Optimal Binary Representation of Mosaic Floorplans and Baxter Permutations

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Abstract. A *floorplan* is a rectangle subdivided into smaller rectangular *blocks* by horizontal and vertical line segments. Two floorplans are considered equivalent if and only if there is a bijection between the blocks in the two floorplans such that the corresponding blocks have the same horizontal and vertical boundaries. *Mosaic floorplans* use the same objects as floorplans but use an alternative definition of equivalence. Two mosaic floorplans are considered equivalent if and only if they can be converted into equivalent floorplans by sliding the line segments that divide the blocks. The *Quarter-State Sequence* method of representing mosaic floorplans uses $4n$ bits, where n is the number of blocks. This paper introduces a method of representing an n-block mosaic floorplan with a $(3n-3)$-bit binary string. It has been proven that the shortest possible binary string representation of a mosaic floorplan has a length of $(3n-o(n))$ bits. Therefore, the representation presented in this paper is asymptotically optimal. *Baxter permutations* are a set of permutations defined by prohibited subsequences. There exists a bijection between mosaic floorplans and Baxter permutations. As a result, the methods introduced in this paper also create an optimal binary string representation of Baxter permutations.

Keywords: Binary Representation, Mosaic Floorplan, Baxter Permutation.

1 Introduction

In this section, the definitions of mosaic floorplans and Baxter permutations are introduced, previous work in the area and their applications are described, and the main result is stated.

1.1 Floorplans and Mosaic Floorplans

Definition 1. *A* floorplan *is a rectangle subdivided into smaller rectangular subsections by horizontal and vertical line segments such that no four subsections meet at the same point.*

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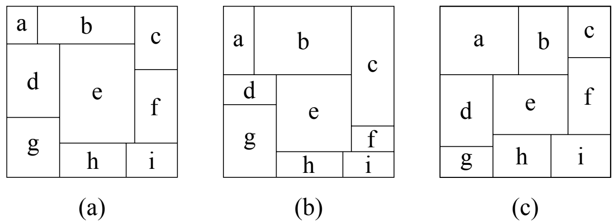

Fig. 1. Three example floorplans

The smaller rectangular subsections are called *blocks*. Figure 1 shows three floorplans, each containing 9 blocks. Note that the horizontal and vertical line segments do not cross each other. They can only form *T-junctions* ($\vdash$, $\perp$, $\dashv$, and $\top$).

The definition of equivalent floorplans does not consider the size of the blocks in the floorplan. Instead, two floorplans are considered equivalent if and only if their corresponding blocks have the same relative position relationships. The formal definition of equivalent floorplans follows.

Definition 2. *Let F_1 be a floorplan with R_1 as its set of blocks. Let F_2 be another floorplan with R_2 as its set of blocks. F_1 and F_2 are considered* equivalent floorplans *if and only if there is a bijection $g : R_1 \to R_2$ such that the following conditions hold:*

1. *For any two blocks $r, r' \in R_1$, r and r' share a horizontal line segment as their common boundary with r above r' if and only if $g(r)$ and $g(r')$ share a horizontal line segment as their common boundary with $g(r)$ above $g(r')$.*
2. *For any two blocks $r, r' \in R_1$, r and r' share a vertical line segment as their common boundary with r to the left of r' if and only if $g(r)$ and $g(r')$ share a vertical line segment as their common boundary with $g(r)$ to the left of $g(r')$.*

In Figure 1, (a) and (b) have the same number of blocks and the position relationships between their blocks are identical. Therefore, (a) and (b) are equivalent floorplans. However, (c) is not equivalent to either.

The objects of *mosaic floorplans* are the same as the objects of floorplans. However, mosaic floorplans use a different definition of equivalence. Informally, two mosaic floorplans are considered equivalent if and only if they can be converted to each other by sliding the horizontal and vertical line segments one at a time. The equivalence of the mosaic floorplans is formally defined by using the *horizontal constraint graph* and the *vertical constraint graph* [9]. The horizontal constraint graph describes the horizontal relationship between the vertical line segments of a floorplan. The vertical constraint graph describes the vertical relationship between the horizontal line segments of a floorplan. The formal definitions of horizontal constraint graphs, vertical constraint graphs, and equivalence of mosaic floorplans follow.

Definition 3. *Let F be a floorplan.*

1. *The* horizontal constraint graph $G_H(F)$ *of F is a directed graph. The vertex set of $G_H(F)$ has a bijection with the set of the vertical line segments of F. For two vertices u_1 and u_2 in $G_H(F)$, there is a directed edge $u_1 \rightarrow u_2$ if and only if there is a block b in F such that the vertical line segment v_1 corresponding to u_1 is on the left boundary of b and the vertical line segment v_2 corresponding to u_2 is on the right boundary of b.*
2. *The* vertical constraint graph $G_V(F)$ *of F is a directed graph. The vertex set of $G_V(F)$ has a bijection with the set of the horizontal line segments of F. For two vertices u_1 and u_2 in $G_V(F)$, there is a directed edge $u_1 \rightarrow u_2$ if and only if there is a block b in F such that the horizontal line segment h_1 corresponding to u_1 is on bottom boundary of b and the horizontal line segment h_2 corresponding to u_2 is on the top boundary of b.*

The graphs in Figure 2 are the constraint graphs of all three floorplans shown in Figure 1. Note that the top, bottom, right, and left boundaries of the floorplan are represented by the north, south, east, and west vertices labeled by N, S, E, and W, respectively, in the constraint graphs. Also note that each edge in $G_H(F)$ and $G_V(F)$ corresponds to a block in the floorplan.

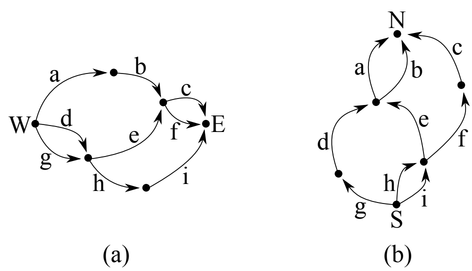

Fig. 2. The constraint graphs representing all three mosaic floorplans in Figure 1. (a) is the horizontal constraint graph. (b) is the vertical constraint graph.

Definition 4. *Two mosaic floorplans are* equivalent mosaic floorplans *if and only if they have identical horizontal constraint graphs and vertical constraint graphs.*

Thus, in Figure 1, (a), (b), and (c) are all equivalent mosaic floorplans. Note that (c) is obtained from (b) by sliding the horizontal line segment between blocks d and g downward, the horizontal line segment between blocks c and f upward, and the vertical line segment between blocks a and b to the right.

1.2 Applications of Floorplans and Mosaic Floorplans

Floorplans and mosaic floorplans are used in the first major stage in the physical design cycle of VLSI (Very Large Scale Integration) circuits [10]. The blocks in a floorplan correspond to the components of a VLSI chip. The floorplanning stage is used to plan the relative position of the circuit components. At this stage, the blocks do not have specific sizes assigned to them yet, so only the position relationship between the blocks are considered.

For a floorplan, the wires between two blocks run cross their common boundary. In this setting, two equivalent floorplans provide the same connectivity between blocks. For a mosaic floorplan, the line segments are the wires. Any block with a line segment on its boundary can be connected to the wires represented by the line segment. In this setting, two equivalent mosaic floorplans provide the same connectivity between blocks.

Binary representations of floorplans and mosaic floorplans are used by various algorithms to generate floorplans in order to solve various VLSI layout optimization problems.

Floorplans are also used to represent rectangular cartograms [15,17]. Rectangular cartograms provide a visual method of displaying statistical data about a set of regions.

1.3 Baxter Permutations

Baxter permutations are a set of permutations defined by prohibited subsequences. They were first introduced in [3]. It was shown in [8] that the set of Baxter permutations has bijections to all objects in the *Baxter combinatorial family*. For example, [4] showed that *plane bipolar orientations* with n edges have a bijection with Baxter permutations of length n. [5] established a relationship between Baxter permutations and pairs of alternating sign matrices.

In particular, it was shown in [1,6,20] that mosaic floorplans are one of the objects in the Baxter combinatorial family. A simple and efficient bijection between mosaic floorplans and Baxter permutations was established in [1,6]. As a result, any binary representation of mosaic floorplans can also be converted to a binary representation of Baxter permutations.

1.4 Previous Work on Representations of Floorplans and Mosaic Floorplans

Because of their applications in VLSI physical design, the representations of floorplans and mosaic floorplans have been studied extensively by mathematicians, computer scientists and electrical engineers. Although their definitions are similar, the combinatorial properties of floorplans and mosaic floorplans are quite different. The following is a list of research on floorplans and mosaic floorplans.

Floorplans

There is no known formula for calculating $F(n)$, the number of n-block floorplans. The first few values of $F(n)$ are $\{1, 2, 6, 24, 116, 642, 3938, \ldots\}$. Researchers have been trying to bound the range of $F(n)$. In [2], it was shown that there exists a constant $c = \lim_{n\to\infty}(F(n))^{1/n}$ and $11.56 < c < 28.3$. This means that

$11.56^n \leq F(n) \leq 28.3^n$ for large n. The upper bound of $F(n)$ was reduced to $F(n) \leq 13.5^n$ in [7].

Algorithms for generating floorplans are presented in [12]. In [18], a $(5n-5)$-bit representation of n-block floorplans is shown. A different $5n$-bit representation of n-block floorplans is presented in [19]. The shortest known binary representation of n-block floorplans uses $(4n-4)$ bits [16].

Since $F(n) \geq 11.56^n$ for large n [2], any binary string representation of n-block floorplans must use at least $\log_2 11.56^n = 3.531n$ bits. Closing the gap between the known $(4n-4)$-bit binary representation and the $3.531n$ lower bound remains an open research problem [16].

Mosaic Floorplans

It was shown in [6] that the set of n-block mosaic floorplans has a bijection to the set of Baxter permutations, and the number of n-block mosaic floorplans equals to the n^{th} *Baxter number* $B(n)$, which is defined as the following:

$$B(n) = \frac{\sum_{r=0}^{n-1} \binom{n+1}{r}\binom{n+1}{r+1}\binom{n+1}{r+2}}{\binom{n+1}{1}\binom{n+1}{2}}$$

In [14], it was shown that $B(n) = \Theta(8^n/n^4)$. The first few Baxter numbers are $\{1, 2, 6, 22, 92, 422, 2074, \ldots\}$.

There is a long list of papers on representation problem of mosaic floorplans. [11] proposed a *Sequence Pair* (SP) representation. Two sets of permutations are used to represent the position relations between blocks. The length of the representation is $2n \log_2 n$ bits.

[9] proposed a *Corner Block List* (CB) representation for mosaic floorplans. The representation consists of a list S of blocks, a binary string L of $(n-1)$ bits, and a binary string T of $2n-3$ bits. The total length of the representation is $(3n + n \log_2 n)$ bits.

[21] proposed a *Twin Binary Sequences* (TBS) representation for mosaic floorplans. The representation consists of 4 binary strings $(\pi, \alpha, \beta, \beta')$, where π is a permutation of integers $\{1, 2, \ldots, n\}$, and the other three strings are n or $(n-1)$ bits long. The total length of the representation is $3n + n \log_2 n$.

A common feature of above representations is that each block in the mosaic floorplan is given an explicit name (such as an integer between 1 and n). They all use at least one list (or permutation) of these names in the representation. Because at least $\log_2 n$ bits are needed to represent every integer in the range $[1, n]$, the length of these representations is inevitably at least $n \log_2 n$ bits.

A different approach using a pair of *Twin Binary Trees* was introduced in [20]. The blocks of the mosaic floorplan are not given explicit names. Instead, the shape of the two trees are used to encode the position relations of blocks. In this representation, each tree consists of $2n$ nodes. Each tree can be encoded by using $4n$ bits, so the total length of the representation is $8n$ bits.

In [13], a representation called *Quarter-State-Sequence* (QSS) was presented. It uses a Q sequence that represents the configuration of one of the corners of the mosaic floorplan. The length of the Q sequence representation is $4n$ bits. This is the best known representation for mosaic floorplans.

The number of n-block mosaic floorplans equals the n^{th} Baxter number, so at least $\log_2 B(n) = \log_2 \Theta(8^n/n^4) = 3n - o(n)$ bits are needed to represent mosaic floorplans.

1.5 Main Result

Theorem 1. *The set of n-block mosaic floorplans can be represented by* $(3n-3)$ *bits, which is optimal up to an additive lower order term.*

Most binary representations of mosaic floorplans discussed in section 1.4 are complex. In contrast, the representation introduced in this paper is very simple.

By using the bijection between mosaic floorplans and Baxter permutations described in [1], the methods in this paper also work on Baxter permutations. Hence, the optimal representation of mosaic floorplans results in an optimal representation of all objects in the Baxter combinatorial family.

2 Optimal Representation of Mosaic Floorplans

In this section, an optimal representation of mosaic floorplans is described.

2.1 Standard Form of Mosaic Floorplans

Let M be a mosaic floorplan. Let h be a horizontal line segment in M. The *upper segment set* of h and the *lower segment set* of h are defined as the following:

ABOVE(h) = the set of vertical line segments above h that intersect h.
BELOW(h) = the set of vertical line segments below h that intersect h.

Similarly, for a vertical line segment v in M, the *left segment set* of v and the *right segment set* of h are defined as the following:

LEFT(v) = the set of horizontal segments on the left of v that intersect v.
RIGHT(v) = the set of horizontal segments on the right of v that intersect v.

Definition 5. *A mosaic floorplan M is in* standard form *if the following hold:*

1. *For every horizontal segment h in M, all vertical segments in ABOVE(h) appear to the right of all vertical segments in BELOW(h). (Figure 3(a))*
2. *For every vertical segment v in M, all horizontal segments in RIGHT(v) appear above all horizontal segments in LEFT(v). (Figure 3(b))*

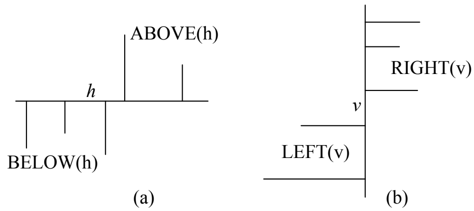

Fig. 3. Standard form of mosaic floorplans

The mosaic floorplan shown in Figure 1 (c) is the standard form of mosaic floorplans shown in Figure 1 (a) and Figure 1 (b).

The standard form M_{standard} of a mosaic floorplan M can be obtained by sliding its vertical and horizontal line segments. Because of the equivalence definition of mosaic floorplans, M_{standard} and M are considered the same mosaic floorplans. For a given M, M_{standard} can be obtained in linear time by using the horizontal constraint graphs and vertical constraint graphs described in [9]. From now on, all mosaic floorplans are assumed to be in standard form.

2.2 Staircases

Definition 6. *A* staircase *is an object that satisfies the following conditions:*

1. *The border is formed by a line segment on the positive x-axis starting from the origin and a line segment on the positive y-axis starting from the origin connected by non-increasing vertical and horizontal line segments.*
2. *The interior is divided into smaller rectangular subsections by horizontal and vertical line segments.*
3. *No four subsections meet at the same point.*

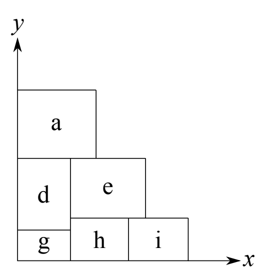

Fig. 4. A staircase with $n = 6$ blocks and $m = 3$ steps that is obtained from the mosaic floorplan in Figure 1 (c) by deleting blocks b, c and f

A *step* of a staircase S is a horizontal line segment on the border of S, excluding the x-axis. Figure 4 shows a staircase with $n = 6$ blocks and $m = 3$ steps. Note that a mosaic floorplan is a staircase with $m = 1$ step.

2.3 Deletable Rectangles

Definition 7. *A* deletable rectangle *of a staircase S is a block that satisfies the following conditions:*

1. *Its top edge is completely contained in the border of S.*
2. *Its right edge is completely contained in the border of S.*

In the staircase shown in Figure 4, the block a is the only deletable rectangle. The concept of deletable rectangles is a key idea for the methods introduced in this paper. This concept was originally defined in [16] for their $(4n-4)$-bit representation of floorplans. However, a modified definition of deletable rectangles is used in this paper to create a $(3n-3)$-bit representation of mosaic floorplans.

Lemma 1. *The removal of a deletable rectangle from a staircase results in another staircase unless the original staircase contains only one block.*

Proof. Let S be a staircase with more than one block and let r be a deletable rectangle in S. Define S' to be the object that results when r is removed from S. Because the removal of r still leaves S' with at least one block, the border of S' still contains a line segment on the x-axis and a line segment on the y-axis, so condition (1) of a staircase holds for S'. Removing r will not cause the remainder of the border to have an increasing line segment because the right edge of r must be completely contained in the border, so condition (2) of a staircase also holds for S'. The removal of r does not form new line segments, so the interior of S' will still be divided into smaller rectangular subsections by vertical and horizontal line segments, and no four subsections in S' will meet at the same point. Thus, conditions (3) and (4) of a staircase hold for S'. Therefore, S' is a staircase.

The basic ideas of the representation can now be outlined. Given a mosaic floorplan M, the deletable rectangles of M are removed one by one. By Lemma 1, this results in a sequence of staircases, until only one block remains. The necessary location information of these deletable rectangles are recorded so that the original mosaic floorplan M can be reconstructed. However, if there are multiple deletable rectangles for these staircases, many more bits will be needed. Fortunately, the following key lemma shows that this does not happen.

Lemma 2. *Let M be a n-block mosaic floorplan in standard form. Let $S_n = M$, and let S_{i-1} ($2 \leq i \leq n$) be the staircase obtained by removing a deletable rectangle r_i from S_i.*

1. *There is a single, unique deletable rectangle in S_i for $1 \leq i \leq n$.*
2. *r_{i-1} is adjacent to r_i for $2 \leq i \leq n$.*

Proof. The proof is by reverse induction.

$S_n = M$ has only one deletable rectangle located in the top right corner.

Assume that S_{i+1} ($i \leq n-1$) has exactly one deletable rectangle r_{i+1}. Let h be the horizontal line segment in S_{i+1} that contains the bottom edge of r_{i+1}, and let v be the vertical line segment in S_{i+1} that contains the left edge of r_{i+1} (Figure 5). Let a be the uppermost block in S_{i+1} whose right edge aligns with v, and let b be the rightmost block in S_{i+1} whose top edge aligns with h. Note that either a or b may not exist, but at least one will exist because $2 \leq i$. After r_{i+1} is removed from S_{i+1}, a and b are the only candidates for deletable rectangles of the resulting staircase S_i. There are two cases:

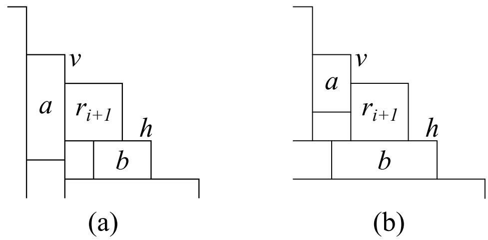

Fig. 5. Proof of Lemma 2

1. The line segments h and v form a $\vdash$-junction (Figure 5 (a)) Then, the bottom edge of a must be below h because M is a standard mosaic floorplan, and a is not a deletable rectangle in S_i. Thus, the block b is the only deletable rectangle in S_i.
2. The line segments h and v form a $\perp$-junction (see Figure 5 (b)) Then, the left edge of b must be to the left of v because M is a standard mosaic floorplan, and b is not a deletable rectangle in S_i. Thus, the block a is the only deletable rectangle in S_i.

In both cases, only one deletable rectangle r_i (which is either a or b) is revealed when the deletable rectangle r_{i+1} is removed. There is only one deletable rectangle in $S_n = M$, so all subsequent staircases contain exactly one deletable rectangle, and (1) is true. In both cases, r_{i+1} is adjacent to r_i, so (2) is true.

Let S be a staircase and r be a deletable rectangle of S whose top side is on the k-th step of S. There are four types of deletable rectangles.

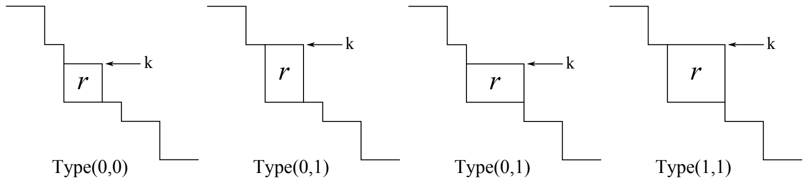

Fig. 6. The four types of deletable rectangles

1. Type $(0, 0)$:
   (a) The upper left corner of r is a $\vdash$-junction.
   (b) The lower right corner of r is a $\perp$-junction.
   (c) The deletion of r decreases the number of steps by one.
2. Type $(0, 1)$:
   (a) The upper left corner of r is a $\top$-junction.
   (b) The lower right corner of r is a $\perp$-junction.
   (c) The deletion of r does not change the number of steps.
3. Type $(1, 0)$:
   (a) The upper left corner of r is a $\vdash$-junction.
   (b) The lower right corner of r is a $\dashv$-junction.
   (c) The deletion of r does not change the number of steps.
4. Type $(1, 1)$:
   (a) The upper left corner of r is a $\top$-junction.
   (b) The lower right corner of r is a $\dashv$-junction.
   (c) The deletion of r increases the number of steps by one.

2.4 Optimal Binary Representation

This binary representation of mosaic floorplans depends on the fact that a mosaic floorplan M is a special case of a staircase and the fact that the removal of a deletable rectangle from a staircase results in another staircase. The binary string used to represent M records the unique sequence of deletable rectangles that are removed in this process. The information stored by this binary string enables the original mosaic floorplan M to be reconstructed.

A 3-bit binary string is used to record the information for each deletable rectangle r_i. The string has two parts: the type and the location of r_i. To record the type of r_i, the bits corresponding to its type is stored directly. To store the location, note that, by Lemma 2, two consecutive deletable rectangles r_i and r_{i-1} are adjacent. Thus, they must share either a horizontal edge or a vertical edge. A single bit can be used to record the location of r_i with respect to r_{i-1}: a 1 if they share a horizontal edge, and a 0 if they share a vertical edge.

Encoding Procedure

Let M be the n-block mosaic floorplan to be encoded. Starting from $S_n = M$, remove the unique deletable rectangles r_i, where $2 \leq i \leq n$, one by one. For each deletable rectangle r_i, two bits are used to record the type of r_i, and one bit is used to record the type of the common boundary shared by r_i and r_{i-1}.

Decoding Procedure

The process starts with the staircase S_1, which is a single rectangle. Each staircase S_{i+1} can be reconstructed from S_i by using the 3-bit binary string for the deletable rectangle r_{i+1}. The 3-bit string records the type of r_{i+1} and the type of edge shared by r_i and r_{i+1}, so r_{i+1} can be uniquely added to S_i. Thus, the decoding procedure can reconstruct the original mosaic floorplan $S_n = M$.

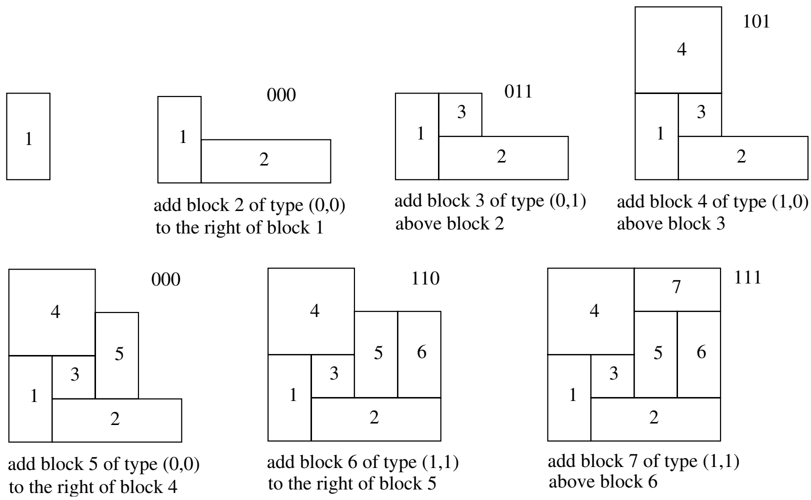

Fig. 7. The decoding of the binary representation (000 011 101 000 110 111)

The lower left block of the mosaic floorplan M (which is the only block of S_1) does not need any information to be recorded. Each of the other blocks of M needs three bits. Thus, the length of the representation of M is $(3n-3)$ bits.

3 Conclusion

In this paper, a binary representation of n-block mosaic floorplans using $(3n-3)$ bits was introduced. Since any representation of n-block mosaic floorplans requires at least $(3n-o(n))$ bits [14], this representation is optimal up to an additive lower term. This representation is very simple and easy to implement.

Mosaic floorplans have a bijection with Baxter permutations, so the optimal representation of mosaic floorplans leads to an optimal $(3n-3)$ bit representation of Baxter permutations and all objects in the Baxter combinatorial family.

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