Problem Set 3
Introduction to Modal Logic

Institute for Logic, Language and Computation
Universiteit van Amsterdam

Due September 28, 2005

1. We say that $\langle \mathfrak{M}, w \rangle$ is bisimilar to $\langle \mathfrak{M}', w' \rangle$ if there exists a bisimulation Z between $\mathfrak{M}$ and $\mathfrak{M}'$ such that wZw'. Show that "being bisimilar to" is a transitive relation on model state pairs. That is, prove that if $\langle \mathfrak{M}_0, w_0 \rangle$ is bisimilar to $\langle \mathfrak{M}_1, w_1 \rangle$ and $\langle \mathfrak{M}_1, w_1 \rangle$ is bisimilar to $\langle \mathfrak{M}_2, w_2 \rangle$, then $\langle \mathfrak{M}_0, w_0 \rangle$ is bisimilar to $\langle \mathfrak{M}_2, w_2 \rangle$. (For the sake of simplicity, use the basic modal language).

2. Exercise 2.2.4, pg. 72. Consider the binary until operator U. In a model $\mathfrak{M} = \langle W, R, V \rangle$ its truth definition reads:

   > $\mathfrak{M}, w \Vdash U(\phi, \psi)$ iff there is a v such that wRv and $\mathfrak{M}, v \Vdash \phi$, and for all u such that wRu and uRv, $\mathfrak{M}, u \Vdash \psi$.

   Prove that u is not definabel in the basic modal language. See page 72 for a hint.

3. Is the following claim true or false?

   > Let $\mathfrak{M}_1$ and $\mathfrak{M}_2$ be two models and suppose that Z and Z' are two bisimulation linking $\mathfrak{M}_1$ and $\mathfrak{M}_2$. Then $Z \cap Z'$ is a bisimulation between $\mathfrak{M}_1$ and $\mathfrak{M}_2$.

   If the claim is true then provide a proof, otherwise provide a counterexample.