Rationality: Two Themes

Rationality is a matter of reasons:

▶ Whether a belief $P$ is rational depends on the reasons for holding $P$

▶ Whether an act $\alpha$ is rational depends on the reason for doing $\alpha$

Rationality is a matter of reliability:

▶ A rational belief is one that is arrived at through a process that reliably produces beliefs that are true.

▶ A act is rational if it is arrived at through a process that reliably achieves specified goals.
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Rationality is a matter of **reasons**:

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- Whether an act \( \alpha \) is *rational* depends on the *reason for doing* \( \alpha \)
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Rationality: Two Themes

“Neither theme alone exhausts our notion of rationality. Reasons without reliability seem empty, reliability without reasons seems blind. In tandem these make a powerful unit, but how exactly are they related and why?”

(Nozick, pg. 64)
Instrumental Rationality

“The notion of instrumental rationality is a powerful and natural one...
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Instrumental Rationality I

What does it mean to be *instrumentally rational*?
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**Rationality as Effectiveness**: Ann’s action $\alpha$ is instrumentally rational iff Ann’s $\alpha$-ing is an effective way for Ann to achieve her goal, desire, end or taste $G$.

Too narrow: Bob checks the forecast with on the local news, weather.com and the local newspaper. They all concur that it will be a gorgeous day. So, Bob leaves without an umbrella and gets soaked in a freak rainstorm.

Too broad: Charles never checks weather reports, but does consult her Ouiji board. On the day that Bob got soaked, Charles' Ouiji board told him to take an umbrella, so he stayed dry. We need to take the agent's beliefs into account.
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*We need to take the agent’s beliefs into account*
Subjective Rationality: Ann’s action $\alpha$ is instrumentally rational iff when she chooses $\alpha$: (1) her choice was based on her beliefs ($B$) and (2) if $B$ were true beliefs, then $\alpha$ would be an effective way to achieve her goals, desires, tastes, etc.
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What constraints should be placed on reasonable beliefs that underlie a rational choice?
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Can goals be irrational?

Hume: Our reason cannot tell us what to desire, so no desire can ever be against reason.

'Tis not contrary to reason to prefer the destruction of the whole world to the scratching of my finger...

Does this mean that “anything goes”? 

▶ constraints on how preferences “hang together”

• transitivity, completeness, etc.

• “a person shows herself to lack “rational integration” if she has some desire for x, yet also desires not to desire x” (Nozick, pg. 139 - 151)

▶ the ultimate goal is happiness, other desires are the manifestation of the pursuit of happiness or pleasure.
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1. the person is willing to switch to preferring y to x for a small gain, or

2. the person has some reason to prefer x to y, or

3. the person has some reason to prefer preferring x to y to not doing that.
Preferences, Desires, Goals

The person's preferences and desires are in equilibrium (with her beliefs about their causes).

The person does not have desires that she knows are impossible to fulfill.

A person will not have a goal for which she knows that there is no feasible route, however long, for her current situation to the achievement of that goal.

Some goals are stable (recall Bratman on plans).

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R. Nozick. “\textit{Rational Preferences}”. in The Nature of Rationality, pgs. 139 - 151.
Economic Rationality

Can we characterize *Homo Economicus* simply in terms of instrumental rationality?

Eg., Ann is eating ice cream.
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*Consumption Rationality*: Ann’s action $\alpha$ is “consumptively rational” only if it is an instance of the $\alpha$-type — a general desire, value, or end of hers.

*Economic Rationality* Ann’s action $\alpha$ is economically rational only if it is (a) instrumentally rational or (b) consumptively rational.
What are preferences?

Preferring or choosing $x$ is different than “liking” $x$: one can prefer $x$ to $y$ but dislike both options.

In utility theory, preferences are always understood as comparative: “preference” is more like “bigger” than “big”.

**Revealed Preferences**: Ann is said to have a preference for $x$ over $y$ iff Ann chooses $x$ over $y$ where choice is conceived of as overt behavior.

**Deliberative Preferences**: A person deliberates and (ideally) ranks all the possible “outcomes”.
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Are preferences over *outcomes* or *options*?
Preliminaries: Orderings

An ordering is a *relation* $R$ on a set $X$: a subset of the set of pairs of elements from $X$: $R \subseteq X \times X$

Write $aRb$ iff $(a, b) \in R$
Preliminaries: Orderings

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Write \( aRb \) iff \((a, b) \in R\)

Properties of orderings:

- Reflexivity: for all \( a \in X \), \( aRa \)
- Transitivity: for all \( a, b, c \in X \), \( aRb \) and \( bRc \) then \( aRc \)
- Symmetry: for all \( a, b \in X \), \( aRb \) implies \( bRa \)
- Asymmetry: for all \( a, b \in X \), \( aRb \) implies not-\( bRa \)
- Completeness: for all \( a, b \in X \), \( aRb \) or \( bRa \) (or \( a = b \))
Preliminaries: Orderings

Let $X$ be the set of outcomes (or options) and $\succeq$ an ordering ($\succeq \subseteq X \times X$).
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Given two outcomes $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \not\succeq x$: The agent strictly prefers $x$ to $y$ ($x \succ y$).
2. $y \succeq x$ and $x \not\succeq y$: The agent strictly prefers $y$ to $x$ ($y \succ x$).
3. $x \succeq y$ and $y \succeq x$: The agent is indifferent between $x$ and $y$ ($x \approx y$).
4. $x \not\succeq y$ and $y \not\succeq x$: The agent cannot compare $x$ and $y$ ($x \perp y$).
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What properties does this preference ordering have?
Ordinal Utility Theory: Axioms

1. The ordering is complete: the agent can always rank options (for any two options $x$ and $y$, either (1) the agent strictly prefers $x$ to $y$, (2) strictly prefers $y$ to $x$ or (3) is indifferent between $x$ and $y$).

2. Strict preference is asymmetric: it is not the case that the agent strictly prefers $x$ to $y$ and strictly prefers $y$ to $x$.

3. Weak preference is reflexive: the agent always thinks $x$ is at least as good as $x$.

4. Weak preference (and hence strict and indifference) is transitive.

Why should we accept these axioms?
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*Why should we accept these axioms?*
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“Rather than trying to provide instrumental or pragmatic justifications for the axioms of ordinal utility, it is better...to see them as constitutive of our conception of a fully rational agent....those disposed to blatantly ignore transitivity are unintelligible to use: we can’t understand their pattern of actions as sensible”

(Gaus [OPPE], pg. 39)
Ordinal Utility Theory

**Fact.** Suppose that $X$ is finite and $\succeq$ is a complete and transitive ordering over $X$, then there is a utility function $u : X \rightarrow \mathbb{R}$ that represents $\succeq$ ($x \succeq y$ iff $u(x) \geq u(y)$).
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**Important point:** consider $x \succ y \succ z$
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**Important point:** consider $x \succ y \succ z$, all three utility functions represent this ordering:

<table>
<thead>
<tr>
<th>Preference</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>3</td>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>$y$</td>
<td>2</td>
<td>5</td>
<td>99</td>
</tr>
<tr>
<td>$z$</td>
<td>1</td>
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</table>
Cardinal Utility Theory

\( x \succ y \succ z \) is represented by both \((3, 2, 1)\) and \((1000, 999, 1)\), so cannot say \( y \) is “closer” to \( x \) than to \( z \).
$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so cannot say $y$ is “closer” to $x$ than to $z$.

Key idea: Ordinal preferences over lotteries allows us to infer a cardinal scale (with some additional axioms).
Axioms of Cardinal Utility

Suppose that $X$ is a set of outcomes and consider lotteries over $X$ (i.e., probability distributions over $X$).
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Suppose that $X$ is a set of outcomes and consider **lotteries over** $X$ (i.e., probability distributions over $X$)

A **compound lottery** is $\alpha L + (1 - \alpha)L'$ meaning “play lottery $L$ with probability $\alpha$ and $L'$ with probability $1 - \alpha$”

Running example: Suppose Ann prefers pizza ($p$) over taco ($t$) over yogurt ($y$) ($p \succ t \succ y$) and consider the different lotteries where the prizes are $p$, $t$ and $y$. 
Cardinal Utility Theory: Continuity

**Continuity**: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1 - p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$. 

Suppose Ann has $t$. Consider the lottery $L = 0.99$ get $y$ and $0.01$ get $p$. Would Ann trade $t$ for $L$?

Consider the lottery $L' = 0.99$ get $p$ and $0.01$ get $y$. Would Ann trade $t$ for $L'$?

Continuity says that there must be some lottery where Ann is indifferent between keeping $t$ and playing the lottery.
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Continuity says that there is must be some lottery where Ann is indifferent between keeping $t$ and playing the lottery.
**Better Prizes**: suppose $L_1$ is a lottery over $(w, x)$ and $L_2$ is over $(y, z)$ suppose that $L_1$ and $L_2$ have the same probability over prizes. The lotteries each have an equal prize in one position they have unequal prizes in the other position then if $L_1$ is the lottery with the better prize then $L_1 \succ L_2$; if neither lottery has a better prize then $L_1 \approx L_2$. 
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Lottery 1 ($L_1$) is 0.6 chance for $p$ and 0.4 chance for $y$
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Since Ann prefers $p$ to $t$, this axiom says that Ann prefers $L_1$ to $L_2$
Cardinal Utility Theory: Better Chances

**Better Chances:** Suppose $L_1$ and $L_2$ are two lotteries which have the same prizes, then if $L_1$ offers a better chance of the better prize, then $L_1 \succ L_2$
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This axioms states that Ann must prefer $L_1$ to $L_2$
Reduction of Compound Lotteries: If the prize of a lottery is another lottery, then this can be reduced to a simple lottery over prizes.
Cardinal Utility Theory: Reduction of Compound Lotteries

**Reduction of Compound Lotteries**: If the prize of a lottery is another lottery, then this can be reduced to a simple lottery over prizes.

This eliminates utility from the thrill of gambling and so the only ultimate concern is the prizes.
Cardinal Utility Theory

**Von Neumann-Morgenstern Theorem.** If an agent satisfies the previous axioms, then the agents ordinal utility function can be turned into cardinal utility function.

Utility is unique only up to linear transformations. So, it still does not make sense to add two different agents cardinal utility functions.

Issue with continuity: $1\text{ EUR} \succ 1\text{ cent} \succ \text{death}$, but who would accept a lottery which is $p$ for 1EUR and $(1-p)$ for death?

Deep issues about how to identify correct descriptions of the outcomes and options.
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Issue with Better Prizes

Suppose you have a kitten, which you plan to give away to either Ann or Bob. Ann and Bob both want the kitten very much. Both are deserving, and both would care for the kitten. You are sure that giving the kitten to Ann ($x$) is at least as good as giving the kitten to Bob ($y$) (so $x \succeq y$). But you think that would be unfair to Bob. You decide to flip a fair coin: if the coin lands heads, you will give the kitten to Bob, and if it lands tails, you will give the kitten to Ann. (J. Drier, “Morality and Decision Theory” in [HR])
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Eric Pacuit: Rationality (Lecture 11)
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Why does this contradict better prizes? consider the lottery which is \(x\) for sure \((L_1)\) and the lottery which is 0.5 for \(y\) and 0.5 for \(x\) \((L_2)\). Better prizes implies \(L_1 \succeq L_2\) but a person concerned with fairness may have \(L_2 \succeq L_1\). But if fairness is important then that should be part of the description of the outcome!
Next week: more about utility theory