Instrumental Rationality

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Decision theory gives the agent some way to determine what is the “best” option, but in general this need not be the option that leads to the highest satisfaction of one’s goals.
Ultimatum Game

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Suppose the players meet only once. It would seem that the Proposer should propose 99\% for herself and 1\% for the Disposer. And if the Disposer is instrumentally rational, then she should accept the offer.
Ultimatum Game

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A typical explanation is that the players’ utility functions are not simply about getting funds to best advance their goals, but about acting according to some norms of fair play. But acting according to norms of fair play does not seem to be a goal: it is a principle to which a person wishes to conform.
Choice Processes and Outcomes


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Choice Processes and Outcomes


“The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines. But maximizing behavior differs from nonvolitional maximization because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person’s preferences over comprehensive outcomes (including the choice process) have to be distinguished form the conditional preferences over culmination outcomes given the act of choice.”

(pg. 745)
Choice Functions

Suppose $X$ is a set of options. And consider $B \subseteq X$ as a choice problem. A **choice function** is any function where $C(B) \subseteq B$. $B$ is sometimes called a menu and $C(B)$ the set of “rational” or “desired” choices.
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A relation $R$ on $X$ **rationalizes a choice function** $C$ if for all $B$ $C(B) = \{x \in B \mid \text{for all } y \in B \ xRy\}$. (i.e., the agent is chooses according to some preference ordering).
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Sen’s $\alpha$: If $x \in C(A)$ and $B \subset A$ and $x \in B$ then $x \in C(B)$
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**Sen’s $\beta$:** If $x, y \in C(A)$, $A \subseteq B$ and $y \in C(B)$ then $x \in C(B)$.
You arrive at a garden party and can readily identify the most comfortable chair. You would be delighted if an imperious host were to assign you that chair. However, if the matter is left to your own choice, you may refuse to rush to it.

Are you still a maximizer? Quite possibly you are, since your preference ranking for choice behavior may well be defined over "comprehensive outcomes", including choice processes (in particular, who does the choosing) as well as the outcomes at culmination (the distribution of chairs).

To take another example, you may prefer mangoes to apples, but refuse to pick the last mango from a fruit basket, and yet be very pleased if someone else were to "force" that last mango on you. (Sen, pg. 747)
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Let $X = \{x, y, z\}$ and consider $B_1 = X$ and $B_2 = \{x, y\}$. Define

$$C(B_1) = C(\{x, y, z\}) = \{x\}$$

$$C(B_2) = C(\{x, y\}) = \{y\}$$

This choice function cannot be rationalized.
Framing effects

*Logicophilia*, a virulent virus, threatens 600 students at Tilburg University

[Adapted from Tversky and Kahneman (1981)]
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1. You must choose between two prevention programs, resulting in:
   
   A: 200 participants will be saved for sure.
   
   B: 33 % chance of saving all of them, otherwise no one will be saved.

72 % of the participants choose A over B.

2. You must choose between two prevention programs, resulting in:
   
   A': 400 will not be saved, for sure.
   
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- Standard decision theory is **extensional**
  - Choosing $A$ and $A \iff B$ implies Choosing $B$.
  - Also true of many formalisms of beliefs:
    - “Believing” $A$ and $\vdash A \iff B$ implies “Believing” $B$. 
Conclusions, I

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- Utility is not a sort of “value”, but simply a representation of one’s ordering of options based on one’s underlying values, ends and principles.
Rational Constraints on Beliefs

- Deductive cogency: an ideal rational agent’s beliefs should be *consistent* and *deductively closed*. (Preface Paradox, Lottery Paradox, Problems with Closure)
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- How should an ideally rational agent *change her beliefs*?
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Savage’s Representation Theorem

A set of states $S$, a set of consequences $O$, acts are functions from $S$ to $O$. 

1. each act/state pair produces a unique consequence that settles every issue the agent cares about
2. she is convinced that her behavior will make no causal difference to which state obtains.

The agent is assumed to have preference ordering $\succeq$ over the set of acts.

Expected Utility:
$$\text{Exp}, u(\alpha) = \sum_{w \in W} P(w) \times u(\alpha, w)$$
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Small Worlds

States: \{\text{the sixth egg is good, the sixth egg is rotten}\}
Consequences \{ 6\text{-egg omelet, no omelet and five good eggs destroyed, 6\text{-egg omelet and a saucer to wash}}.\}
Acts: \{ break egg into bowl, break egg into saucer, throw egg away\}
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Representation

EU-coherence: There must be at least on probability $P$ defined on states and one utility function for consequences that represent the agent’s preferences in the sense that, for any acts $\alpha$ and $\beta$, she strictly (weakly) prefers $\alpha$ to $\beta$ only if $\text{Exp}_{P,u}(\alpha)$ is greater (as great as) $\text{Exp}_{P,u}(\beta)$. 
Axioms of Preference

For all acts $\alpha, \beta, \gamma$ and events $X, Y$
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4. **Wagers** For consequences $O_1$ and $O_2$ and any event $X$, there is an act $[O_1 \text{ if } X, O_2 \text{ else}]$ that produces $O_1$ in any state that entails $X$ and $O_2$ in any state that entails $\neg X$
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5. **Savage’s P4** If the agent prefers $[O_1$ if $X$, $O_2$ else] to $[O_1$ if $Y$, $O_2$ else] when $O_1$ is more desirable than $O_2$, then she will also prefer $[O_1^*$ if $X$, $O_2^*$ else] to $[O_1^*$ if $Y$, $O_2^*$ else] for any other outcomes such that $O_1^*$ is more desirable than $O_2^*$. 
The Sure-Thing Principle

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Representation Theorem

If an agent satisfies all of the above postulates (including some technical ones not discussed), then the agent acts as if she is maximizing an expected utility.

These axioms (along with a few others) guarantee the existence of a unique probability $P$ and utility $u$, unique up to the arbitrary choice of a unit and zero-point, whose associated expectation represents the agent’s preferences.
Defining Beliefs from Preferences

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**Definition** A practically rational agent **believes** \(X\) **more strongly than she believes** \(Y\) if and only if she strictly prefers \([O_1 \text{ if } X, \ O_2 \text{ else}]\) to \([O_1 \text{ if } Y, \ O_2 \text{ else}]\) for some (hence any by P4) outcome with \(O_1\) more desirable than \(O_2\).
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If \(O_1\) is preferred to \(O_2\) then the agent has a good reason for preferring \([O_1 \text{ if } X, O_2 \text{ else}]\) to \([O_1 \text{ if } Y, O_2 \text{ else}]\) exactly if she is more confident in \(X\) than in \(Y\).
Are the Axioms Requirements of Practical Rationality?

Issues

- Decision makers are sensitive to *risk* and *ambiguity* in ways that contradict standard expected utility calculations (Allais, Ellsberg)

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Decision makers are sensitive to framing effects.

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- The decision makers expected utility calculations should be sensitive to an agent’s judgements about the probable causal consequences of the available options. (*Newcomb’s Paradox*)
Newcomb’s Paradox

Two boxes in front of you, \( A \) and \( B \).

Box \( A \) contains $1,000 and box \( B \) contains either $1,000,000 or nothing.
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Box \( A \) contains $1,000 and box \( B \) contains either $1,000,000 or nothing.

Your choice: either open both boxes, or else just open \( B \). (You can keep whatever is inside any box you open, but you may not keep what is inside a box you do not open).
A very powerful being, who has been invariably accurate in his predictions about your behavior in the past, has already acted in the following way:

1. If he has predicted that you will open just box $B$, he has in addition put $1,000,000$ in box $B$
2. If he has predicted you will open both boxes, he has put nothing in box $B$.

What should you do?

Newcomb’s Paradox

<table>
<thead>
<tr>
<th></th>
<th>( B = 1 \text{M} )</th>
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## Newcomb's Paradox

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There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).
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Dominance reasoning is appropriate only when probability of outcome is independent of choice.

(A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize? Whether or not the nephew is cut from the will may depend on whether or not he apologizes.)

What the Predictor did yesterday is probabilistically dependent on the choice today, but causally independent of today’s choice.
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What the Predictor did yesterday is *probabilistically dependent* on the choice today, but *causally independent* of today’s choice.
Newcomb’s Problem: Causal Decision Theory

\[ V(A) = \sum_w V(w) \cdot P_A(w) \]

(the expected value of act \( A \) is a probability weighted average of the values of the ways \( w \) in which \( A \) might turn out to be true)
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Orthodox Bayesian Decision Theory: \( P_A(w) := P(w \mid A) \)
(Probability of \( w \) given \( A \) is chosen)

Causal Decision theory: \( P_A(w) = P(A \Box \rightarrow w) \) (Probability of if \( A \) were chosen then \( w \) would be true)
Newcomb’s Problem: Causal Decision Theory

Suppose 99% confidence in predictors reliability.

\[ V(B_1) = V(M) P(M|B_1) + V(N) P(N|B_1) \]
\[ = 1000000 \cdot 0.99 + 0 \cdot 0.01 = 990000 \]

\[ V(B_2) = V(L) P(L|B_2) + V(K) P(K|B_2) \]
\[ = 1001000 \cdot 0.01 + 1000 \cdot 0.99 = 1100000 \]

\( B_1 \): one-box (open box \( B \))
\( B_2 \): two-box choice (open both \( A \) and \( B \))
\( N \): receive nothing
\( K \): receive $1,000
\( M \): receive $1,000,000
\( L \): receive $1,001,000
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$V(B_1) = V(M)P(M \mid B_1) + V(N)P(N \mid B_1) = 10,000,000 \cdot 0.99 + 0 \cdot 0.01 = 990,000$
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Let $\mu$ be the assigned to the conditional $B_1 \square \rightarrow M$ (and $B_2 \square \rightarrow L$) (both conditional are true iff the Predictor put $1,000,000$ in box $B$ yesterday).

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Conclusions, II

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- We need an account of which distinctions are relevant and which are not...what justifies a preference.

- Utility theory is a way to formalize and model rational action, but it is not itself a complete theory of rational action.

Next Week: Game Theory