Preference (Modal) Logics

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Preference Modalities \( \langle \succeq \rangle \phi \): “there is a world at least as good (as the current world) satisfying \( \phi \)”

\[ \mathcal{M}, w \models \langle \succeq \rangle \phi \text{ iff there is a } v \succeq w \text{ such that } \mathcal{M}, v \models \phi \]
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Preference Modalities $\langle \succeq \rangle \varphi$: "there is a world at least as good (as the current world) satisfying $\varphi$"

$\mathcal{M}, w \models \langle \succeq \rangle \varphi$ iff there is a $v \succeq w$ such that $\mathcal{M}, v \models \varphi$

$\mathcal{M}, w \models \langle \succ \rangle \varphi$ iff there is $v \succeq w$ and $w \not\succeq v$ such that $\mathcal{M}, v \models \varphi$
Preference Lifting, I

Given a preference ordering $\preceq$ over a set of objects $X$, we want to **lift** this to an ordering $\preceq' \supseteq$ over $\mathcal{P}(X)$.

Given $\preceq$, what reasonable properties can we infer about $\preceq'$?

You know that $x \prec y \prec z$
Can you infer that $\{x, y\} \hat{\prec} \{z\}$?
Preference Lifting, II

- You know that \( x \prec y \prec z \)
  Can you infer that \( \{x, y\} \hat{\prec} \{z\} \)?

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  Can you infer anything about \( \{y\} \) and \( \{x, z\} \)?
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- You know that $w \prec x \prec y \prec z$
  Can you infer that $\{w, x, y\} \hat{\leq} \{w, y, z\}$?
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Can you infer that $\{w, x, y\} \hat{\preceq} \{w, y, z\}$?

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Can you infer that $\{w, x\} \hat{\prec} \{y, z\}$?
Preference Lifting, III

There are different interpretations of $X \lesssim Y$:

- You will get one of the elements, but cannot control which.
- You can choose one of the elements.
- You will get the full set.
Preference Lifting, IV

Kelly Principle

(EXT) $\{x\} \prec \supseteq \{y\}$ provided $x \prec y$

(MAX) $A \prec \text{Max}(A)$

(MIN) $\text{Min}(A) \prec A$

Preference Lifting, IV

Gärdenfors Principle

\[(G1) \quad A \hat{\succ} A \cup \{x\} \text{ if } a \prec x \text{ for all } a \in A\]
\[(G2) \quad A \cup \{x\} \hat{\succ} A \text{ if } x \prec a \text{ for all } a \in A\]

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Independence

\[(\text{IND}) \quad A \cup \{x\} \preceq B \cup \{x\} \text{ if } A \preceq B \text{ and } x \notin A \cup B\]
Theorem (Kannai and Peleg). If $|X| \geq 6$, then no weak order satisfies both the Gärdenfors principle and independence.

From Worlds to Sets, I

\[ M, w \models \varphi \leq \exists \psi \text{ iff there is } s, t \text{ such that } M, s \models \varphi \text{ and } M, t \models \psi \text{ and } s \preceq t \]
From Worlds to Sets, I

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From Worlds to Sets, I

$\mathcal{M}, w \models \varphi \leq_{\exists \exists} \psi$ iff there is $s, t$ such that $\mathcal{M}, s \models \varphi$ and $\mathcal{M}, t \models \psi$ and $s \leq t$

$\mathcal{M}, w \models \varphi \prec_{\exists \exists} \psi$ iff there is $s, t$ such that $\mathcal{M}, s \models \varphi$ and $\mathcal{M}, t \models \psi$ and $s < t$

$\mathcal{M}, w \models \varphi \leq_{\forall \exists} \psi$ iff for all $s$ there is a $t$ such that $\mathcal{M}, s \models \varphi$ implies $\mathcal{M}, t \models \psi$, and $s \leq t$
From Worlds to Sets, I

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From Worlds to Sets, II

\[ \varphi \leq E \exists \psi := E (\varphi \land \Box \preceq \psi) \]
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\( \varphi \preceq \exists \psi \; := \; E(\varphi \land \Diamond \preceq \psi) \)

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\[ \varphi \prec \forall \exists \psi := A(\varphi \rightarrow \lozenge \prec \psi) \]
\( \mathcal{M}, w \models \varphi \preceq\forall \psi \) iff for all \( s, t \), \( \mathcal{M}, s \models \varphi \) and \( \mathcal{M}, t \models \psi \) implies \( s \preceq t \)
From Worlds to Sets, III

\[ \mathcal{M}, w \models \varphi \preceq \forall \forall \psi \text{ iff for all } s, \text{ for all } t, \mathcal{M}, s \models \varphi \text{ and } \mathcal{M}, t \models \psi \]
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implies \( s \prec t \)
From Worlds to Sets, IV

$$\varphi \preceq \forall \psi \; : = \; A(\psi \rightarrow \square \preceq \neg \varphi)$$
From Worlds to Sets, IV

\[ \varphi \preceq \forall \psi := A(\psi \rightarrow \Box \preceq \neg \varphi) \]

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From Worlds to Sets, IV

\[ \varphi \leq \forall \psi := A(\psi \rightarrow \Box \preceq \neg \varphi) \]

\[ \varphi \prec \forall \psi := A(\psi \rightarrow \Box \prec \neg \varphi) \]

*We must assume the ordering \( \preceq \) is total*
From Sets to Worlds

\[ P_1 \gg P_2 \gg P_3 \gg \cdots \gg P_n \]

\( x > y \) iff \( x \) and \( y \) differ in at least one \( P_i \) and the first \( P_i \) where this happens is one with \( P_i x \) and \( \neg P_i y \)

Once a semantics and language are fixed, then standard questions can be asked: eg. develop a proof theory, completeness, decidability, model checking.
General Issues

How should we *compare* the different logical systems?

- Embedding one logic in another:

  - Coalition logic is a fragment of ATL ($\langle C \phi \rangle = \langle\langle C \rangle\rangle_{\tau} \phi$)
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- Embedding one logic in another: \textit{coalition logic} is a fragment of ATL ($tr([C] \varphi) = \langle\langle C\rangle\rangle \circ \varphi$).

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▶ Compare different models for a fixed language:
  - Alternating-Time Temporal Logics: Three different semantics for the ATL language.


▶ Comparing different frameworks: eg. PDL vs. Temporal Logic, PDL vs. STIT, STIT vs. ATL, etc.
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How should we *merge* the different logical systems?

- Combining logics is hard! ▶ Explain


**Theorem** $\square \varphi \leftrightarrow \varphi$ is provable in combinations of Epistemic Logics and PDL with certain “cross axioms” ($\square[a]\varphi \leftrightarrow [a]\square \varphi$) (and full substitution).

Merging Logics of Rational Agency

- Entangling Knowledge/Beliefs and Preferences
- “Epistemizing” Logics of Action and Ability
- BDI (Belief + Desires + Intentions) Logics
Logics of Knowledge and Preference

\( K(\varphi \succeq \psi) \): “Ann knows that \( \varphi \) is at least as good as \( \psi \)”

\( K\varphi \succeq K\psi \): “knowing \( \varphi \) is at least as good as knowing \( \psi \)”
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J. van Eijck. Yet more modal logics of preference change and belief revision. manuscript, 2009.

$A(\psi \rightarrow \langle \preceq \rangle \varphi)$ vs. $K(\psi \rightarrow \langle \preceq \rangle \varphi)$
General Issues

\[ A(\psi \to \langle \preceq \rangle \varphi) \quad \text{vs.} \quad K(\psi \to \langle \preceq \rangle \varphi) \]

Should preferences be restricted to information sets?
$A(\psi \rightarrow \langle \succeq \rangle \varphi) \text{ vs. } K(\psi \rightarrow \langle \succeq \rangle \varphi)$

Should preferences be restricted to information sets?

$\mathcal{M}, w \models \langle \succeq \cap \sim \rangle \varphi$ iff there is a $v$ with $w \sim v$ and $w \preceq v$ such that $\mathcal{M}, v \models \varphi$

$K(\psi \rightarrow \langle \succeq \cap \sim \rangle \varphi)$
Merging Logics of Rational Agency

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Consider the following game: Two cards, Ace and Joker, lie face down and the agent $i$ must choose one. The Ace wins, the Joker loses.
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