The Dutch Book Argument

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4. **Dutch Book Theorem.** An agent who tries to maximize her subjective expected utility using beliefs that violate the laws for probability will freely preform an act that is sure to leave her worse off than some alternative act would in all circumstances.
Betting Behavior

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If she is indifferent between 63, 81 EUR and [100 EUR if it rains, 0 EUR otherwise], then she believes to degree 0.6381 that it will rain.
Dutch Book

An agent will swap an (set of) wagers with the (sum of) their fair prices.
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Consider $W_1 = \{0.6, W_X, W_Y\}$ and $W_2 = \{0.5, W_{X \lor Y}\}$

**Indifferent between $W_1$ and $W_2**

But $W_2$ is always better:

- If $X$ is true $\text{payoff}(W_1) = 1.6 > \text{payoff}(W_2) = 1.5$
- If $Y$ is true $\text{payoff}(W_1) = 1.6 > \text{payoff}(W_2) = 1.5$
- If neither $X$ nor $Y$ is true $\text{payoff}(W_1) = 0.6 > \text{payoff}(W_2) = 0.5$
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$X$ and $Y$ are indifferent between $W_1$ and $W_2$

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**Dutch Book Theorem**

**Theorem.** Imagine an EU-maximizer who satisfies 1-3 and has a precise degree of belief for every proposition she considers. If these beliefs violate the laws of probability, then she will make Dutch Book against herself.
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allow agents to have incomplete or imprecise preferences
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*justify probabilistic coherence and EU simultaneously*
Converse Dutch Book Theorem

If you violate $\Phi$, then you are susceptible to Dutch Book.

We need: "If you obey $\Phi$, then you are not susceptible to a Dutch Book (or possibly you are not susceptible to a Dutch Book)."
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Converse Dutch Book Theorem. If your degrees of beliefs (fair betting prices) satisfy the laws of probability, then there does not exist a Dutch Book consisting of bets at those prices.
Savage’s Representation Theorem

A set of states $S$, a set of consequences $O$, acts are functions from $S$ to $O$. 

1. each act/state pair produces a unique consequence that settles every issue the agent cares about
2. she is convinced that her behavior will make no causal difference to which state obtains.

The agent is assumed to have preference ordering $\succeq$ over the set of acts.

Expected Utility: $\text{Exp} P, u(\alpha) = \sum_{w \in W} P(w) \times u(\alpha, w)$
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Small Worlds

States: \{\text{the sixth egg is good, the sixth egg is rotten}\}
Consequences \{ \text{6-egg omelet, no omelet and five good eggs destroyed, 6-egg omelet and a saucer to wash….} \}
Acts: \{ \text{break egg into bowl, break egg into saucer, throw egg away}\}
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<table>
<thead>
<tr>
<th>Good Egg</th>
<th>Rotten Egg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break into bowl</td>
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</tr>
</tbody>
</table>
Representation

EU-coherence: There must be at least one probability $P$ defined on states and one utility function for consequences that represent the agent's preferences in the sense that, for any acts $\alpha$ and $\beta$, she strictly (weakly) prefers $\alpha$ to $\beta$ only if $Exp_{P,u}(\alpha)$ is greater (as great as) $Exp_{P,u}(\beta)$. 
Axioms of Preference

For all acts $\alpha, \beta, \gamma$ and events $X, Y$
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4. **Wagers** For consequences $O_1$ and $O_2$ and any event $X$, there is an act $[O_1$ if $X$, $O_2$ else] that produces $O_1$ in any state that entails $X$ and $O_2$ in any state that entails $\neg X$
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5. **Savage’s P4** If the agent prefers $[O_1$ if $X,$ $O_2$ else] to $[O_1$ if $Y,$ $O_2$ else] when $O_1$ is more desirable than $O_2$, then she will also prefer $[O_1^*$ if $X,$ $O_2^*$ else] to $[O_1^*$ if $Y,$ $O_2^*$ else] for any other outcomes such that $O_1^*$ is more desirable than $O_2^*$. 
A businessman contemplates buying a certain piece of property. He considers the outcome of the next presidential election relevant.
The Sure-Thing Principle

A businessman contemplates buying a certain piece of property. He considers the outcome of the next presidential election relevant. So, to clarify the matter to himself, he asks whether he would buy if he knew that the Democratic candidate were going to win, and decides that he would. Similarly, he considers whether he would buy if he knew a Republican candidate were going to win, and again he finds that he would. Seeing that he would buy in either event, he decides that he should buy, even though he does not know which event obtains, or will obtain, as we would ordinarily say. (Savage, 1954)
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If an agent satisfies all of the above postulates (including some technical ones not discussed), then the agent acts as if she is maximizing an expected utility.

These axioms (along with a few others) guarantee the existence of a unique probability $P$ and utility $u$, unique up to the arbitrary choice of a unit and zero-point, whose associated expectation represents the agent’s preferences.
Defining Beliefs from Preferences

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*Definition* A practically rational agent **believes** \(X\) more strongly than she **believes** \(Y\) if and only if she strictly prefers \([O_1 \text{ if } X, \; O_2 \text{ else}]\) to \([O_1 \text{ if } Y, \; O_2 \text{ else}]\) for some (hence any by P4) outcome with \(O_1\) more desirable than \(O_2\).
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If \(O_1\) is preferred to \(O_2\) then the agent has a good reason for preferring \([O_1 \text{ if } X, \ O_2 \text{ else}]\) to \([O_1 \text{ if } Y, \ O_2 \text{ else}]\) exactly if she is more confident in \(X\) than in \(Y\).
Are the Axioms Requirements of Practical Rationality?

Epistemic vs. Practical Rationality

“...The fact (if it is a fact) that one will be bound to lose money unless one’s degrees of belief [obey the laws of probability] just isn’t relevant to the truth of those beliefs” (Kennedy and Chihara, 1979)
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> There is nothing more to the rationality of beliefs than their propensity to produce practically rational actions.
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$W_2 = [O_1 \text{ if } X \lor Z, \text{ else } O_2]$

Suppose $P_1 \succeq P_2$ but $W_1 \succeq W_2$. This is inconsistent as they are simply rephrasing of the same event!

Howson and Urbach: Define beliefs as the betting odds the agent believes are fair.

Joyce: relate probability consistency to accuracy of graded beliefs.
Epistemic vs. Practical Rationality

Skyrms: “you evaluate a betting arrangement independent of how it is described.”

The agent believes $X$ more likely than $Y$ and $X \lor Z$ more likely than $Y \lor Z$ and suppose that $O_1$ strictly preferred to $O_2$

$P_1 = \{[O_1 \text{ if } X, \text{ else } O_2], [O_1 \text{ if } Z, \text{ else } O_2]\}$

$P_2 = \{[O_1 \text{ if } Y, \text{ else } O_2], [O_1 \text{ if } Z, \text{ else } O_2]\}$

$W_1 = [O_1 \text{ if } Y \lor Z, \text{ else } O_2]$

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Next Week: Decision-Theoretic Paradoxes, Changing Beliefs