

# Rationality

## Lecture 8

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*We need to take the agent's beliefs into account*

## Instrumental Rationality II

**Subjective Rationality:** Ann's action  $\alpha$  is instrumentally rational iff when she chooses  $\alpha$ : (1) her choice was based on her beliefs ( $B$ ) and (2) *if*  $B$  were true beliefs, then  $\alpha$  would be an effective way to achieve her goals, desires, tastes, etc.

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What constraints should be placed on reasonable beliefs that underlie a rational choice?

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  - “a person shows herself to lack “rational integration” if she has some desire for  $x$ , yet also desires not to desire  $x$ ” (Nozick, pg. 139 - 151)
- ▶ the ultimate goal is *happiness*, other desires are the manifestation of the pursuit of happiness or pleasure

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3. the person has some reason to prefer preferring  $x$  to  $y$  to not doing that.

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R. Nozick. "*Rational Preferences*". in *The Nature of Rationality*, pgs. 139 - 151.

## Economic Rationality

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*Economic Rationality* Ann's action  $\alpha$  is economically rational only if it is (a) instrumentally rational or (b) consumptively rational.

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Are preferences over *outcomes* or *options*?



## Preliminaries: Orderings

An ordering is a *relation*  $R$  on a set  $X$ : a subset of the set of pairs of elements from  $X$ :  $R \subseteq X \times X$

Write  $aRb$  iff  $(a, b) \in R$

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Properties of orderings:

- ▶ Reflexivity: for all  $a \in X$ ,  $aRa$
- ▶ Transitivity: for all  $a, b, c \in X$ ,  $aRb$  and  $bRc$  then  $aRc$
- ▶ Symmetry: for all  $a, b \in X$ ,  $aRb$  implies  $bRa$
- ▶ Asymmetry: for all  $a, b \in X$ ,  $aRb$  implies not- $bRa$
- ▶ Completeness: for all  $a, b \in X$ ,  $aRb$  or  $bRa$  (or  $a = b$ )

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4.  $x \not\succeq y$  and  $y \not\succeq x$ : The agent *cannot compare*  $x$  and  $y$  ( $x \perp y$ )



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What properties does this preference ordering have?

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# Ordinal Utility Theory: Axioms

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“Rather than trying to provide instrumental or pragmatic justifications for the axioms of ordinal utility, it is better...to see them as constitutive of our conception of a fully rational agent....those disposed to blatantly ignore transitivity are unintelligible to use: we can't understand their pattern of actions as sensible” (Gaus [OPPE], pg. 39)

## Ordinal Utility Theory

**Fact.** Suppose that  $X$  is finite and  $\succeq$  is a complete and transitive ordering over  $X$ , then there is a utility function  $u : X \rightarrow \mathfrak{R}$  that represents  $\succeq$  ( $x \succeq y$  iff  $u(x) \geq u(y)$ )

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**Important point:** consider  $x \succ y \succ z$ , all three utility functions represent this ordering:

Preference	$u_1$	$u_2$	$u_3$
$x$	3	10	1000
$y$	2	5	99
$z$	1	0	1

## Cardinal Utility Theory

$x \succ y \succ z$  is represented by both  $(3, 2, 1)$  and  $(1000, 999, 1)$ , so cannot say  $y$  is “closer” to  $x$  than to  $z$ .

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Key idea: Ordinal preferences over *lotteries* allows us to infer a cardinal scale (with some additional axioms).

John von Neumann and Oskar Morgenstern. *The Theory of Games and Economic Behavior*. Princeton University Press, 1944.



## Axioms of Cardinal Utility

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Running example: Suppose Ann prefers pizza ( $p$ ) over taco ( $t$ ) over yogurt ( $y$ ) ( $p \succ t \succ y$ ) and consider the different lotteries where the prizes are  $p$ ,  $t$  and  $y$ .

## Cardinal Utility Theory: Continuity

**Continuity:** for all options  $x, y$  and  $z$  if  $x \succeq y \succeq z$ , there is some lottery  $L$  with probability  $p$  of getting  $x$  and  $(1 - p)$  of getting  $y$  such that the agent is indifferent between  $L$  and  $z$ .

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Continuity says that there is must be some lottery where Ann is indifferent between keeping  $t$  and playing the lottery.

## Cardinal Utility Theory: Better Prizes

**Better Prizes:** suppose  $L_1$  is a lottery over  $(w, x)$  and  $L_2$  is over  $(y, z)$  suppose that  $L_1$  and  $L_2$  have the same probability over prizes. The lotteries each have an equal prize in one position they have unequal prizes in the other position then if  $L_1$  is the lottery with the better prize then  $L_1 \succ L_2$ ; if neither lottery has a better prize then  $L_1 \approx L_2$ .

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Since Ann prefers  $p$  to  $t$ , this axiom says that Ann prefers  $L_1$  to  $L_2$

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This axiom states that Ann must prefer  $L_1$  to  $L_2$



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This eliminates utility from the thrill of gambling and so the only ultimate concern is the prizes.

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- ▶ Deep issues about how to identify correct descriptions of the outcomes and options.

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Suppose you have a kitten, which you plan to give away to either Ann or Bob. Ann and Bob both want the kitten very much. Both are deserving, and both would care for the kitten. You are sure that giving the kitten to Ann ( $x$ ) is at least as good as giving the kitten to Bob ( $y$ ) (so  $x \succeq y$ ). But you think that would be unfair to Bob. You decide to flip a fair coin: if the coin lands heads, you will give the kitten to Bob, and if it lands tails, you will give the kitten to Ann. (J. Drier, “Morality and Decision Theory” in [HR])

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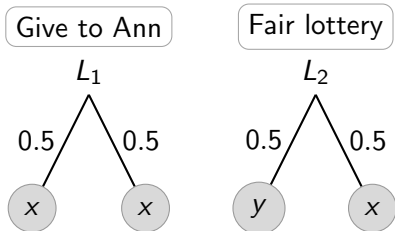
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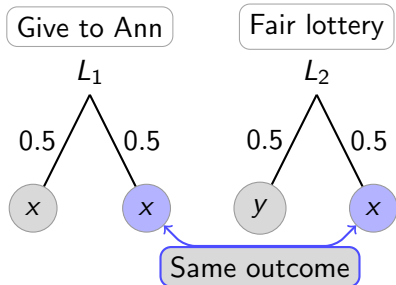
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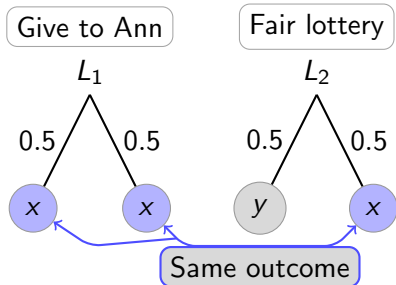
Why does this contradict better prizes? consider the lottery which is  $x$  for sure ( $L_1$ ) and the lottery which is 0.5 for  $y$  and 0.5 for  $x$  ( $L_2$ ). Better prizes implies  $L_1 \succeq L_2$  but a person concerned with fairness may have  $L_2 \succeq L_1$ . *But if fairness is important then that should be part of the description of the outcome!*



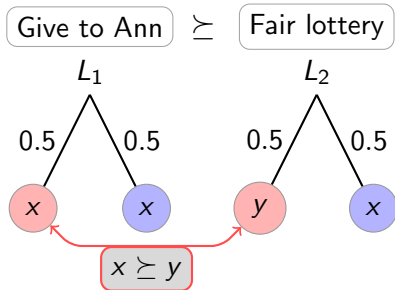
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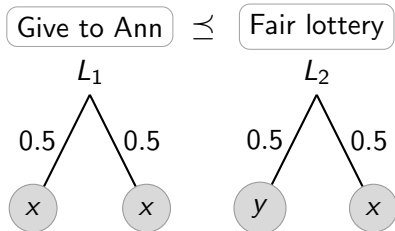


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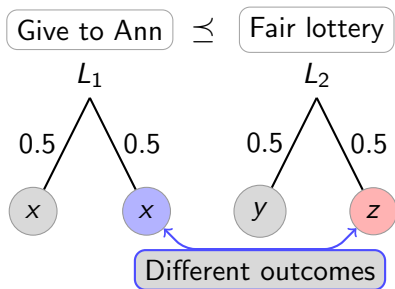


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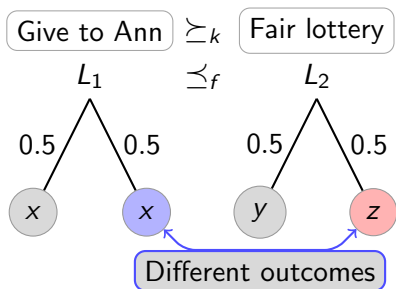




- ▶  $x$  is the outcome “Ann gets the kitten, *in a fair way*”
- ▶  $y$  is the outcome “Bob gets the kitten”



- ▶  $x$  is the outcome "Ann gets the kitten"
- ▶  $z$  is the outcome "Ann gets the outcome, *fairly*"
- ▶  $y$  is the outcome "Bob gets the kitten, *fairly*"



If all the agent cares about is who gets the kitten, then  $L_1 \succeq L_2$

If all the agent cares about is being fair, then  $L_1 \preceq L_2$

## Allais Paradox, Again

	Options	Red (1)	White (89)	Blue (10)
$S_1$	$A$	$1M$	$1M$	$1M$
	$B$	$0$	$1M$	$5M$
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Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

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Decision theory gives the agent some way to determine what is the “best” option, but in general this need not be the option that leads to the highest satisfaction of one’s goals.



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Suppose the players meet only once. It would seem that the Proposer should propose 99% for herself and 1% for the Disposer. And if the Disposer is instrumentally rational, then she should accept the offer.

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A typical explanation is that the players' utility functions are not simply about getting funds to best advance their goals, but about acting according to some norms of fair play. But acting according to norms of fair play does not seem to be a goal: it is a principle to which a person wishes to conform.

## Choice Processes and Outcomes

A. Sen. *Maximization and the Act of Choice*. *Econometrica*, Vol. 65, No. 4, 1997, 745 - 779.

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## Choice Functions

Suppose  $X$  is a set of options. And consider  $B \subseteq X$  as a choice problem. A **choice function** is any function where  $C(B) \subseteq B$ .  $B$  is sometimes called a menu and  $C(B)$  the set of “rational” or “desired” choices.

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Sen's  $\beta$ : If  $x, y \in C(A)$ ,  $A \subset B$  and  $y \in C(B)$  then  $x \in C(B)$ .



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To take another example, you may prefer mangoes to apples, but refuse to pick the last mango from a fruit basket, and yet be very pleased if someone else were to “force” that last mango on you. ” (Sen, pg. 747)

Let  $X = \{x, y, z\}$  and consider  $B_1 = X$  and  $B_2 = \{x, y\}$ . Define

$$C(B_1) = C(\{x, y, z\}) = \{x\}$$

$$C(B_2) = C(\{x, y\}) = \{y\}$$

*This choice function cannot be rationalized.*

## Framing effects

*Logicophilia*, a virulent virus, threatens 600 students at Tilburg University

[Adapted from Tversky and Kahneman (1981)]

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1. You must choose between two prevention programs, resulting in:
  - A: 200 participants will be saved for sure.
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2. You must choose between two prevention programs, resulting in:
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2. You must choose between two prevention programs, resulting in:
    - A': 400 will not be saved, for sure.
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- 78 % of the participants choose B' over A'.

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## Framing effects

The Experiment:	
<b>A:</b> 0 + 200 for sure.	<b>B:</b> (33% 600) + (66% 0).
⇒ 72 % of the participants choose A over B.	
<b>A':</b> 600 - 400 for sure.	<b>B':</b> (33% 600) + (66% 0).
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- ▶ Standard decision theory is **extensional**
    - Choosing  $A$  and  $A \leftrightarrow B$  implies Choosing  $B$ .
- Also true of many formalisms of beliefs:
- “Believing”  $A$  and  $\vdash A \leftrightarrow B$  implies “Believing”  $B$ .

## Conclusions, I

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- ▶ *Instrumental rationality* is a fundamental account of “rationality”, but it is not necessarily the “whole of rationality”
- ▶ Utility is not a sort of “value”, but simply a representation of one’s ordering of options based on one’s underlying values, ends and principles.



## Conclusions, II

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- ▶ We need an account of which distinctions are relevant and which are not...what justifies a preference.
- ▶ Utility theory is a way to formalize and model rational action, but it is not itself a complete theory of rational action.

J. Pollock. *Rational Choice and Action Omnipotence*. The Philosophical Review, Vol. 111, No. 1 (2002), pgs. 1 - 23.

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Next week: more on decision theory