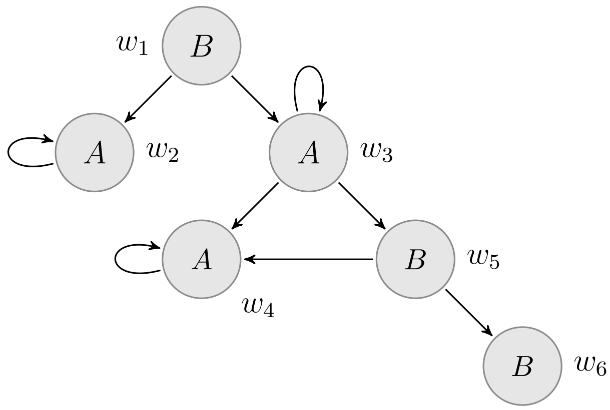
Homework 7
Due 12/5/07

1. Consider the following Kripke structure.



For each formula below, list the states where the formula is true.

(a) $\Box A \rightarrow \Box\Box A$

(b) $\Box\Box A \rightarrow \Box A$

(c) $\Diamond(\Diamond A \wedge \Diamond B)$

(d) $\Diamond\Box(A \wedge \neg A)$

(e) $\Box(\Box A \rightarrow A) \rightarrow \Box A$

2. Using $\Box P$ to mean "the agent knows P" and $\Diamond P$ to mean "it is consistent with what the agent knows that P", translate the following sentences:

(a) If P is true, then it is consistent with what the agent knows that she knows P.

(b) If the agent knows P, then it is consistent with what the agent knows that she knows P

(c) If it is consistent with what the agent knows that P and consistent with what the agent knows that Q, then it is consistent with what the agent knows that P and Q.

Which of these seem plausible principles concerning knowledge and possibility?

3. Consider the following two formula:
   (a) $\Box(\Box P \to P)$
   (b) $\Box P \to \Diamond P$

   For each formula, give an interpretation in English for each of the Alethic, Deontic and Epistemic interpretations of modal logic. For each interpretation, argue whether the formula under consideration is a plausible principle.

4. Recall Aristotle's sea battle argument:
   "If I give the order to attack, then, necessarily, there will be a sea battle tomorrow. If not, then, necessarily, there will not be one. Now, I give the order or I do not. Hence, either it is necessary that there is a sea battle tomorrow or it is necessary that none occurs."

   In class we gave two ways to formalize this argument:

$$\begin{array}{l} A \to \Box B \\ A \to \Box\neg B \\ A \vee \neg A \\ \hline \Box B \vee \Box\neg B \end{array} \qquad\qquad \begin{array}{l} \Box(A \to B) \\ \Box(A \to \neg B) \\ A \vee \neg A \\ \hline \Box B \vee \Box\neg B \end{array}$$

   Explain (informally) whether each argument is valid.

5. Recall the argument that $\Box P \wedge \Box Q \to \Box(P \wedge Q)$ is true at any state in any Kripke structure:

   Suppose that $\Box P \wedge \Box Q$ true at a state w in a Kripke structure. Then both $\Box P$ and $\Box Q$ is true at w. This means that in all accessible worlds, P is true and it is the case that in all accessible worlds Q is true. Therefore, in all accessible worlds both P and Q are true. Hence, $\Box(P \wedge Q)$ is true at w.

   Give a similar argument that $(\Box(P \to Q) \wedge \Box P) \to \Box Q$ is true at any state in any Kripke structure. What about the formula $\Diamond(P \to Q) \wedge \Diamond P) \to \Diamond Q$?

6. Read the two articles "Intelligent Interaction: dynamic trends in today's logic" and "Logic, Rational Agency, and Intelligent Interaction" by Johan van Benthem (available on the course website).