Social Choice Theory for Logicians

Lecture 1

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ai.stanford.edu/~epacuit/classes/nasslli2012/logsoc.html
Context of Decision Making

- Individual decision making and individual action against nature.
  - Example: gambling.
- Individual decision making in interaction.
  - Example: playing chess.
- Collective decision making.
  - Example: carrying a piano, voting.
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Main Question

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the “will of the group”? 
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Typical Examples:

- Electing government officials
- Department meetings
- Deciding where to go to dinner with friends
- ....
Which candidate *should* be chosen?

<table>
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<th>3</th>
<th>5</th>
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<tbody>
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A few observations:

- More people rank A first than any other candidate
Which candidate *should* be chosen?

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A few observations:
- More people rank A first than any other candidate
- But, a stronger majority ranks A last
Which candidate *should* be chosen?

Marquis de Condorcet (1743 - 1794)  VS.  Jean-Charles de Borda (1733 - 1799)
Which candidate *should* be chosen?

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A few observations:

- More people rank A first than any other candidate
- In pairwise elections, C beats every other candidate (C is the Condorcet winner)
- B and C are the only candidates not ranked last by anyone
A few observations:

- More people rank A first (last) than any other candidate
- In pairwise elections, C beats every other candidate (C is the Condorcet winner)
- Taking into account the entire ordering, B has the most “support” (B is the Borda winner)
Which candidate *should* be chosen?

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A few observations:
- More people rank A first (last) than any other candidate
- In pairwise elections, C beats every other candidate (C is the *Condorcet winner*)
- B gets 13 (vs. A) + 10 (vs. C) + 21 (vs. D) = 44 points
Which candidate *should* be chosen?

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- In pairwise elections, C beats every other candidate (C is the Condorcet winner)
- B gets 13 (vs. A) + 10 (vs. C) + 21 (vs. D) = 44 points
- C gets 13 (vs. A) + 11 (vs. B) + 14 (vs. D) = 38 points
Which candidate *should* be chosen?

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Which candidate *should* be chosen?

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Conclusion: *many ways to answer the above question!*
Choosing How to Choose
Choosing How to Choose

Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Condorcet-consistent methods based on the simple majority graph (e.g., Cup Rule/Voting Trees, Copeland, Banks, Slater, Schwartz, and the basic Condorcet rule itself), rules based on the weighted majority graph (e.g., Maximin/Simpson, Kemeny, and Ranked Pairs/Tideman), rules requiring full preference information (e.g., Bucklin, Dodgson, and Young); Majoritarian Judgment; Cumulative Voting; Range Voting

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Choosing How to Choose

**Plurality Vote**: Each voter selects one candidate (or none if voters can abstain) and the candidate(s) with the most votes win.

**Plurality with Runoff**: If there is a candidate with an absolute majority then that candidate wins, otherwise the top two candidates move on to round two. The candidate with the most votes in the second round wins.
Choosing How to Choose

**Approval Voting**: Each voter selects a *subset* of the candidates (empty set means the voter abstains) and the candidate(s) with the most votes win.

**Borda Count**: Each voter provides a linear ordering of the candidates. The candidate(s) with the most total *points* wins, where points are calculated as follows: if there are $n$ candidates, $n - 1$ points are given to the highest ranked candidates, $n - 2$ to the second highest, etc..
How should we judge different social decision methods?

Pragmatic concerns: Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

Behavioral considerations: Do the different procedures really lead to different outcomes in practice?

Information required from the voters: What type of information do the ballots convey? Eg., Choosing a single alternative, linearly rank all the alternatives, report something about the “intensity” of preference.

Axiomatics: Characterize the different social decision methods in terms of normative principles of group decision making.
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What properties do we want?

- **Condorcet Candidate**: Always choose the candidate that beats every other candidate in head-to-head elections.
- **Monotonicity**: A candidate receiving more support shouldn’t make her worse off.
- **Independence**: The winner should not depend on “irrelevant” spoiler candidates.
- The outcome of a vote should not be “surprising” given the data.
What properties do we want?

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## Condorcet Paradox

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Does the group prefer **A** over **B**? Yes

Does the group prefer **B** over **C**? Yes

Does the group prefer **A** over **C**? No

*(this conflict with transitivity)*
**Condorcet Paradox**

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- Does the group prefer A over B? Yes
- Does the group prefer B over C? Yes
- Does the group prefer A over C? No
  (this conflicts with **transitivity**)
Failure of monotonicity: plurality with runoff

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Winner: A

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Winner: C
## Failure of monotonicity: plurality with runoff

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Winner: C

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**Eric Pacuit: The Logic Behind Voting**
Failure of monotonicity: plurality with runoff

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Winner: A

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Winner: C

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Winner: C

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Winner: C
Failure of monotonicity: plurality with runoff

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**Winner:** A

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**Winner:** C
## No-show paradox

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B: $417 + 82 = 499$

H: $143 + 357 = 500$

W: $285 + 324 = 609$
## No-show paradox

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H Wins
No-show paradox

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Suppose two more people show up with the ranking B H W
No-show paradox

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B: \(419 + 82 = 501\)
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W Wins!
Failure of Independence

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The BC ranking is:

\[
A > B > C
\]

Add a new (undesirable) candidate X

The new BC ranking is:

\[
C > B > A > X
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Failure of Independence

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- The BC ranking is: $A (8) > B (7) > C (6)$
Failure of Independence

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- The BC ranking is: $A (8) > B (7) > C (6)$
- Add a new (undesirable) candidate $X$
- The new BC ranking is: $C (13) > B (12) > A (11) > X (6)$
Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

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Outcome by majority vote

Proposition 1: N (7 - 6)

Proposition 2: N (7 - 6)

Proposition 3: N (7 - 6)

But there is no support for NNN

## Multiple Elections Paradox

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**Outcome by majority vote**

**Proposition 1:** \( N \ (7 - 6) \)

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Outcome by majority vote

**Proposition 1:** \(N\) (7 - 6)
**Proposition 2:** \(N\) (7 - 6)
**Proposition 3:** \(N\) (7 - 6)

---

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

<table>
<thead>
<tr>
<th>YYY</th>
<th>YYN</th>
<th>YNY</th>
<th>YNN</th>
<th>NYY</th>
<th>NYN</th>
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<th>NNN</th>
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<td>1</td>
<td>1</td>
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</table>

Outcome by majority vote

Proposition 1: $N (7 - 6)$
Proposition 2: $N (7 - 6)$
Proposition 3: $N (7 - 6)$

But there is no support for NNN!

## Anscombe’s Paradox

<table>
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<tr>
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</tr>
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</tr>
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<td>Voter 4</td>
<td>Yes</td>
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</tr>
<tr>
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<td>Yes</td>
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<td>Yes</td>
</tr>
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Voters 4 & 5 support the outcome on a majority of issues.

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Voters 4 & 5 support the outcome on a majority of issues

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Voters 4 & 5 support the outcome on a majority of issues.
Voters 1,2 & 3 do not support the outcome on a majority of issues.

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- Voters 4 & 5 support the outcome on a majority of issues.
- Voters 1, 2 & 3 do not support the outcome on a majority of issues.

A *majority of voters do not support the majority outcome on a majority of issues.*

---

What properties do we want?

- **Condorcet Candidate**: Always choose the candidate that beats every other candidate in head-to-head elections.

- **Monotonicity**: A candidate receiving more support shouldn’t make her worse off.

- **Independence**: The winner should not depend on “irrelevant” spoiler candidates.

- The outcome of a vote should not be “surprising” given the data.
Arrow’s Theorem


Also, see


Sen’s Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley’s Lover*, consider
Sen’s Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley’s Lover*, consider

\[ l \]: Lewd reads the book;
\[ p \]: Prude reads the book;
\[ l \rightarrow p \]: If Lewd reads the book, then so does Prude.
Sen’s Liberal Paradox
Sen’s Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude’s moral outlook being corrupted)
Sen’s Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude’s moral outlook being corrupted)

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.
### Sen’s Liberal Paradox

<table>
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<tr>
<th>l</th>
<th>p</th>
<th>l → p</th>
</tr>
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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere.  
   - l is Lewd's case,  
   - p is Prude's case.

2. Unanimous desires of all individuals must be respected.  
   - So, society must be inconsistent!
Sen’s Liberal Paradox

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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere, e.g., Lewd’s case, Prude’s case.

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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere. \( l \) is Lewd’s case, \( p \) is Prude’s case.

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Muller-Satterthwaite Theorem
