Rationality

Lecture 15

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Shared cooperative activity

What is a team?

Any group?

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- Surely not. But interesting phenomena at this level already.

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Then a group with:

i A certain (hierarchical) structure?

ii Whose members identify with the group (c.f. Gold 2005)?

  - Information about who's in and who's out.
  - Reasoning and acting as group members.

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iii. Team- or group objectives/aims/preferences?
    - Shared by the members?

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iv Shared commitments? (Bratman, 1999, Gilbert 1989, Tuomela, 2007)

  - Shared intentions.
  - Sanctions for lapsing?
  - Shared praise[blame] for success[failure]?

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v Common knowledge (beliefs?) of (i-iv)?

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iv. Shared commitments? (Bratman, 1999, Gilbert 1989, Tuomela, 2007)
v. Common knowledge (beliefs?) of (i-iv)?

Note: *None of these are necessary conditions*!

What is a team?

Acting as a team (at least) involves:

- Adopting the team's preferences. (Preference transformation).
- Team-reasoning (Agency Transformation).

What is a team?

1. Group identification.
    - Information about who's in and who's out.
    - Reasoning as group members.
    - Shared goal.
        - Group preference / utilities.
2. Shared commitments.
    - Shared intentions.
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Intentions: Recap

Motivational attitudes which:

- Are relatively stable.
- Are conduct-controlling, i.e. commit to action.
- Constraint further practical reasoning.

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Commitments and Intentions

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Intentions and Teamwork:

M. Gilbert. *On Social Facts*. Princeton UP, 1989.

J. Searle. *The Construction of Social Reality*. Free Press, 1995.

M. Bratman. *Faces of Intentions*. Cambridge UP, 1999.

R. Tuomela. *The Philosophy of Sociality*. Oxford UP, 2010.

Shared Intentions

A The Intention part:

1. Me:
   1.1 I intend that we J.
   1.2 I intend that we J in accordance with and because of meshing subplans of (1.1) and (2.1).

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3. Additional requirements:
   3.1 The intentions in (1) and in (2) are not coerced by the other participant.
   3.2 The intentions in (1) and (2) are minimally cooperatively stable.

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B: The epistemic part:

1. It is common knowledge between us that (A).

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Main Question

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the "will of the group"?

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Typical Examples:

- Electing government officials
- Department meetings
- Deciding where to go to dinner with friends
- ....

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- **Neutrality**: The names of the candidates, or options, do not matter (if two candidate are exchanged in every ranking, then the outcome changes accordingly)
- **Monotonicity**: Moving up in the rankings is always better

Majority Rules

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If there are only **two** options, then majority voting is the "best" procedure: Choosing the outcome with the most votes (allowing for ties) is the *only* group decision method satisfying the previous properties.

K. May. *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20 (1952).

Generalizing May's Theorem

In May's Theorem, the agents are making a single binary choice between two alternatives. What about more general situations?

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In May's Theorem, the agents are making a single binary choice between two alternatives. What about more general situations?

- Agents choose between between more than two alternatives.
- There are multiple interconnected propositions on which simultaneous decisions are to be made.

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- Many proposed group decision methods (voting methods) with very little agreement about how to compare them.

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Suppose that there are three agents have preferences over there options $\{a, b, c\}$.

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The Logic of Group Decisions

Even if all the agents in a group have rational beliefs, the groups beliefs may not be rational

The Logic of Group Decisions

Fundamental Problem: groups are inconsistent!

The Logic of Group Decisions: The Doctrinal "Paradox" (Kornhauser and Sager 1993)

p: a valid contract was in place

q: there was a breach of contract

r: the court is required to find the defendant liable.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

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Should we accept r?

	p	q	$(p \wedge q) \leftrightarrow r$	r
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2	yes	no	yes	no
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Should we accept r? No, a simple majority votes no.

	p	q	$(p \wedge q) \leftrightarrow r$	r
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2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions: The Doctrinal “Paradox” (Kornhauser and Sager 1993)

Should we accept r? Yes, a majority votes yes for p and q and $(p \wedge q) \leftrightarrow r$ is a legal doctrine.

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Conclusion: Groups are inconsistent, difference between 'premise-based' and 'conclusion-based' decision making, ...

Many Variants!

See
`http://personal.lse.ac.uk/LIST/doctrinalparadox.htm`
for many generalizations!

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Arrow's Theorem

K. Arrow. *Social Choice & Individual Values*. 1951.

Also, see

J. Geanakoplos. *Three Brief Proofs of Arrow's Impossibility Theorem.*. Economic Theory, **26**, 2005.

A. Taylor. *Social Choice and The Mathematics of Manipulation*. Cambridge University Press, 2005.

W. Gaertner. *A Primer in Social Choice Theory*. Oxford University Press, 2006.

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- Notation: write $\vec{P}$ for the tuple $(P_1, P_2, \ldots, P_n)$.

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If each agent ranks x above y, then so does the social welfare function

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If for each $i \in \mathcal{A}$, xP_iy then $xF(\vec{P})y$

Universal Domain

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F is a total function.

Independence of Irrelevant Alternatives

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If for each $i \in \mathcal{A}$, xP_iy iff xP'_iy, then $xF(\vec{P})y$ iff $xF(\vec{P}')y$.

Dictatorship

There is an individual $d \in \mathcal{A}$ such that the society strictly prefers x over y whenever d strictly prefers x over y.

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There is an individual $d \in \mathcal{A}$ such that the society strictly prefers x over y whenever d strictly prefers x over y.

There is a $d \in \mathcal{A}$ such that $xF(\vec{P})y$ whenever xP_dy.

Arrow's Theorem

Theorem (Arrow, 1951) Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

Sen's Liberal Paradox

A. Sen. *The Impossibility of a Paretian Liberal*. The Journal of Political Economy, **78**, pgs. 152 - 157, 1970.

Franz Dietrich and Christian List. *A Liberal Paradox for Judgment Aggregation*. Forthcoming.

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l: Lewd reads the book;
p: Prude reads the book;
$l \rightarrow p$: If Lewd reads the book, then so does Prude.

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Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.

Sen's Liberal Paradox

	l	p	$l \to p$

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	l	p	$l \to p$
Lewd	True	True	True

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	l	p	$l \to p$
Lewd	True	True	True
Prude	False	False	True

Sen's Liberal Paradox

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Lewd	True	True	True
Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
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2. Unanimous desires of all individuals must be respected.

So, society must be inconsistent!

Fair Division

S. Brams, P. Edelman and P. Fishburn. *Paradoxes of Fair Division*. Journal of Philosophy, **98:6**, pgs. 300-314.

J. Robertson and W. Webb. *Cake-Cutting Algorithms: Be Fair if You Can*. A.K. Peters, 1998.

S. Brams and A. Taylor. *Fair Division: From cake-cutting to dispute resolution*. Cambridge University Press, 1998.

S. Brams and A. Taylor. *The Win-Win Solution*. W. W. Norton & Company, 2000.

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- **Equitable:** each player values its allocation the same *according to its own valuation function*.
- **Efficiency:** there is no other division better for everybody, or better for some players and not worse for the others

Fair Division of Indivisible Goods

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- Players cannot compensate each other with side payments
- All players have positive values for every item
- A player prefers a set S to different set T if
  - S has as many elements as T
  - for every item in $t \in T - S$ there is a distinct item $s \in S - T$ that the player prefers to t.

Envy-Freeness and Efficiency

A unique envy-free division may be inefficient

A :	1	2	3	4	5	6
B :	4	3	2	1	5	6
C :	5	1	2	6	3	4

$A : \{1, 3\}$
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This is the unique *envy-free* outcome.

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However, $(12, 34, 56)$ is **not** (necessarily) envy-free

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There is no other division, including an efficient one, that guarantees envy-freeness.

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There may be no envy-free division, even when all players have different preference rankings

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Trivial if all players have the same preference.

Envy-Freeness and Efficiency

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Three divisions are efficient: $(1,3,2)$, $(2,1,3)$ and $(3,1,2)$.
However, none of them are envy-free.

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Three divisions are efficient: $(1,3,2)$, $(2,1,3)$ and $(3,1,2)$. However, none of them are envy-free.

In fact, there is **no** envy-free division.

Group Rationality Constraints

- Defining a group's preferences and beliefs:
  - Even if all the agents in a group have rational preferences, the groups preference may not be rational.
  - Even if all the agents in a group have rational beliefs, the groups beliefs may not be rational.
- Different normative constraints on group decision making are in conflict.
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  - Puzzles of Fair Division
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Next: Voting Theory and Conclusions