Models of Strategic Reasoning
Lecture 1

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Lecture 1: Introduction, Motivation and Background

Lecture 2: The Dynamics of Rational Deliberation

Lecture 3: Reasoning to a Solution: Common Modes of Reasoning in Games

Lecture 4: Reasoning to a Model: Iterated Belief Change as Deliberation

Lecture 5: Reasoning in Specific Games: Experimental Results
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Lecture 5: Reasoning in Specific Games: Experimental Results
ai.stanford.edu/~epacuit/esslli2012/stratreas.html
Plan for Today

- Introductory Remarks
- Equilibrium Selection and Harsanyi’s tracing procedure
- Higher-order beliefs in games
- General comments on common knowledge and beliefs
The “Axiom” of Game Theory

Common Knowledge of Rationality
The “Axiom” of Game Theory

Common Knowledge of Rationality

(graded) belief, strong/robust belief, take for granted...
The “Axiom” of Game Theory

Common Knowledge of Rationality

(graded) belief, strong/robust belief, take for granted...

“it is completely transparent to the players that...”
The “Axiom” of Game Theory

Common Knowledge of Rationality

(graded) belief, strong/robust belief, take for granted...

“it is completely transparent to the players that...”

“Bayesian decision theory” (optimize)
Dynamics

“The economist’s predilection for equilibria frequently arises from the belief that some underlying dynamic process (often suppressed in formal models) moves a system to a point from which it moves no further.”

Dynamics

“The economist’s predilection for equilibria frequently arises from the belief that some underlying dynamic process (often suppressed in formal models) moves a system to a point from which it moves no further.”  


“It is not just a question of what common knowledge obtains at the moment of truth, but also how common knowledge is preserved, created, or destroyed in the deliberational process which leads up to the moment of truth.”

Substantive vs. Procedural Rationality

“Behavior is **substantively rational** when it is appropriate to the achievement of given goals within the limits imposed by given conditions and constraints. [...] Given these goals, the rational behavior is determined entirely by the characteristics of the environment in which it takes place.”

Substantive vs. Procedural Rationality

“Behavior is **substantively rational** when it is appropriate to the achievement of given goals within the limits imposed by given conditions and constraints. [...] Given these goals, the rational behavior is determined entirely by the characteristics of the environment in which it takes place. ”

“Behavior is **procedurally rational** when it is the outcome of appropriate deliberation. Its procedural rationality depends on the process that generated it. When psychologists use the term rational, it is usually procedural rationality they have in mind. ”

Substantive vs. Procedural Rationality, II

“The human mind is programmable: it can acquire an enormous variety of different skills, behavior patterns, problem-solving repertoires, and perceptual habits. [...]
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“The human mind is programmable: it can acquire an enormous variety of different skills, behavior patterns, problem-solving repertoires, and perceptual habits. [...] We can expect substantive rationality only in situations that are sufficiently simple as to be transparent to this mind. In all other situations, we must expect that the mind will use such imperfect information as it has, will simplify and represent the situation as it can, and will make such calculations as are within it powers. We cannot expect to predict what it will do in such situations unless we know what information it has, what forms of representations it prefers, and what [calculation rules] are available to it.”

Rationality: Two Themes

Rationality is a matter of reasons:

▶ The rationality of a belief $P$ depends on the reasons for holding $P$.

▶ The rationality of act $\alpha$ depends on the reason for doing $\alpha$.

Rationality is a matter of reliability:

▶ A rational belief is one that is arrived at through a process that reliably produces beliefs that are true.

▶ An act is rational if it is arrived at through a process that reliably achieves specified goals.
Rationality: Two Themes

Rationality is a matter of reasons:

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Rationality: Two Themes

Rationality is a matter of **reasons**: 
- The rationality of a belief $P$ depends on the *reasons for holding* $P$
- The rationality of act $\alpha$ depends on the *reason for doing* $\alpha$

Rationality is a matter of **reliability**: 
- A rational belief is one that is arrived at through a process that reliably produces beliefs that are true. 
- A act is rational if it is arrived at through a process that reliably achieves specified goals.
Rationality: Two Themes

“Neither theme alone exhausts our notion of rationality. Reasons without reliability seem empty, reliability without reasons seems blind. In tandem these make a powerful unit, but how exactly are they related and why?”

(Nozick, pg. 64)

A game is a description of strategic interaction that includes:

- actions the players *can* take
- description of the players’ interests (i.e., preferences),
- description of the “structure” of the decision problem
“We adhere to the classical point of view that the game under consideration fully describes the real situation — that any (pre) commitment possibilities, any repetitive aspect, any probabilities of error, or any possibility of jointly observing some random event, have already been modeled in the game tree.” (pg. 1005)

From Game Models to Models of Games

“Formally, a game is defined by its strategy sets and payoff functions. But in real life, many other parameters are relevant; there is a lot more going on. Situations that substantively are vastly different may nevertheless correspond to precisely the same strategic game. For example, in a parliamentary democracy with three parties, the winning coalitions are the same whether the parties each hold a third of the seats in parliament, or, say, 49 percent, 39 percent, and 12 percent, respectively. But the political situations are quite different. The difference lies in the attitudes of the players, in their expectations about each other, in custom, and in history, though the rules of the game do not distinguish between the two situations. (pg. 72, our emphasis)

Epistemic Program in Game Theory


Epistemic Program in Game Theory

Game $G$
Epistemic Program in Game Theory

Game $G$ \rightarrow \text{Strategy Space}
Epistemic Program in Game Theory

Game $G$ → Strategy Space

$S$

Game Model
Epistemic Program in Game Theory

Game $G$ → Strategy Space

Game Model
Epistemic Program in Game Theory

Game Model

Strategy Space

Game G

Rat

¬Rat
Epistemic Program in Game Theory

Game $G$

Strategy Space

$b$

$a$

$s$

Game Model

$\text{Rat}$

$\neg \text{Rat}$
Models of Hard and Soft Information

Epistemic Model: \( \mathcal{M} = \langle W, \{\sim_i\}_{i \in \mathcal{A}}, V \rangle \)
- \( w \sim_i v \) means \( i \) cannot rule out \( v \) according to her information.

Language:

\[ \varphi := p \mid \neg \varphi \mid \varphi \wedge \psi \mid K_i \varphi \]

Truth:
- \( \mathcal{M}, w \models p \) iff \( w \in V(p) \) (\( p \) an atomic proposition)
- Boolean connectives as usual
- \( \mathcal{M}, w \models K_i \varphi \) iff for all \( v \in W \), if \( w \sim_i v \) then \( \mathcal{M}, v \models \varphi \)
Models of Hard and Soft Information

Epistemic-Plausibility Model: \( \mathcal{M} = \langle W, \{\sim_i\}_{i \in A}, \{\preceq_i\}_{i \in A}, \mathcal{V} \rangle \)

- \( w \preceq_i v \) means \( v \) is at least as plausibility as \( w \) for agent \( i \).

Language: \( \varphi := p \mid \neg \varphi \mid \varphi \land \psi \mid K_i \varphi \mid B^\varphi \psi \mid [\preceq_i] \varphi \)

Truth:

- \( \llbracket \varphi \rrbracket_\mathcal{M} = \{ w \mid \mathcal{M}, w \models \varphi \} \)
- \( \mathcal{M}, w \models B^\varphi_i \psi \) iff for all \( v \in \text{Min}_{\preceq_i}(\llbracket \varphi \rrbracket_\mathcal{M} \cap [w]_i) \), \( \mathcal{M}, v \models \psi \)
- \( \mathcal{M}, w \models [\preceq_i] \varphi \) iff for all \( v \in W \), if \( v \preceq_i w \) then \( \mathcal{M}, v \models \varphi \)
Epistemic-Plausibility Model: \( M = \langle W, \{\sim_i\}_{i \in A}, \{\pi_i\}_{i \in A}, V \rangle \)

\[ \pi_i : W \to [0, 1] \text{ is a probability measure} \]

Language: \( \varphi := p | \neg \varphi | \varphi \land \psi | K_i \varphi | B^p \psi \)

Truth:

1. \( [\varphi]_M = \{w \mid M, w \models \varphi\} \)
2. \( M, w \models B^p \varphi \text{ iff } \pi_i([\varphi]_M \cap [w]_i) = \frac{\pi_i([\varphi]_M \cap [w]_i)}{\pi_i([w]_i)} \geq \ p, M, v \models \psi \)
3. \( M, w \models K_i \varphi \text{ iff for all } v \in W, \text{ if } w \sim_i v \text{ then } M, v \models \varphi \)
Models of Hard and Soft Information

- *Describing* what the agents know and believe rather than *defining* the agents’ knowledge (and beliefs) in terms or more primitive notions (representational vs. explanatory models)
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“Discussions of substantive rationality take place in an essentially static framework. Thus, equilibrium is discussed without explicit reference to any dynamic process by means of which the equilibrium is achieved. Similarly, prior beliefs are taken as given, without reference to how a rational agent acquires these beliefs. Indeed, all questions of the procedure by means of which rational behavior is achieved are swept aside by a methodology that treats this procedure as completed and reifies the supposed limiting entities by categorizing them axiomatically.” (pg. 180)

What does it mean to choose “rationally”? 

“A glance at any dictionary will confirm that economists, firmly entrenched in the static viewpoint described above, have hijacked this word and used it to mean something for which the word *consistent* would be more appropriate. ” (pg. 181)

Models of Strategic *Reasoning*

- Brian Skyrms’ models of “dynamic deliberation”
- Ken Binmore’s analysis using Turing machines to “calculate” the rational choice
- Robin Cubitt and Robert Sugden’s “common modes of reasoning”
- Johan van Benthem et col.’s “virtual rationality *announcements*”
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Different frameworks, common thought: the “rational solutions” of a game are the result of individual deliberation about the “rational” action to choose.
Reasoning in Games: A Heuristic Treatment
Background: Setting the Stage

(Subjective) Probability ("betting interpretation"), Von Neumann-Morgenstern Utilities, Expected Utility, Extensive/Strategic Games, Nash Equilibrium
Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they only care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.
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Diagram:

```
Ann
  └── Bob
    └── Bob
      └── Ann's preference
        └── Ann's preference
```

- **Ann cuts one piece bigger**: Bob takes bigger piece (Ann's preference)
- **Bob cuts almost even**: Bob takes smaller piece (Ann's preference)
- **Ann takes bigger piece**: Bob takes bigger piece (Ann's preference)
- **Bob takes smaller piece**: Bob takes smaller piece (Ann's preference)

Eric Pacuit: Models of Strategic Reasoning 19/72
Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they only care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.
Ann

cut one piece bigger

cut almost even

Bob

take bigger piece

take smaller piece

2,3

Bob

take bigger piece

take smaller piece

3,2

Bob

take bigger piece

take smaller piece

1,4

4,1
What should Ann do?

<table>
<thead>
<tr>
<th></th>
<th>Bob</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TB</strong></td>
<td>1,4</td>
</tr>
<tr>
<td><strong>TS</strong></td>
<td>4,1</td>
</tr>
<tr>
<td><strong>CB</strong></td>
<td>2,3</td>
</tr>
<tr>
<td><strong>CE</strong></td>
<td>3,2</td>
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</table>

It depends on what she expects Bob to do, but this depends on what she thinks Bob expects her to do, and so on...
What should Ann do? Bob best choice in Ann’s worst choice
What should Ann do? *maximize over each row and choose the maximum value*
What should Bob do? minimize over each column and choose the maximum value.
Theorem (Von Neumann). For every two-player zero-sum game with finite strategy sets $S_1$ and $S_2$, there is a number $v$, called the value of the game such that:

1. $v = \max_{p \in \Delta(S_1)} \min_{q \in \Delta(S_2)} U_1(p, q) = \min_{q \in \Delta(S_2)} \max_{p \in \Delta(S_1)} U_1(p, q)$

2. The set of mixed Nash equilibria is nonempty. A mixed strategy profile $(p, q)$ is a Nash equilibrium if and only if

   $$p \in \operatorname{argmax}_{p \in \Delta(S_1)} \min_{q \in \Delta(S_2)} U_1(p, q)$$

   $$q \in \operatorname{argmax}_{q \in \Delta(S_2)} \min_{p \in \Delta(S_1)} U_1(p, q)$$

3. For all mixed Nash equilibria $(p, q)$, $U_1(p, q) = v$
Why play such an equilibrium?

“Let us now imagine that there exists a complete theory of the zero-sum two-person game which tells a player what to do, and which is absolutely convincing. If the players knew such a theory then each player would have to assume that his strategy has been “found out” by his opponent. The opponent knows the theory, and he knows that the player would be unwise not to follow it... a satisfactory theory can exist only if we are able to harmonize the two extremes...strategies of player 1 ‘found out’ or of player 2 ‘found out.’ ” (pg. 148)

“Von Neumann and Morgenstern are assuming that the payoff matrix is common knowledge to the players, but presumably the players’ subjective probabilities might be private. Then each player might quite reasonably act to maximize subjective expected utility, believing that he will not be found out, with the result not being a Nash equilibrium.”

(Skyrms, pg. 14)
Suppose that Ann believes Bob will play TB with probability 1/4, for whatever reason. Then, $1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$.

But, TB maximizes expected utility no matter what belief Bob may have:

$$p + 3 = 4 \times p + 3 \times (1 - p) \geq 1 \times p + 2 \times (1 - p) = 2 - p.$$
Suppose that Ann believes Bob will play $TB$ with probability $1/4$, for whatever reason. Then,

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But, $TB$ is maximizes expected utility no matter what belief Bob may have:

$$p + 3 = 4 \times p + 3 \times (1 - p) \geq 1 \times p + 2 \times (1 - p) = 2 - p$$
“The rules of a game and its numerical data are seldom sufficient for logical deduction alone to single out a unique choice of strategy for each player. ‘**To do so one requires either richer information (such as institutional detail or perhaps historical precedent for a certain type of behavior) or bolder assumptions about how players choose strategies.**’ Putting further restrictions on strategic choice is a complex and treacherous task. But one’s intuition frequently points to patterns of behavior that cannot be isolated on the grounds of consistency alone.”

---

Finding the rational choice...

What is a rational choice for Ann (Bob)?
Finding the rational choice...

What is a rational choice for Ann (Bob)? *Flip a coin!*
Finding the rational choice...

What is a rational choice for Ann (Bob)?
Finding the rational choice...

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<thead>
<tr>
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<tbody>
<tr>
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<td>C1</td>
<td>C2</td>
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<tr>
<td>Ann</td>
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<td></td>
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<tr>
<td>P1</td>
<td>1,-1</td>
<td>-1,1</td>
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<tr>
<td>P2</td>
<td>-1,1</td>
<td>1,-1</td>
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</tr>
<tr>
<td>P2</td>
<td>1,-1</td>
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What is a rational choice for Ann (Bob)? *Play a different game!*
Prisoner’s Dilemma

Two people commit a crime.
Prisoner’s Dilemma

Two people commit a crime. They are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing.
Prisoner’s Dilemma

Two people commit a crime. They are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it.
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Prisoner’s Dilemma

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Prisoner’s Dilemma

Two options: Confess ($C$), Don’t Confess ($D$)
Prisoner’s Dilemma

Two options: Confess ($C$), Don’t Confess ($D$)

Possible outcomes:
Prisoner’s Dilemma

Two options: Confess ($C$), Don’t Confess ($D$)

Possible outcomes: We both confess ($C, C$),
Prisoner’s Dilemma

Two options: Confess ($C$), Don’t Confess ($D$)

Possible outcomes: We both confess ($C, C$), I confess but my partner doesn’t ($C, D$),
Prisoner’s Dilemma

Two options: Confess ($C$), Don’t Confess ($D$)

Possible outcomes: We both confess ($C, C$), I confess but my partner doesn’t ($C, D$), My partner confesses but I don’t ($D, C$),
Prisoner’s Dilemma

Two options: Confess ($C$), Don’t Confess ($D$)

Possible outcomes: We both confess ($C, C$), I confess but my partner doesn’t ($C, D$), My partner confesses but I don’t ($D, C$), neither of us confess ($D, D$).
Prisoner’s Dilemma

Bob

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>C</th>
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<tbody>
<tr>
<td>D</td>
<td>3,3</td>
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<tr>
<td>C</td>
<td>4,1</td>
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Ann
**Prisoner’s Dilemma**

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*Ann’s preferences*
Prisoner’s Dilemma

Bob’s preferences

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<td>Ann D</td>
<td>3</td>
<td>1</td>
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Prisoner’s Dilemma

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What should Ann (Bob) do?
Dominance Reasoning

A

B
Dominance Reasoning
## Dominance Reasoning

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<tr>
<td>B</td>
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The diagram illustrates the concept of dominance reasoning with arrows indicating the dominance of strategies in decision-making scenarios.
Prisoner’s Dilemma

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What should Ann (Bob) do?
Prisoner’s Dilemma

What should Ann (Bob) do? Dominance reasoning
Prisoner’s Dilemma

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<tr>
<td>Ann</td>
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<tr>
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<td>3,3</td>
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<tr>
<td>C</td>
<td>4,1</td>
<td>2,2</td>
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</tbody>
</table>

What should Ann (Bob) do? *Dominance reasoning*
Prisoner’s Dilemma

What should Ann (Bob) do? Dominance reasoning is not Pareto!
Prisoner’s Dilemma

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<tr>
<td>Ann</td>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>Bob</td>
<td>2.5</td>
<td>2</td>
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What should Ann (Bob) do? *Think as a group!*
Prisoner’s Dilemma

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<td>4,1</td>
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What should Ann (Bob) do? *Play against your mirror image!*
Prisoner’s Dilemma

What should Ann (Bob) do? *Play against your mirror image!*
Prisoner's Dilemma

What should Ann (Bob) do? *Change the game* (eg., Symbolic Utilities)
What should/will Ann (Bob) do?
What should/will Ann (Bob) do?
What should/will Ann (Bob) do?
Nozick: Symbolic Utility

“Yet the symbolic value of an act is not determined solely by that act.”
Nozick: Symbolic Utility

“Yet the symbolic value of an act is not determined solely by *that* act. The act’s meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties.”
Nozick: Symbolic Utility

“Yet the symbolic value of an act is not determined solely by that act. The act’s meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties. What the act symbolizes is something it symbolizes when done in that particular situation, in preference to those particular alternatives.
Nozick: Symbolic Utility

“Yet the symbolic value of an act is not determined solely by that act. The act’s meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties. What the act symbolizes is something it symbolizes when done in that particular situation, in preference to those particular alternatives. If an act symbolizes “being a cooperative person,” it will have that meaning not simply because it has the two possible payoffs it does
Nozick: Symbolic Utility

“Yet the symbolic value of an act is not determined solely by that act. The act’s meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties. What the act symbolizes is something it symbolizes when done in that particular situation, in preference to those particular alternatives. If an act symbolizes “being a cooperative person,” it will have that meaning not simply because it has the two possible payoffs it does but also because it occupies a particular position within the two-person matrix — that is, being a dominated action that (when joined with the other person’s dominated action) yield a higher payoff to each than does the combination of dominated actions. ” (pg. 55)
What should/will Ann (Bob) do?

Prisoner’s Dilemma

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What should/will Ann (Bob) do?
What should/will Ann (Bob) do?

Prisoner’s Dilemma

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Bob
Prisoner’s Dilemma

What should/will Ann (Bob) do?
What should/will Ann (Bob) do?
What should/will Ann (Bob) do?

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<td>Ann D</td>
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<td>Ann C</td>
<td>4,1</td>
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Prisoner’s Dilemma
What should/will Ann (Bob) do?
What should/will Ann (Bob) do?
What should/will Ann (Bob) do?
“Game theorists think it just plain wrong to claim that the Prisoners’ Dilemma embodies the essence of the problem of human cooperation.
“Game theorists think it just plain wrong to claim that the Prisoners’ Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner’s Dilemma, we wouldn’t have evolved as social animals!

---

“Game theorists think it just plain wrong to claim that the Prisoners’ Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner’s Dilemma, we wouldn’t have evolved as social animals! .... No paradox of rationality exists. Rational players don’t cooperate in the Prisoners’ Dilemma, because the conditions necessary for rational cooperation are absent in this game.”

( pg. 63)

Equilibrium are not interchangeable

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<tr>
<td><strong>D</strong></td>
<td><strong>S</strong></td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>-4,-4</td>
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<tr>
<td><strong>S</strong></td>
<td>-1,1</td>
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</table>
Equilibrium are not interchangeable
Perfect equilibrium

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<tr>
<th>Ann</th>
<th>L</th>
<th>R</th>
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<tbody>
<tr>
<td>U</td>
<td>1,1</td>
<td>0,0</td>
</tr>
<tr>
<td>D</td>
<td>0,0</td>
<td>0,0</td>
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Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability $\epsilon$-

$\epsilon$-perfect equilibrium: a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than $\epsilon$-

Perfect equilibrium: the mixed strategy profile that is the limit as $\epsilon$ goes to 0 of $\epsilon$-perfect equilibria.
**Perfect equilibrium**

A perfect equilibrium is a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than \( \epsilon \). Prefect equilibrium is the mixed strategy profile that is the limit as \( \epsilon \) goes to 0 of \( \epsilon \)-perfect equilibria.

The game matrix is:

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<tr>
<td><strong>Ann</strong></td>
<td>L</td>
<td>R</td>
</tr>
<tr>
<td><strong>U</strong></td>
<td>1,1</td>
<td>0,0</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>0,0</td>
<td>0,0</td>
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</table>
Isn’t \((U, L)\) more “reasonable” than \((D, R)\)?
Perfect equilibrium

**Completely mixed strategy**: a mixed strategy in which every strategy gets some positive probability
Perfect equilibrium

<table>
<thead>
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<th></th>
<th>Bob</th>
<th></th>
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</table>
| **L** | 1,1 | 0,
| **R** | 0,0 | 0,0 |

**Completely mixed strategy**: a mixed strategy in which every strategy gets some positive probability

**$\epsilon$-perfect equilibrium**: a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than $\epsilon$

**Prefect equilibrium**: the mixed strategy profile that is the limit as $\epsilon$ goes to 0 of $\epsilon$-prefect equilibria.
Proper equilibrium

$$\begin{array}{c|ccc}
 & \text{Bob} & \text{C} & \text{R} \\
\hline
\text{U} & -9,-9 & -7,-7 & -7,-7 \\
\text{M} & 0,0 & 0,0 & -7,-7 \\
\text{D} & 1,1 & 0,0 & -9,-9 \\
\end{array}$$

$\epsilon$-perfect equilibrium: a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than $\epsilon$.

Perfect equilibrium: the mixed strategy profile that is the limit as $\epsilon$ goes to 0 of $\epsilon$-perfect equilibria.
Proper equilibrium

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<th>Bob</th>
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<tr>
<td></td>
<td>L</td>
<td>C</td>
<td>R</td>
</tr>
<tr>
<td>Ann</td>
<td>-9,-9</td>
<td>-7,-7</td>
<td>-7,-7</td>
</tr>
<tr>
<td>M</td>
<td>0,0</td>
<td>0,0</td>
<td>-7,-7</td>
</tr>
<tr>
<td>D</td>
<td>1,1</td>
<td>0,0</td>
<td>-9,-9</td>
</tr>
</tbody>
</table>

Proper equilibrium: the mixed strategy profile that is the limit as $\epsilon$ goes to 0 of $\epsilon$-perfect equilibria.
Proper equilibrium

\[ \begin{array}{ccc}
U & L & C & R \\
U & -9,-9 & -7,-7 & -7,-7 \\
M & 0,0 & 0,0 & -7,-7 \\
D & 1,1 & 0,0 & -9,-9 \\
\end{array} \]

\textbf{\( \epsilon \)-proper equilibrium}: a completely mixed strategy profile such that if strategy \( s \) is a better response than \( s' \), then \( \frac{p(s)}{p(s')} < \epsilon \)

\textbf{Proper equilibrium}: the mixed strategy profile that is the limit as \( \epsilon \) goes to 0 of \( \epsilon \)-proper equilibria.
Proper equilibrium

\[ \begin{array}{ccc}
  & \text{Bob} & \\
  \text{U} & -9,-9 & -7,-7 & -7,-7 \\
  \text{Ann} & 0,0 & 0,0 & -7,-7 \\
  \text{D} & 1,1 & 0,0 & -9,-9 \\
\end{array} \]

**\(\epsilon\)-proper equilibrium**: a completely mixed strategy profile such that if strategy \(s\) is a better response than \(s'\), then \(\frac{p(s)}{p(s')} < \epsilon\).

**Proper equilibrium**: the mixed strategy profile that is the limit as \(\epsilon\) goes to 0 of \(\epsilon\)-proper equilibria.
“There cannot be any mistakes if the players are absolutely rational. Nevertheless, a satisfactory interpretation of equilibrium points in extensive games seems to require that the possibility of mistakes is not completely excluded. This can be achieved by a point of view which looks at complete rationality as the limiting case of incomplete rationality.”  

Tracing Procedure


Tracing Procedure


The Tracing Procedure

Suppose there is a common prior that Ann will choose with probability 0.5 and Bob will choose \( L \) with probability 0.5.

Consider the modified game where the utilities are the expected utilities of the first game. For each \( t \in [0, 1] \), the game \( G_t \) is defined so that the payoffs of \( u_t^i(x, y) = x \cdot u_0^i(x, y) + (1 - x) \cdot u_1^i(x, y) \).
The Tracing Procedure

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<tr>
<td><strong>U</strong></td>
<td>4,1</td>
<td>0,0</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>0,0</td>
<td>1,4</td>
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- Suppose there is a common prior that Ann will choose **U** with probability 0.5 and Bob will choose **L** with probability 0.5.
The Tracing Procedure

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<tr>
<td>D</td>
<td>0,0</td>
<td>1,4</td>
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Suppose there is a common prior that Ann will choose $U$ with probability 0.5 and Bob will choose $L$ with probability 0.5.

Consider the modified game where the utilities are the expected utilities of the first game ($G^1$).
The Tracing Procedure

Suppose there is a common prior that Ann will choose $U$ with probability 0.5 and Bob will choose $L$ with probability 0.5.

Consider the modified game where the utilities are the expected utilities of the first game ($G^1$).

This game has a unique Nash equilibrium.
The Tracing Procedure

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<td>4,1</td>
<td>0,0</td>
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<td>$D$</td>
<td>0,0</td>
<td>1,4</td>
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<th>$L$</th>
<th>$R$</th>
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<tbody>
<tr>
<td>$U$</td>
<td>2,0.5</td>
<td>2,2</td>
</tr>
<tr>
<td>$D$</td>
<td>0.5,0.5</td>
<td>0.5,2</td>
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For each $t \in [0, 1]$, the game $G^t$ is defined so that the payoffs of $u^t_i(x, y) = x \cdot u^0_i(x, y) + (1 - x) \cdot u^1_i(x, y)$
The Tracing Procedure

For each $t \in [0, 1]$, the game $G^t$ is defined so that the payoffs of $u_i^t(x, y) = x \cdot u_i^0(x, y) + (1 - x) \cdot u_i^1(x, y)$

A graph of the equilibrium points as $t$ varies from 0 to 1 will show a connected path from equilibria in $G^0$ to equilibria in $G^1$
For each \( t \in [0, 1] \), the game \( G^t \) is defined so that the payoffs of 
\[
u_i^t(x, y) = x \cdot u_i^0(x, y) + (1 - x) \cdot u_i^1(x, y)
\]
A graph of the equilibrium points as \( t \) varies from 0 to 1 will show a connected path from equilibria in \( G^0 \) to equilibria in \( G^1 \)
This process almost always leads to a unique equilibrium in \( G^1 \) (modifying the payoffs with a logarithmic term guarantees uniqueness)
Harsanyi and Selten also propose a method to calculate a prior given the game.
Harsanyi and Selten also propose a method to *calculate* a prior given the game.

The picture is of deliberators who are “computationally adept but, initially at least, strategically naive”
Harsanyi and Selten also propose a method to calculate a prior given the game.

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They can identify game-theoretic equilibria instantaneously.
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They can identify game-theoretic equilibria instantaneously.

At $t = 0$ each contemplates jumping to the conclusion that the act the with maximum expected utility according to the common prior is the correct one.
Harsanyi and Selten also propose a method to calculate a prior given the game.

The picture is of deliberators who are “computationally adept but, initially at least, strategically naive”

They can identify game-theoretic equilibria instantaneously.

At $t = 0$ each contemplates jumping to the conclusion that the act the with maximum expected utility according to the common prior is the correct one.

At later times, the hypothesis that the other players will make their best response gets stronger and stronger, until at $t = 1$ only an equilibrium point of the original game remains.
Two general issues

- The relation between sequential and normal form games/decisions
- What are the players deliberating/reasoning about?
Normal form vs. Extensive Form

Explicitly modeling deliberation transforms a *single* choice into a situation of *sequential* choice.
Normal form vs. Extensive form

Eric Pacuit: Models of Strategic Reasoning
Normal form vs. Extensive form

Eric Pacuit: Models of Strategic Reasoning
Normal form vs. Extensive form

A

B

a1

0,0

a2

b1

b2

-1,-1

1,1

b1 if a1

b2 if a1

a1

-1,-1

1,1

a2

0,0

0,0

(Cf. the various notions of sequential equilibrium)

Eric Pacuit: Models of Strategic Reasoning 46/72
Normal form vs. Extensive form

Eric Pacuit: Models of Strategic Reasoning 46/72
Normal form vs. Extensive form

(Cf. the various notions of *sequential equilibrium*)
What are the players deliberating/reasoning about?
What are the players deliberating/reasoning about?

Their preferences?
What are the players deliberating/reasoning about?

Their preferences? The model?
What are the players deliberating/reasoning about?

Their preferences? The model? The other players?
What are the players deliberating/reasoning about?

Their preferences? The model? The other players? What to do?
The Cost of Thinking

“A person required to risk money on a remote digit of $\pi$ would have to compute that digit in order to comply fully with the theory, though this would really be wasteful if the cost of computation were more than the prize involved.
“A person required to risk money on a remote digit of \( \pi \) would have to compute that digit in order to comply fully with the theory, though this would really be wasteful if the cost of computation were more than the prize involved. For the postulates of the theory imply that you should behave in accordance with the logical implications of all that you know.

---

The Cost of Thinking

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The Cost of Thinking

“A person required to risk money on a remote digit of π would have to compute that digit in order to comply fully with the theory, though this would really be wasteful if the cost of computation were more than the prize involved. For the postulates of the theory imply that you should behave in accordance with the logical implications of all that you know. Is it possible to improve the theory in this respect, making allowance within it for the cost of thinking, or would that entail paradox, as I am inclined to believe but unable to demonstrate? If the remedy is not in changing the theory but rather in the way in which we attempt to use it, clarification is still to be desired.”

(pg.308)

Deliberation in Decision Theory

“deliberation crowds out prediction”


▶ Conclusion
Meno’s Paradox

1. If you know what you’re looking for, inquiry is unnecessary.
2. If you do not know what you’re looking for, inquiry is impossible.
Therefore, inquiry is either unnecessary or impossible.
Meno’s Paradox

1. If you know what you’re looking for, inquiry is unnecessary.
2. If you do not know what you’re looking for, inquiry is impossible.

Therefore, inquiry is either unnecessary or impossible.

Levi’s Argument

1. If you have access to self-knowledge and logical omniscience to apply the principles of rational choice to determine which options are admissible, then the principles of rational choice are vacuous for the purposes of deciding what to do.

2. If you do not have access to self-knowledge and logical omniscience in this sense, then the principles of rational choice are inapplicable for the purposes of deciding what to do.

Therefore, the principles of rational choice are either unnecessary or impossible.
If \( X \) takes the sentence “Sam behaves in manner \( R \)” to be an act description vis-à-vis a decision problem faced by Sam, then \( X \) is in a state of full belief that has the following contents:

1. **Ability Condition**: Sam has the ability to choose that Sam will \( R \) on a trial of kind \( S \), where the trial of kind \( S \) is a process of deliberation eventuating in choice.
2. **Deliberation Condition**: Sam is subject to a trial of kind \( S \) at time \( t \); that is, Sam is deliberating at time \( t \).
3. **Efficaciousness Condition**: Adding the claim that Sam chooses that he will \( R \) to \( X \)’s current body of full beliefs entails that Sam will \( R \).
4. **Serious Possibility**: For each feasible option for Sam, nothing in \( X \)’s state of full belief is incompatible with Sam’s choosing that option.
If $X$ takes the sentence “Sam behaves in manner $R$” to be an act description vis-à-vis a decision problem faced by Sam, then $X$ is in a state of full belief that has the following contents:

1. **Ability Condition**: Sam has the ability to choose that Sam will $R$ on a trial of kind $S$, where the trial of kind $S$ is a process of deliberation eventuating in choice.
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Foreknowledge of Rationality

Let $A$ be a set of **feasible** options and $C(A) \subseteq A$ the **admissible** options.
Foreknowledge of Rationality

Let $A$ be a set of **feasible** options and $C(A) \subseteq A$ the **admissible** options.

1. **Logical Omniscience**: The agent must have enough logical omniscience and computational capacity to use his principles of choice to determine the set $C(A)$ of admissible outcomes.
Foreknowledge of Rationality

Let $A$ be a set of \textbf{feasible} options and $C(A) \subseteq A$ the \textbf{admissible} options.

1. \textit{Logical Omniscience}: The agent must have enough logical omniscience and computational capacity to use his principles of choice to determine the set $C(A)$ of admissible outcomes.

2. \textit{Self-Knowledge}: The agent must know “enough” about his own values (goal, preferences, utilities) and beliefs (both full beliefs and probability judgements).
Foreknowledge of Rationality

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1. *Logical Omniscience*: The agent must have enough logical omniscience and computational capacity to use his principles of choice to determine the set $C(A)$ of admissible outcomes.

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2. **Self-Knowledge**: The agent must know “enough” about his own values (goal, preferences, utilities) and beliefs (both full beliefs and probability judgements).

3. **Smugness**: The agent is certain that in the deliberation taking place at time $t$, $X$ will choose an admissible option.

If all the previous conditions are satisfied, then no inadmissible option is feasible from the deliberating agent’s point of view when deciding what to do: $C(A) = A$. 

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“Though this result is not contradictory, it implies the vacuousness of principles of rational choice for the purpose of deciding what to do...If they are useless for this purpose, then by the argument of the previous section, they are useless for passing judgement on the rationality of choice as well.” (L, pg. 10)
“Though this result is not contradictory, it implies the vacuousness of principles of rational choice for the purpose of deciding what to do...If they are useless for this purpose, then by the argument of the previous section, they are useless for passing judgement on the rationality of choice as well.” (L, pg. 10)

(Earlier argument: “If X is merely giving advice, it is pointless to advise Sam to do something X is sure Sam will not do...The point I mean to belabor is that passing judgement on the rationality of Sam’s choices has little merit unless it gives advice to how one should choose in predicaments similar to Sam’s in relevant aspects”)
**Weak Thesis**: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

**Strong Thesis**: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.
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“...the probability assignment to A may still be available to the subject in his purely doxastic capacity but not in his capacity of an agent or practical deliberator. The agent *qua* agent must abstain from assessing the probability of his options.” (Rabinowicz, pg. 3)
“(...) probabilities of acts play no role in decision making. (...) The decision maker chooses the act he likes most be its probability as it may. But if this is so, there is no sense in imputing probabilities for acts to the decision maker.” (Spohn (1977), pg. 115)
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▶ Levi “I never deliberate about an option I am certain that I am not going to choose”. If I have a low probability for doing some action A, then I may spend less time and effort in deliberation…
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- Levi “I never deliberate about an option I am *certain* that I am not going to choose”. If I have a low probability for doing some action A, then I may spend less time and effort in deliberation…

- Deliberation as a *feedback* process: change in inclinations causes a change in probabilities assigned to various options, which in turn may change my inclinations towards particular options….
Discussion

Logical Omniscience/Self-Knowledge: “decision makers do not know their preferences at the time of deliberation” (Schick). “If decision makers never have the capacities to apply the principles of rational choice and cannot have their capacities improved by new technology and therapy, the principles are inapplicable. Inapplicability is no better a fate than vacuity.”

Drop smugness: “the agent need not assume he will choose rationally...the agent should be in a state of suspense as to which of the feasible options will be chosen” (Levi)

Implications for game theory (common knowledge of rationality implies, in particular, that agents satisfy Smugness).

Conclusion

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Discussion

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- **Implications for game theory** (*common knowledge of rationality* implies, in particular, that agents satisfy *Smugness*).
Weak Thesis: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.
\[ b^A_{C,S} \]: A bet on proposition \( A \) that costs \( C \) to buy and pays \( S \) if won.

A bet is fair if, and only if, the agent is prepared to take each side of the bet (buy it, if offered, and sell it, if asked).
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Identification of credences with betting rates: $P(A) = C/S$
\[ EU(\text{Buy } b^A_{C,S}) = P(A) \cdot (S - C) + P(\overline{A})(-C) \]
$$EU(\text{Buy } b^A_{C,S}) = P(A) \cdot (S - C) + P(\overline{A})(-C) = \frac{C}{S} \cdot (S - C) - \frac{C}{S} \cdot C$$
$EU(\text{Buy } b_A^{C,S}) = P(A) \cdot (S - C) + P(\overline{A})(-C) = C/S \cdot (S - C) - C/S \cdot C = 0$
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$$EU(\text{Sell } b^A_{C,S}) = P(A) \cdot (-S + C) + P(\overline{A})(C)$$
\[ EU(\text{Buy } b^A_{C,S}) = P(A) \cdot (S - C) + P(\overline{A})(-C) = \]
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\[ EU(\text{Sell } b^A_{C,S}) = P(A) \cdot (-S + C) + P(\overline{A})(C) = C/S \cdot (C-S) + C/S \cdot C = 0 \]
Suppose that $A$ and $B$ are alternative actions available to the agent.

$EU(A)$ and $EU(B)$ are their expected utilities for the agent *disregarding any bets that he might place on the actions themselves.*

The “gain” $G$ for an agent who accepts and wins a bet $b^A_{C,S}$ is the *net gain* $S - C$.

If he takes a bet on $A$ with a net gain $G$, his expected utility of $A$ will instead be $EU(A) + G$. The reason is obvious: If that bet is taken, then, if $A$ is performed, the agent will receive $G$ in addition to $EU(A)$. 
“The agents readiness to accept a bet on an act does not depend on the betting odds but only on his gain. If the gain is high enough to put this act on the top of his preference order of acts, he will accept it, and if not, not. The stake of the agent is of no relevance whatsoever.” (Spohn, 1977, p. 115)
“The agents readiness to accept a bet on an act does not depend on the betting odds but only on his gain. If the gain is high enough to put this act on the top of his preference order of acts, he will accept it, and if not, not. The stake of the agent is of no relevance whatsoever.” (Spohn, 1977, p. 115)

Take the bet “I will do action A” provided $EU(A) + G > EU(B)$ and if not, do not take the bet. *This has nothing to do with the ratio C/S.*
Suppose $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$ iff $G > 4$.
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Let $C = 10$ and $S = 15$. Then, $G = 15 - 10 = 5$, which rationalizes taking the bet on $A$. 
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However, if $C$ and $S$ drop down to 4 and 6, respectively, the quotient $C/S$ remains unchanged but $G$ falls below 4.
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However, if $C$ and $S$ drop down to 4 and 6, respectively, the quotient $C/S$ remains unchanged but $G$ falls below 4.

The agent is certain that if he takes the bet on doing the action, then he will do that action.

Betting on an action is not the same thing as deciding to do an action.
Argument II

Suppose that $P(A)$ is well-defined and $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$
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The agent considers it *probable* that if he is offered a bet on $A$, then he will take it (but not necessarily *certain*).  

Argument II

Suppose that $P(A)$ is well-defined and $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$

The agent considers it probable that if he is offered a bet on $A$, then he will take it (but not necessarily certain).

If no bet on $A$ is offered, then the agent does not think it is probable that he will perform $A$, so $P(A)$ is relatively low.
Argument II

If a bet on $A$ is offered with net gain $G$, then $P(A)$ increases.

The agent thinks it probable that he will perform $A$ if he takes the bet (since $EU(A + G) > EU(B)$, we have $EU(A + G) > EU(B) - C$ and the agent thinks it probable that he will take the bet (because of the above assumption and the fact that $EU(A + G) > EU(B)$).

Thus, the probability of an action depends on whether the bet is offered or not.
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*Thus, the probability of an action depends on whether the bet is offered or not.*
If probabilities correspond to betting rates, then this cannot depend on whether or not the bets are offered.
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_forgetfulness_
We must choose between the following 4 complex options:

1. take the bet on $A$ & do $A$
2. take the bet on $A$ & do $B$
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**Claim 1:** If an agent is certain that he won’t perform an option, then this option is not *feasible*
We must choose between the following 4 complex options:

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**Claim 1:** If an agent is certain that he won’t perform an option, then this option is not *feasible*

**Claim 2:** If the agent assigns probabilities to options, then, on pain of incoherence, his probabilities for inadmissible (= irrational) options, as revealed by his betting dispositions, must be zero.
Consider two alternatives $A$ and $B$ with $EU(A) > EU(B)$. 
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Suppose $P(A), P(B)$ are well-defined with $P(A) = \text{the betting rate for } A = x$. 

$G = S - C \geq 0$. 

$S \geq C \geq 0$. 

Thus, $G = S - C \geq 0$. 

$1 \geq x \geq 0$.
Consider two alternatives $A$ and $B$ with $EU(A) > EU(B)$.

Suppose $P(A), P(B)$ are well-defined with $P(A)$ = the betting rate for $A = x$

Suppose that the agent is offered a fair bet $b$ on $A$, with a positive stake $S$ and a price $C$. Since $b$ is fair, $C/S = x$. Since $1 \geq x \geq 0$ and $S > 0$, it follows that $S \geq C \geq 0$.

Thus, $G = S - C \geq 0$. 
Expected utilities of the complex actions:

- $EU(b \& A) = EU(A) + G$
- $EU(\neg b \& A) = EU(A)$
- $EU(b \& B) = EU(B) - C$
- $EU(\neg b \& B) = EU(B)$

At least one of $b \& A$ and $\neg b \& B$ is admissible.

$$EU(b \& A) = EU(A) + G > EU(B) = EU(\neg b \& B)$$

This holds even if the agent’s net gain is 0 (i.e., $G = S - C = 0$).
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This holds even if the agent’s net gain is 0 (i.e., $G = S - C = 0$).

But then it follows that the agent should be willing to accept the bet on $A$ even if $S = C$. Thus, the (fair) betting rate $x$ for $A$ must equal 1 ($P(A) = 1$), which implies, on pain of incoherence, that $P(B) = 1 - P(A) = 0$. The inadmissible option has probability zero.
Do we have to conclude that probabilities for ones current options must lack any connection at all to ones potential betting behavior?

Rabinowicz: Suppose that the agent is offered an opportunity to make a *betting commitment* with respect to $A$ at stake $S$ and price $C$. The agent makes a commitment (to buy or sell) not knowing whether he will be required to sell or to buy the bet.

A betting commitment is *fair* if the agent is willing to accept the commitment even if he is *radically uncertain* about what will be required of him.
Game Plan

**Today**: Towards a model of deliberation in games, equilibrium refinement program, general comments about reasoning and deliberation in game and decision theory
Game Plan

**Today:** Towards a model of deliberation in games, equilibrium refinement program, general comments about reasoning and deliberation in game and decision theory

**Tomorrow:** Skyrms’ dynamic model of Bayesian deliberators.