka proposed a similar necessary and sufficient condition for the PSDP with stability in both discrete and continuous time cases.

Conditions for PSDP to be solved, given in [10] and [7], do not take into account any dimension issue for the subspace resolving the problem. Furthermore, to the best of our knowledge, the problem of reducing the dimension of such subspace has not been thoroughly investigated in literature.

In this paper a new solution for the PSDP with stability is proposed. It is based on a controlled invariant whose dimensions are reduced if compared with the solution proposed in [10] and [7]. This is due to the use of a particular class of controlled invariant subspaces known as self-bounded, which were introduced by Basile and Marro in [4, 9]. The set of self bounded subspaces enjoys interesting properties, the most important of which is to be a lattice instead of a semi-lattice, hence to admit an infimum other than a supremum.

This paper deals with discrete-time systems. After reviewing the well known concepts of minimum conditioned invariant, a structural condition for PSDP and a condition for PSDP with stability are presented. The proofs of the theorems are constructive and are developed in a geometric framework [2].

The structure of the compensator, whereby the signal decoupling of previewed signals is obtained, is discussed. It consists of a preaction and a postaction units. A synthesis procedure, based on geometric approach algorithms, will be provided for both of these units.

The following notation is used. **R** stands for the field of real numbers. Sets, vector spaces and subspaces are denoted by script capitals like, e.g. $\mathcal{X}, \mathcal{I}, \mathcal{V}$. Matrices and linear maps are denoted by slanted capitals like A, B, etc., the image and the null space of the matrix or linear transformation A by $\mathrm{im}A$ and $\ker A$, respectively. The transpose of matrix A is represented by A^T and its pseudoinverse by $A^\#$.

The paper is organized as follows. Section 2 presents the structural conditions for the PSDP. Section 3 proposes new necessary and sufficient conditions for the PSDP with stability. In Section 4 a synthesis procedure for the decoupling compensator is reported and finally in Section 5 an illustrative example is discussed.

2 Structural conditions for PSDP

Let us consider the discrete-time system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Hh(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $x \in \mathcal{X}$ ($= \mathbf{R}^n$), $u \in \mathbf{R}^m$, $h \in \mathbf{R}^h$ and $y \in \mathbf{R}^q$ denote the state, the manipulable input, the signal to be decoupled and the regulated output, respectively. In the following, the short notation $\mathcal{B} := \mathrm{im}B$, $\mathcal{C} := \ker C$ and $\mathcal{H} := \mathrm{im}H$ will be used.

In this paper we deal with the signal decoupling problem when a certain degree of knowledge of signal $h(k)$ is available. In particular we assume that signal $h(k)$ is previewed, i.e. it is known p steps in advance, or analytically the sample $h(k)$ is known at step $k-p$. Note that measurable disturbance can be thought as 0-steps previewed signals.

Our aim here is to use the preview on $h(k)$ to "prepare" the system dynamics to localize signal $h(k)$ on the nullspace of the output matrix C. This is formalized in the following statement.

Problem 1 (Previewed signal decoupling) *Refer to system (1) with zero initial condition and assume that input $h(k)$ is previewed by p steps, $p \geq 0$. Determine a control law that, using this preview, maintains the output $y(k)$ identically zero.*

The key tool to analyze the structural conditions for the signal decoupling problem, is the well-known [2] algorithm computing $\mathcal{S}^* := \min \mathcal{S}(A, \mathcal{C}, \mathcal{B})$, the minimal $(\mathcal{A}, \mathcal{C})$-conditioned invariant containing $\mathcal{B}$, here reported for the reader convenience:

$$\mathcal{S}_0 := \mathcal{B} \tag{2}$$

$$\mathcal{S}_i := \mathcal{B} + A(\mathcal{S}_{i-1} \cap \mathcal{C}). \tag{3}$$

Structural conditions to solve Problem 1 for p-previewed signals are given in the following.

Property 1 *Necessary and sufficient condition for Problem 1 to be solved is that*

$$\mathcal{H} \subseteq \mathcal{V}^* + \mathcal{S}_p. \tag{4}$$

where $\mathcal{V}^* := \max \mathcal{V}(A, \mathcal{B}, \mathcal{C})$ is the maximal controlled invariant contained in $\mathcal{C}$.

Remark 1 *Structural condition (4) in Property 1, is similar to that proposed in [10] for the continuous-time case but less restrictive since condition (4) does not consider stability. It is worth noting that the case of measurable inputs is accounted for by condition (4). In fact measurable signals correspond to previewed signals with $p = 0$ and therefore (4) turns into the well known condition*

$$\mathcal{H} \subseteq \mathcal{V}^* + \mathcal{B}.$$

Similarly, the lack of any preview leads to the structural condition for unknown (disturbance) signals

$$\mathcal{H} \subseteq \mathcal{V}^*.$$

Observe that, being

$$\mathcal{V}^* \subseteq \mathcal{V}^* + \mathcal{B} \subseteq \mathcal{V}^* + \mathcal{S}_p,$$

a less conservative decoupling condition corresponds to a larger preview time.

The following property characterizes the minimum number of preview steps necessary to decouple previewed signals for a given disturbance matrix H.

Property 2 *Consider system (1) and let r be the minimum number of steps necessary to obtain convergence of algorithm for $\min \mathcal{S}(A, \mathcal{C}, \mathcal{B})$. The minimum positive integer $p \leq r$, such that condition (4) holds, corresponds to the minimum number of previewed steps for $h(k)$ necessary to decouple signal $h(k)$. Moreover, if for $p = r$ condition (4) is not satisfied, the PSDP has no solution for the given disturbance matrix H.*

3 Previewed signal decoupling problem with stability

The p–previewed signal decoupling problem with stability is investigated.

Problem 2 (Previewed signal decoupling with stability) *Refer to system (1) with zero initial condition and assume that it is stabilizable and that input $h(k)$ is previewed by p steps, $p \geq 0$. Determine a compensator that, using the preview, maintains the output $y(k)$ identically zero and the state bounded.*

The Previewed signal decoupling with stability is approached by means of self-bounded controlled invariants [4, 9]. A special attention is devoted to the dimension of the resolving subspace.

Let us introduce the lattice of the $(A,\mathcal{B})$-controlled invariant subspaces self bounded with respect to $\mathcal{C}$

$$\Phi = \Phi(\mathcal{B},\mathcal{C}) = \{\mathcal{V} \mid \; A\,\mathcal{V} \subseteq \mathcal{V}+\mathcal{B},\; \mathcal{V}\subseteq \mathcal{C},\; \mathcal{V}^*\cap\mathcal{B}\subseteq\mathcal{V}\} \tag{5}$$

whose infimum is given by

$$\mathcal{V}_m = \mathcal{V}^* \cap \min\mathcal{S}(A,\mathcal{C},\mathcal{B}) \tag{6}$$

and supremum by $\mathcal{V}^*$.

Lemma 1 *The set*

$$\Phi_2 = \{\mathcal{V} \mid \mathcal{V}\in\Phi, \mathcal{H}\subseteq\mathcal{V}+\mathcal{S}_p\} \tag{7}$$

enjoys the following properties:

1. *is a sub-lattice of* Φ*;*

2. $\forall\mathcal{V}\in\Phi,\; \mathcal{H}\subseteq\mathcal{V}+\mathcal{S}_p \Leftrightarrow \mathcal{V}^*\cap(\mathcal{H}+\mathcal{S}_p)\subseteq\mathcal{V}$*, i.e.* $\Phi_2 \equiv \{\mathcal{V}\mid\mathcal{V}\in\Phi, \mathcal{V}^*\cap(\mathcal{H}+\mathcal{S}_p)\subseteq\mathcal{V}\}$*;*

3. *the infimum of* Φ_2 *is given by*

$$\mathcal{V}_{m2} = \mathcal{V}^*\cap\min\mathcal{S}(A,\mathcal{V}^*,\mathcal{H}+\mathcal{S}_p) \tag{8}$$

Proof: *(Property 1.)* We want to show that given two generic elements $\mathcal{V}_1$ and $\mathcal{V}_2$ of set Φ_2 their sum and intersection still belongs to the same set. Such proof appears trivial for the subspace obtained by summing the two given subspaces. Let's consider now element $\mathcal{V}_1\cap\mathcal{V}_2$. By assumption, since both $\mathcal{V}_1$ and $\mathcal{V}_2$ belong to Φ_2 it is obvious that

$$\mathcal{H}\subseteq\mathcal{V}_1+\mathcal{S}_p \tag{9}$$
$$\mathcal{H}\subseteq\mathcal{V}_2+\mathcal{S}_p \tag{10}$$

which lead to

$$\mathcal{H}\subseteq(\mathcal{V}_1+\mathcal{S}_p)\cap(\mathcal{V}_2+\mathcal{S}_p).$$

By intersecting both terms with $\mathcal{V}^*+\mathcal{S}_p$ we obtain

$$\mathcal{H}\subseteq((\mathcal{V}_1+\mathcal{S}_p)\cap(\mathcal{V}_2+\mathcal{S}_p))\cap(\mathcal{V}^*+\mathcal{S}_p) \tag{11}$$

since the structural condition (4) holds, and then

$$\mathcal{H}\subseteq(\mathcal{V}^*\cap(\mathcal{V}_1+\mathcal{S}_p))\cap(\mathcal{V}^*\cap(\mathcal{V}_2+\mathcal{S}_p))+\mathcal{S}_p$$

using the distributive property, being $\mathcal{S}_p$ included in $(\mathcal{V}_1+\mathcal{S}_p)\cap(\mathcal{V}_2+\mathcal{S}_p)$. Analogously we get

$$\mathcal{H}\subseteq((\mathcal{V}_1\cap\mathcal{V}^*)+(\mathcal{V}^*\cap\mathcal{S}_p))\cap((\mathcal{V}_2\cap\mathcal{V}^*)+(\mathcal{V}^*\cap\mathcal{S}_p))+\mathcal{S}_p$$

and finally

$$\mathcal{H}\subseteq(\mathcal{V}_1\cap\mathcal{V}_2)+\mathcal{S}_p$$

being $\mathcal{V}_1$ and $\mathcal{V}_2$ both included in $\mathcal{V}^*$ and both including $\mathcal{V}^*\cap\mathcal{S}_p$.

(Property 2.)
$(\Rightarrow)$

$$\mathcal{V}\in\Phi_1\Rightarrow\mathcal{H}\subseteq\mathcal{V}+\mathcal{S}_p\Rightarrow\mathcal{H}+\mathcal{S}_p\subseteq\mathcal{V}+\mathcal{S}_p$$

and therefore intersecting both members with $\mathcal{V}^*$ we obtain

$$\mathcal{V}^*\cap(\mathcal{H}+\mathcal{S}_p)\subseteq\mathcal{V}^*\cap(\mathcal{V}+\mathcal{S}_p)=\mathcal{V}+(\mathcal{V}^*\cap\mathcal{S}_p)=\mathcal{V}$$

being $(\mathcal{V}^*\cap\mathcal{S}_p)\subseteq(\mathcal{V}^*\cap\mathcal{S}^*)$ which is the infimum of Φ and therefore is contained in all $\mathcal{V}\in\Phi$.
$(\Leftarrow)$

$$\mathcal{V}^*\cap(\mathcal{H}+\mathcal{S}_p)\subseteq\mathcal{V}$$

summing $\mathcal{S}_p$ to both members we obtain

$$\mathcal{S}_p+(\mathcal{V}^*\cap(\mathcal{H}+\mathcal{S}_p))\subseteq\mathcal{V}+\mathcal{S}_p$$

from which using the distributive property we obtain

$$(\mathcal{S}_p+\mathcal{V}^*)\cap(\mathcal{S}_p+\mathcal{H})\subseteq\mathcal{V}+\mathcal{S}_p$$

and being $\mathcal{H}\subseteq\mathcal{S}_p+\mathcal{H}\subseteq\mathcal{S}_p+\mathcal{V}^*$ we obtain

$$(\mathcal{S}_p+\mathcal{H})\subseteq\mathcal{V}+\mathcal{S}_p$$

from which obviously

$$\mathcal{H}\subseteq\mathcal{V}+\mathcal{S}_p$$

(Property 3.)
The proof will be developed in two steps:

(A.) Any element of Φ_2 contains $\mathcal{V}_{m2}=\mathcal{V}^*\cap\mathcal{S}_2^*$ where

$$\mathcal{S}_2^*=\min\mathcal{S}(A,\mathcal{V}^*,\mathcal{H}+\mathcal{S}_p); \tag{12}$$

(B.) $\mathcal{V}^*\cap\mathcal{S}_2^*$ is an element of Φ_2

(Step A.) Consider the sequence that defines $\mathcal{S}_2^*$:

$$\mathcal{Z}'_0 := \mathcal{S}_p+\mathcal{H} \tag{13}$$
$$\mathcal{Z}'_i := \mathcal{S}_p+\mathcal{H}+A\,(\mathcal{Z}'_{i-1}\cap\mathcal{V}^*)\quad(i=1,\dots) \tag{14}$$

Let $\mathcal{V}$ be a generic element of Φ_2, so that

$$A\,\mathcal{V}\subseteq\mathcal{V}+\mathcal{B},\quad \mathcal{V}\supseteq\mathcal{V}^*\cap\mathcal{B}.$$

We proceed by induction: clearly

$$\mathcal{Z}'_0\cap\mathcal{V}^*\subseteq\mathcal{V}$$

since by assumption $\mathcal{V}^*\cap(\mathcal{S}_p+\mathcal{H})\subseteq\mathcal{V}$, and from

$$\mathcal{Z}'_{i-1}\cap\mathcal{V}^*\subseteq\mathcal{V}$$

it follows that

$$A(\mathcal{Z}'_{i-1}\cap\mathcal{V}^*)\subseteq A\mathcal{V}\subseteq\mathcal{V}+\mathcal{B}$$

since $\mathcal{V}$ is an $(A,\mathcal{B})$-controlled invariant. Adding $\mathcal{S}_p+\mathcal{H}$ to both members yields

$$\mathcal{S}_p+\mathcal{H}+A\,(\mathcal{Z}'_{i-1}\cap\mathcal{V}^*)\subseteq\mathcal{V}+\mathcal{S}_p+\mathcal{H}$$

where the first term of the last inclusion is by definition subspace $\mathcal{Z}'_i$ and, by intersecting with $\mathcal{V}^*$, we finally obtain

$$\mathcal{Z}'_i\cap\mathcal{V}^*\subseteq(\mathcal{V}+(\mathcal{S}_p+\mathcal{H}))\cap\mathcal{V}^*=\mathcal{V}+(\mathcal{S}_p+\mathcal{H})\cap\mathcal{V}^*=\mathcal{V}$$

which completes the induction argument and the proof of Step A.

(Step B.) Note that

1. $\mathcal{S}_1^* \cap \mathcal{V}^*$ is an $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{C}$;

2. $\mathcal{S}_1^* \cap \mathcal{V}^*$ is self bounded with respect to $\mathcal{C}$

3. $\mathcal{H} \subseteq (\mathcal{S}_1^* \cap \mathcal{V}^*) + \mathcal{S}_p$

To prove 1. note that

$$A\,\mathcal{V}^* \subseteq \mathcal{V}^* + \mathcal{B}$$
$$A\,(\mathcal{S}_1^* \cap \mathcal{V}^*) \subseteq \mathcal{S}_1^*$$

which simply expresses $\mathcal{V}^*$ as an $(A, \mathcal{B})$-controlled invariant and $\mathcal{S}_1^*$ to be an $(A, \mathcal{V}^*)$-conditioned invariant. By intersection it follows that

$$A\,(\mathcal{S}_1^* \cap \mathcal{V}^*) \subseteq \mathcal{S}_1^* \cap (\mathcal{V}^* + \mathcal{B}) = \mathcal{S}_1^* \cap \mathcal{V}^* + \mathcal{B}$$

being $\mathcal{B} \subseteq \mathcal{S}_1^*$. Then $\mathcal{S}_1^* \cap \mathcal{V}^*$ is an $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{C}$.

To prove 2. note that

$$\mathcal{V}^* \cap \mathcal{B} \subseteq \mathcal{V}^* \cap \mathcal{S}_1^*.$$

Finally, to prove 3. note that being $\mathcal{S}_p \subseteq \mathcal{S}_1^*$ and $\mathcal{H} \subseteq \mathcal{S}_1^*$ it follows that

$$\mathcal{H} \subseteq (\mathcal{V}^* \cap \mathcal{S}_p) + \mathcal{S}_1^* = (\mathcal{V}^* \cap \mathcal{S}_1^*) + \mathcal{S}_p.$$

■

Introducing the sub-lattice Φ_2 is functional to the proof of Theorem 1 because of the self boundedness of its elements which satisfy this fundamental property proved in [2].

Property 3 *Let $\bar{\mathcal{V}}$ and $\mathcal{V}$ be a couple of any $(A, \mathcal{B})$-controlled invariant subspaces self bounded with respect to $\mathcal{C}$ such that $\mathcal{V} \subseteq \bar{\mathcal{V}}$. Let F be a matrix such that $(A + BF)\bar{\mathcal{V}} \subseteq \bar{\mathcal{V}}$. Then $(A + BF)\mathcal{V} \subseteq \mathcal{V}$ holds.*

Theorem 1 *The p-steps previewed signal decoupling, stated in Problem 2, is solvable if and only if the structural condition (4) is satisfied and*

$$\mathcal{V}_{m2} = \mathcal{V}^* \cap \min\mathcal{S}(A, \mathcal{V}^*, \mathcal{H} + \mathcal{S}_p), \tag{15}$$

the infimum of Φ_2, is internally stabilizable.

Proof: According to Property 1, the purpose of the preaction is to cancel, at the generic time instant k, the component of $Hh(k)$ on $\mathcal{S}_p$ in order to force the state dynamics (excited by the remaining component) on a subspace $\mathcal{V}$ satisfying the following properties:

1. $\mathcal{V}$ is an $(A, \mathcal{B})$ controlled invariant included in $\mathcal{C}$;

2. $\mathcal{V}$ is such that $\mathcal{H} \subseteq \mathcal{V} + \mathcal{S}_p$;

3. $\mathcal{V}$ is internally stabilizable.

We will now prove the necessity of the statement, i.e. that if a subspace $\mathcal{V}$ exists that solves Problem 2 (with stability) then $\mathcal{V}_{m2}$ is internally stabilizable. Consider subspace

$$\bar{\mathcal{V}} := \mathcal{V} + \mathcal{R}_{\mathcal{V}^*}$$

where $\mathcal{V}$ is a subspace satisfying Properties 1, 2 and 3 and $\mathcal{R}_{\mathcal{V}^*} = \mathcal{V}^* \cap \mathcal{S}^*$ represents the constrained reachability subspace on $\mathcal{C}$. It is clear that $\bar{\mathcal{V}}$ satisfies Properties 1, 2 and 3 because

- $\bar{\mathcal{V}}$ is an $(A, \mathcal{B})$ controlled invariant contained in $\mathcal{C}$, being the sum of two controlled invariants contained in $\mathcal{C}$;
- $\mathcal{H} \subseteq \bar{\mathcal{V}} + \mathcal{S}_p$, being $\mathcal{H} \subseteq \mathcal{V}$;
- $\bar{\mathcal{V}}$ is internally stabilizable, being the sum of two internally stabilizable subspaces.

Subspace $\bar{\mathcal{V}}$ is an element of Φ_2 defined in (7), since $\mathcal{V}^* \cap \mathcal{B} \subseteq \mathcal{R}_{\mathcal{V}^*} \subseteq \bar{\mathcal{V}}$. Being $\bar{\mathcal{V}}$ internally stabilizable a state feedback matrix F exists that stabilizes such subspace. Because of Property 3, such matrix stabilizes every subspace $\mathcal{V} \in \Phi_2$ included in $\bar{\mathcal{V}}$, and therefore also its infimum $\mathcal{V}_{m2}$ being all of these subspaces self bounded.

For the sufficiency part, simply note that if $\mathcal{V}_{m2}$ is internally stabilizable than it satisfies Properties 1, 2 and 3 at once. ■

Remark 2 *Condition for PSDP, stated in Theorem 1, is an improvement of the well known condition first stated in [7]*

$$\mathcal{H} \subseteq \mathcal{V}_g^* + \mathcal{S}_\rho$$

where $\mathcal{V}_g^*$ denotes the restriction of $\mathcal{V}^*$ having only "good" modes, in the Wonham's notation [12].

From the algorithmic standpoint Theorem 1 provides a resolving subspace $\mathcal{V}_{m2}$ which is a minimal-dimension self-bounded controlled invariant, thus implying order reduction of the derived decoupling compensators.

Let us now present an alternative solution for the PSDP with stability. It is based on the following decomposition of matrix H

$$H = H_V + H_S \tag{16}$$
$$\mathcal{H}_V := \mathrm{im}(H_V) \subseteq \mathcal{V}^* \tag{17}$$
$$\mathcal{H}_S := \mathrm{im}(H_S) \subseteq \mathcal{S}_p. \tag{18}$$

which is always feasible if the structural condition (4) holds

Components of $Hh(k)$ lying on $\mathcal{S}_p$ will be canceled through a *preaction unit*, while signal $H_V h(k)$ will be localized in the nullspace of the output matrix according to standard decoupling techniques as described in Section 4.

The following Theorem, proved in [1], is based on decomposition (16).

Theorem 2 *The p-steps previewed signal decoupling problem, stated in Problem 2, is solvable if and only if the structural condition (4) is satisfied and subspace*

$$\mathcal{V}_{m1} := \mathcal{V}^* \cap \min\mathcal{S}(A, \mathcal{C}, \mathcal{H}_V + \mathcal{B}) \tag{19}$$

is internally stabilizable.

The following result is alternative to that given in Theorem 1 and is more readable. Equivalence of Theorem 1 and 2 is proven in [1]. Theorem 2 follows directly from decomposition (16) and from standard solution [2] of the DDP.

An algorithm for the synthesis of the decoupling controller is presented in the following Section.

4 Synthesis procedure for the controller solving the PSDP

Assume that necessary and sufficient conditions of Theorem 2 hold. Due to equation (16) it is always possible to divide the effect of $Hh(k)$ on system dynamics into two separate parts. The effect of $H_V h(k)$ can be nulled using a postaction unit, being $\mathcal{H}_V \subseteq \mathcal{V}^*$. The effect of $H_S h(k)$ can be nulled using a preaction unit, being $\mathcal{H}_S \in \mathcal{S}_p$.

4.1 Postaction unit

Due to (16) system (1) can be rewritten as

$$\begin{cases} x(k+1) &= Ax(k) + Bu(k) + H_V h(k) + H_S h(k) \\ y(k) &= Cx(k) \end{cases} \tag{20}$$

where $u(k) = u_{\text{pos}}(k) + u_{\text{pre}}(k)$. The purpose of the postaction unit is to decouple $\mathcal{H}_v$. It is an easy matter to show that $\mathcal{H}_V \subseteq \mathcal{V}_{m1}$. Applying Theorem 2, a stabilizing state feedback matrix F exists such that the state trajectory excited by $H_V h(k)$ evolve onto $\mathcal{V}_{m1} \in \mathcal{C}$. Therefore the postaction unit is simply given by

$$u_{\text{pos}}(k) = Fx(k) \tag{21}$$

as shown in Fig. 1.

It is worth noting that, since the system dynamics evolve on a known subspace, the postaction unit can also be implemented as a feedforward unit.

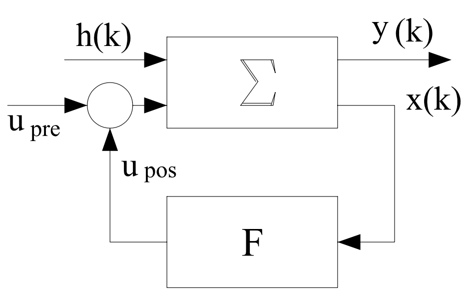


Figure 1: Postaction $u_{\text{pos}}(k)$ is synthesized through a state feedback matrix F.

4.2 Preacion unit

In order to design the preaction unit, subspace $\mathcal{S}_p$ must be interpreted as a special reachability subspace.

Property 4 *Subspace $\mathcal{S}_p$ corresponds to the set of states reachable in p $(p \geq 0)$ steps from initial condition $x_0 = 0$, with the state trajectory constrained to evolve onto $\mathcal{C}$ in the preceding p-steps interval $[0, p-1]$.*

Analytically, matrices Ω_0, Ω_1 ... Ω_p exist such that

$$\mathcal{S}_p = \text{im}\begin{bmatrix} B & AB & A^2B & \cdots & A^pB \end{bmatrix} \begin{bmatrix} \Omega_0 \\ \Omega_1 \\ \Omega_2 \\ \vdots \\ \Omega_p \end{bmatrix}$$

and

$$\begin{bmatrix} CB & CAB & \cdots & CA^{p-1}B \\ 0 & CB & \cdots & CA^{p-2}B \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & CB \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \vdots \\ \Omega_p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Taking into account postaction (21), system (20) can be rewritten as

$$x(k+1) = A_f x(k) + Bu_{\text{pre}}(k) + H_s h(k) + H_v h(k) \tag{22}$$

where

$$A_f := A + BF.$$

To decouple the effects of $H_S h(k)$ on system dynamics (22), a preaction unit is built as

$$u_{\text{pre}}(k) = \sum_{l=0}^{p} \Phi(l) h(k+l) \tag{23}$$

where gains of the preaction unit are computed as

$$\begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_p \end{bmatrix} = M^{\#} \begin{bmatrix} -H_s \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{24}$$

being

$$M = \begin{bmatrix} B & A_f B & A_f^2 B & \cdots & A_f^p B \\ 0 & CB & CA_f B & \cdots & CA_f^{p-1} B \\ 0 & 0 & CB & \cdots & CA_f^{p-2} B \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & CB \end{bmatrix}. \tag{25}$$

Consistency of (24) is guaranteed by Property 4 and by inclusion (18).

The preaction unit in eq. (23) is reported in Fig. 2. Preaction consists of a p-step FIR system which, previewing the signal $h(k)$ p-steps in advance, is able to prepare system dynamics to cancel component $H_S h(k)$ when it presents as input to the system (at the time instant k).

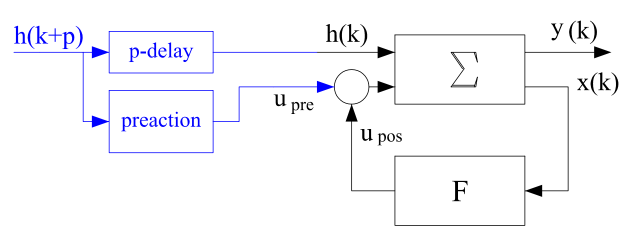


Figure 2: The decoupling compensator consists of a preaction unit and a postaction unit.

5 An illustrative example

Let us consider the previewed signal decoupling problem for system (1) with

$$A = 0.1 \begin{bmatrix} 1 & 2 & 1 & -1 & -2 \\ 0 & -1 & 2 & 1 & 1 \\ 0 & 3 & 1 & -1 & -1 \\ 1 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0.5774 \\ 0 & 0 & -0.5774 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5774 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Algorithms for $\mathcal{V}^*$ and $\mathcal{S}^*$ converge to

$$\mathcal{V}^* = \mathrm{im} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{S}^* = \mathrm{im} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0.5774 \\ 0 & 0 & -0.5774 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5774 \end{bmatrix}.$$

Note that

$$\mathcal{S}_1 = \mathcal{S}^*$$

and that

$$\mathcal{H} \subseteq \mathcal{V}^* + \mathcal{S}_1 = \mathcal{V}^* + \mathcal{S}^*.$$

Being $\mathcal{H} \not\subseteq \mathcal{V}^* + \mathcal{B}$, $p = 1$ is the minimum number of previewed steps necessary to solve the decoupling problem.

According to (16), matrix H is divided in two matrices

$$H_V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad H_S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5774 \\ 0 & 0 & -0.5774 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5774 \end{bmatrix}.$$

The resolving subspace is evaluated as

$$\mathcal{V}_{m1} = \mathcal{V}_{m2} = \mathrm{im} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which results internally stabilizable.

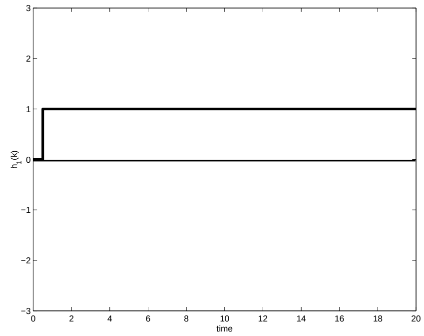
It is worth noting that that subspace $\mathcal{V}_g^*$ proposed in [10] has, in this case, a dimension which is double of $\mathcal{V}_{m1}$.

Postaction unit is given by the state feedback $u_{\text{pos}} = Fx(k)$ where

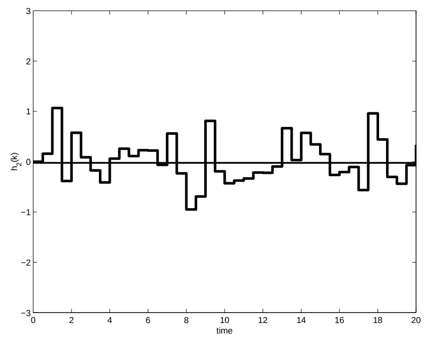
$$F = \begin{bmatrix} -0.1 & 0 & 0 & 0.1 & 0 \\ -0.1 & 0 & 0 & -0.2 & 0 \end{bmatrix}$$

while for preaction unit we obtain from (24) and (25)

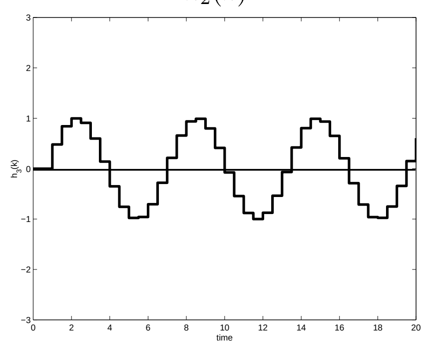
$$\Phi_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -5.7735 \end{bmatrix}, \quad \Phi_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$



$h_1(k)$



$h_2(k)$



$h_3(k)$

Figure 3: The three components of 1-previewed signal $h(k)$.

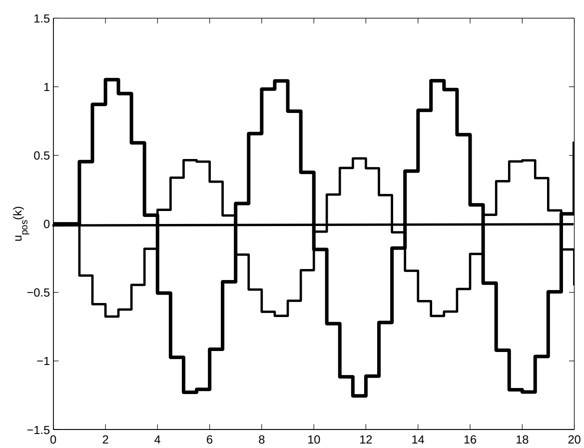


Figure 4: The two components of postaction control $u_{\text{pos}}(k)$.

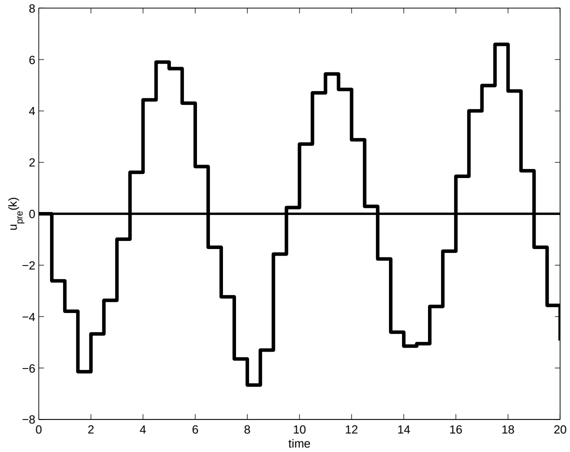


Figure 5: The two components of preaction control $u_{\text{pre}}(k)$.

Simulation are executed by applying a signal

$$h(k) = \left[\begin{array}{ccc} h_1(k) & h_2(k) & h_3(k) \end{array} \right]^T$$

where $h_1(k)$ is a step signal, $h_2(k)$ a band-limited white noise and $h_3(k)$ a sine wave, as reported in Fig.3.

The two components of postaction and preaction signals $u_{\text{pos}}(k)$ are reported in Fig. 4 and Fig. 5, respectively. Preaction-postaction compensator perfectly decouples signal $h(k)$ which has a 1-step preview and maintains the state trajectory bounded.

6 Conclusions

A new solution for previewd signal decoupling problems with stability has been proposed. It is based on two necessary and sufficient constructive conditions, one is structural while the other deals with the stability requirement. The problem has been approached through self-bounded controlled invariants, thus allowing to reduce the dimension of the resolving subspace which corresponds to the infimum of a lattice. A systematic algorithmic procedure has been presented for synthesizing the compensator which consists of a preaction and a postaction units.

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