Dialogical Logic

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1 Introduction

Dialogue games are one of the earliest examples of games in logic. They were introduced by Lorenzen [1] in the 1950s; since then, major players have been Lorezen's student Lorenz [2], and *his* student Rahman [3–4]. The field seems to have been stagnant for several years, but is enjoying a small-scale revival these days. (Within the LogICCC framework, there is a three-part project, involving master's, Ph.D., and postdoctoral students based in Amsterdam, Tübingen, and Lisbon devoted in large part to dialogues and argumentation; I'll be joining the Lisbon part of this going concern in the fall.)

Dialogues are represented as games involving two players, **P**, who lays down an initial *thesis*, and **O**, who (in some sense) argues against the thesis. Moves are played in turns; after laying down the thesis, **O** plays first. Play continues until one of the players has nothing legitimate to say (the precise winning condition will be defined later).

There are two kinds of moves: one can *attack* an assertion by one of the players, or one can *defend* against an attack.

2 Organization

2.1 Particle Rules

Particle rules govern how the players may respond to previous assertions based on the logical form of those assertions.

Formula	Attack	Defense
$\alpha \wedge \beta$	?L	α
	?R	β
$\alpha \vee \beta$	?	α or β
$\neg\alpha$	α	—
$\alpha \rightarrow \beta$	α	β
$\exists x \alpha$	?	$\phi(c)$
$\forall x \alpha$	c	$\phi(c)$

These are the rules suitable for dialogues involving first-order formulas. For other logics, one needs to provide alternative particle rules.

2.2 Procedural Rules

Procedural rules govern how the players may respond to previous assertions *in general*; except for the first rule, they do not depend on the logical form of assertions:

a. proponent **P** may assert an atomic formula only after opponent **O** has asserted it;
b. a play may defend against only the most recent attack that has not yet been responded to;
c. an attack may be answered at most once;
d. an assertion made by **P** may be attacked at most once.

2.3 Winning

A player *loses* a dialogue game when he can make no further moves in accordance with the particle and structural rules. The other player wins.

3 Examples

To illustrate how dialogue games unfold, consider the following examples.

3.1 First example: a valid formula

1	**P**	$p \wedge \neg(p \wedge q) \rightarrow \neg q$	
2	**O**	$p \wedge \neg(p \wedge q)$	[A,1]
3	**P**	?L	[A,2]
4	**O**	p	[D,3]
5	**P**	?R	[A,2]
6	**O**	¬(p ∧q)	[D,5]
7	**P**	¬q	[D,2]
8	**O**	q	[A,7]
9	**P**	$p \wedge q$	[A,6]
10	**O**	?L	[A,9]
11	**P**	p	[D,10]
12	**O**	?R	[A,10]
13	**P**	q	[D,12]

At this stage, **O** loses.

Proponent **P** can lose even when the formula under discussion is valid. For example, **P** can lay down the valid formula $p \rightarrow (p \vee \neg q)$ and play in such a way that he loses:

1	**P**	$p \rightarrow (p \vee \neg q)$	
2	**O**	p	[A,1]
3	**P**	$p \vee \neg q$	[D,2]
4	**O**	?	[A,3]
5	**P**	$\neg q$	[D,4]
6	**O**	q	[A,5]

When **O** asked **P** at stage 4 what disjunct he had in mind when he assert $p \vee \neg q$ at stage 3, proponent chose $\neg q$. He loses, because **O** can challenge $\neg q$, to which **P** has no response. If **P** had chosen p instead of $\neg q$, he could have won the game.

3.2 Second example: a contingent formula

Consider the following example of a contingent (satisfiable but not valid) formula. (This example is taken from the entry on dialogical logic [5]in the *Stanford Encyclopedia of Philosophy*.)

1	**P**	$(p \vee q) \rightarrow p$	
2	**O**	$p \vee q$	[A,1]
3	**P**	?	[A,2]
4	**O**	p	[D,3]
5	**P**	p	[D,2]

P wins this game. But had **O** chosen to respond to the attack in stage 3 by choosing q instead of p, **P** would have lost.

3.3 Third example: double negation laws and excluded middle

The structural rules give dialogical games a close connection with intuitionistic logic. For example, consider the intuitionistically valid principle $p \rightarrow \neg\neg p$. Then:

1	**P**	$p \rightarrow \neg\neg p$	
2	**O**	p	[A,1]
3	**P**	$\neg\neg p$	[D,2]
4	**O**	$\neg p$	[A,3]
5	**P**	p	[A,4]

O cannot continue playing, so he loses.

Contrast this with the converse formula $\neg\neg p \rightarrow p$.

1	**P**	$\neg\neg p \rightarrow p$	
2	**O**	$\neg\neg p$	[A,1]
3	**P**	$\neg p$	[A,2]
4	**O**	p	[A,3]

P would like to assert p, to defend against **O**'s attack in stage 2. But the structural rules require that **P** respond to the *last* attack. **O** wins.

P could have won this game if we were to relax the structural rules in such a way that he is allowed to return to earlier, unanswered attacks. In that case, the previous game could be continued:

5	**P**	p	[D,2]

In this modified game, the tables have turned and **P** wins, since **O** cannot make any further moves.

Finally, consider the principle of the excluded middle, $p \vee \neg p$. How might a game for that go?

1	**P**	$p \vee \neg p$	
2	**O**	?	[A,1]
3	**P**	$\neg p$	[D,2]
4	**O**	p	[A,3]

The procedural rules prevent **P** from advancing any further; he loses. However, if the structural rules were relaxed in such a way that **P** would be allowed to respond to *earlier* attacks by **O**, then the game could be continued: at stage 5, **P** could assert p as a defense against the attack in stage 2, thereby winning.

4 Results about Dialogues

Theorem: For first-order formulas $\phi_1, \ldots, \phi_n, \psi$, **P** has a winning strategy in the dialogue game for $\phi_1 \wedge \cdots \wedge \phi_n \rightarrow \psi$ iff $\phi_1 \wedge \cdots \wedge \phi_n \rightarrow \psi$ is valid in intuitionistic logic.

A more explicit form of the theorem is available:

Theorem: There exist effective procedures that take intuitionistic deductions to winning strategies for **P** in dialogue games, and vice versa.

5 Comments

Dialogue games, as presented here, are intended to be a mathematical model of a certain kind of stylized discussion. Even acknowledging the utility of the model (as given by the preceding results), the dialogues nonetheless still feel somewhat tainted by the artifice of the structural rules.

It strikes me as somewhat unpleasant that the rules can conflict with one another: for exaple, it seems naturl to lay down the particule rules for disjunction: to attack a disjunction is to ask for one of the disjuncts to be argued for, and to defend against such an attack amounts to choosing one of the disjuncts and continuing with the dialogue. This particule rules seems valid. But notice, in the case of the excluded middle example, that **P** could *not* freely choose which disjunct to be defended, because one of them was atomic, and one of the structural rules says that atomic formulas cannot be asserted by **P** unless they were first conceded by **O**.

(Not that it would have helped matters had **P** chosen p rather than $\neg p$ in the dialogue game for the excluded middle $p \vee \neg p$; intuitively, **O** still would have won. I'm using this example to illustrate what I take to be a conflict between two plausible rules.)

6 Further Directions

Without claiming that the following directions for further research are unexplored (surely they all are), the following topics strike me as especially interesting.

6.1 Structure theory

Recall the examples illustrating how the plain structural rules give rise to intuitionistic logic, and how small adjustments give rise to classical logic. These strike me as 'accidental' results of rule fiddling in the sense that they do not seem to have anything to do with the difference between intuitionistic and classical logic.

There are results from the SEP entry on dialogical logic [5]that further illustrate how small variations with the structural rules give rise to subtly different logics (ones with or without weakening, contraction, and exchange).

It would be satisfying if there were a general *structural theory* of dialogues which helped us to understand the logical landscape that unfolds as we vary the dialogical rules.

6.2 Extension: dialogues involving knowledge and belief

The dialogues so far have focused on assertions; the players lay down claims and attack of defend them. In addition to dialogues of 'pure' statements, one could explore other dimensions by investigating the role of knowledge and belief. One could explore dialogues in which the assertions are allowed to contain formulas such as $K\,\phi$ or $B\,\psi$; perhaps the modalities could be subscripted to refer to the discussants. One would then need to investigate particle and structural rules to govern dialogue involving such statements.

6.3 Inversion, rigor, and computer-aided formal proofs

Given a sequence of assertions, can we understand it as the record of a dialogue? In other words, given a sequence of assertions, can we infer a dialogue (if any) from which it came?

Carrying out such an 'inverse analysis' on, say, informal mathematical arguments might provide an understanding of what kinds of attacks the mathematician wants to defend against. Such a project might help us to give a dialogue-theoretic model of rigorous argumentation. It would tell us what kind of particle and structural rules are important. (The inspiration for this kind of idea comes from Avigad [6–7].)

One can see the process of formalizing proofs using a mechanical proof checker as a dialogue between a human formalizer and an artificial checker. The human formalizer submits an argument to the checker, which attempts to certify its validity; when the result is found to be invalid, the checker reports gaps or otherwise unchecked steps, which the formalizer must repair. The process aims at a valid formal proof, not unlike dialogues.

6.4 Zero-knowledge proofs

The concept of a zero-knowledge proof arose in the context of cryptography, but one can study it as a certain kind of dialogical interaction between two agents, one of whom, A, has to convince the other, B, that A has a certain kind of knowledge without revealing what the knowledge is. The term 'zero knowledge' comes from the fact that B can know (or believe with a very high degree of reliability) that A knows something without actually knowing himself what that is.

One might wish to investigate to what extent zero-knowledge proofs can be understood as dialogues intertwined with epistemic elements of (implicit and explicit) knowledge and belief. The aim is not so much to contribute to cryptography, but to understand features of dialogues that permit zero-knowledge to take place.

7 References

[1] P. Lorenzen *Logik und agon*, in *Arti del XII Congresso Internationale de Filosofia* (1958), pp. 187–194.

[2] P. Lorenzen and K. Lorenz, *Dialogische Logik.* (Wissenschaftliche Buchgesellschaft, 1978).

[3] S. Rahman and L. Keiff, *On how to be a dialogician: A short overview on recent developments on dialogues and games.* In *Logic, Thought and Action* (Springer, 2005), p. 359–408.

[4] S. Rahman and T. Tulenheimo *From games to dialogues and back: towards a general frame for validity*, in *Games: Unifying Logic, Language, and Philosophy*, edited by O. Majer, A. Pietarinen, and T. Tulenheimo (2007), pp. 153–208.

[5] L. Keiff, *Dialogical logic.* In *The Stanford Encyclopedia of Philosophy*, edited by E. N. Zalta (Spring 2009). Available online at http://plato.stanford.edu/archives /spr2009/entries/logic-dialogical/.

[6] J. Avigad, *Mathematical method and proof*, Synthese **153** (2006), 105–159

[7] J. Avigad, *Understanding proofs.* In *The Philosophy of Mathematical Practice* .