An Invitation to Modal Logic: Lecture 4

Philosophy 150

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Plan

- ✓ Motivating Examples
- ✓ Formalizing the muddy children puzzle, Introduction to Modal Logic
- ✓ More about truth of modal formulas

12/3: Focus on Epistemic Logic.
Digression: A small experiment.

12/5: Multiagent Epsitemic Logic, Dynamics in Logic

12/7: Dynamics in Logic II

Plan for Today

First half of the lecture: Epsitemic Logic.

Second half of the lecture: a small experiment.

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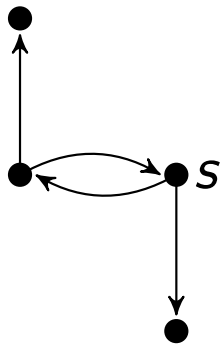
Truth:

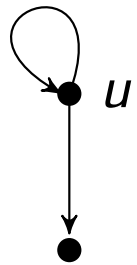
- $w \models \Box P$ iff for all v, if wRv then $v \models P$
- $w \models \Diamond P$ iff there exists v such that wRv and $v \models P$.

Two issues to remember

1. Modal formulas are interpreted locally.



$\mathbb{K}$



$\mathbb{N}$

No modal formula can distinguish between s and u.

Can you think of a first-order formula that can distinguish the Kripke structures?

Two issues to remember

1. Modal formulas are interpreted locally.

2. Modal logic can express interesting properties of Kripke structures.
   - $\Box P \rightarrow P$ corresponds to the reflexivity property.
   - $\Box P \rightarrow \Box\Box P$ corresponds to the transitivity property.

Next lecture: Dynamics in logic.

Questions?
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