Judgement Aggregation

Stanford University
ai.stanford.edu/~epacuit/lmh

Fall, 2008
The Logic of Group Decisions

*Fundamental Problem:* groups are inconsistent!
The Logic of Group Decisions: The Doctrinal “Paradox” (Kornhauser and Sager 1993)

\[ p \land q \leftrightarrow r \]

\( p \): a valid contract was in place
\( q \): there was a breach of contract
\( r \): the court is required to find the defendant liable.

\[
\begin{array}{cccc}
 & p & q & (p \land q) \leftrightarrow r & r \\
1 & yes & yes & yes & yes \\
2 & yes & no & yes & no \\
3 & no & yes & yes & no \\
\end{array}
\]
The Logic of Group Decisions: The Doctrinal “Paradox” (Kornhauser and Sager 1993)

Should we accept $r$?

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The Logic of Group Decisions: The Doctrinal “Paradox”
(Kornhauser and Sager 1993)

Should we accept $r$? No, a simple majority votes no.

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Should we accept \( r \)? Yes, a majority votes yes for \( p \) and \( q \) and \( (p \land q) \leftrightarrow r \) is a legal doctrine.

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Discursive Dilemma

\( a: \) “Carbon dioxide emissions are above the threshold \( x \)”
Introduction

Discursive Dilemma

\[ a: \text{“Carbon dioxide emissions are above the threshold } x \text{”} \]
\[ a \rightarrow b: \text{“If carbon dioxide emissions are above the threshold } x, \text{ then there will be global warming”} \]
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\(b \) “There will be global warming”
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Conclusion: Groups are inconsistent, difference between ‘premise-based’ and ‘conclusion-based’ decision making, ...
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**Conclusion:** Groups are inconsistent, difference between ‘premise-based’ and ‘conclusion-based’ decision making, ...
The Judgement Aggregation Model: The Propositions

**Propositions**: Let $\mathcal{L}$ be a logical language (called *propositions* in the literature) with the usual boolean connectives.
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Consistency: The standard notion of logical consistency.
The Judgement Aggregation Model: The Propositions

**Propositions**: Let $\mathcal{L}$ be a logical language (called *propositions* in the literature) with the usual boolean connectives.

**Consistency**: The standard notion of logical consistency.

**Aside**: We actually need

1. $\{p, \neg p\}$ are inconsistent
2. all subsets of a consistent set are consistent
3. $\emptyset$ is consistent and each $S \subseteq \mathcal{L}$ has a consistent maximal extension (not needed in all cases)
Definition A set $Y \subseteq \mathcal{L}$ is **minimally inconsistent** if it is inconsistent and every proper subset $X \subsetneq Y$ is consistent.
The Judgement Aggregation Model: The Agenda

**Definition** The agenda is a non-empty set $X \subseteq \mathcal{L}$, interpreted as the set of propositions on which judgments are made, with $X$ is a union of proposition-negation pairs $\{p, \neg p\}$. 
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Example: In the discursive dilemma: $X = \{a, \neg a, b, \neg b, a \rightarrow b, \neg(a \rightarrow b)\}$. 
The Judgement Aggregation Model: The Judgement Sets

**Definition**: Given an agenda $X$, each individual $i$’s judgement set is a subset $A_i \subseteq X$.  

Rationality Assumptions:
1. $A_i$ is consistent
2. $A_i$ is complete, if for each $p \in X$, either $p \in A_i$ or $\neg p \in A_i$.
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2. $A_i$ is **complete**, if for each $p \in X$, either $p \in A_i$ or $\neg p \in A_i$
Let $X$ be an agenda, $N = \{1, \ldots, n\}$ a set of voters, a **profile** is a tuple $(A_1, \ldots, A_n)$ where each $A_i$ is a judgement set. An **aggregation function** is a map from profiles to judgment sets. I.e., $F(A_1, \ldots, A_n)$ is a judgement set.

**Examples:**

- **Propositionwise majority voting:** for each $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = \{p \in X | |\{i | p \in A_i\}| \geq |\{i | p \notin A_i\}|\}$.
- **Dictator of $i$:** $F(A_1, \ldots, A_n) = A_i$.
- **Reverse Dictator of $i$:** $F(A_1, \ldots, A_n) = \{\neg p | p \in A_i\}$. 


The Judgement Aggregation Model: Aggregation Rules

Let $X$ be an agenda, $N = \{1, \ldots, n\}$ a set of voters, a profile is a tuple $(A_1, \ldots, A_n)$ where each $A_i$ is a judgement set. An aggregation function is a map from profiles to judgment sets. I.e., $F(A_1, \ldots, A_n)$ is a judgement set.

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- **Dictator of $i$**: $F(A_1, \ldots, A_n) = A_i$

- **Reverse Dictator of $i$**: $F(A_1, \ldots, A_n) = \{\neg p \mid p \in A_i\}$
Universal Domain: The domain of $F$ is the set of all possible profiles of consistent and complete judgement sets.

Collective Rationality: $F$ generates consistent and complete collective judgment sets.
The Judgement Aggregation Model: Aggregation Rules

**Unanimity**: For all profiles \((A_1, \ldots, A_n)\) if \(p \in A_i\) for each \(i\) then \(p \in F(A_1, \ldots, A_n)\)
The Judgement Aggregation Model: Aggregation Rules

**Unanimity:** For all profiles \((A_1, \ldots, A_n)\) if \(p \in A_i\) for each \(i\) then \(p \in F(A_1, \ldots, A_n)\)

**Independence:** For any \(p \in X\) and all \((A_1, \ldots, A_n)\) and \((A_1^*, \ldots, A_n^*)\) in the domain of \(F\),

\[
\text{if } \forall i \in N, \ p \in A_i \iff p \in A_i^* \text{ then } [p \in F(A_1, \ldots, A_n) \iff p \in F(A_1^*, \ldots, A_n^*)].
\]
The Judgement Aggregation Model: Aggregation Rules

**Systematicity**: For any \( p, q \in X \) and all \((A_1, \ldots, A_n)\) and \((A_1^*, \ldots, A_n^*)\) in the domain of \( F \),

\[
\text{if } \left[ \text{for all } i \in N, \ p \in A_i \iff q \in A_i^* \right] \\
\text{then } \left[ p \in F(A_1, \ldots, A_n) \iff q \in F(A_1^*, \ldots A_n^*) \right].
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**The Judgement Aggregation Model: Aggregation Rules**

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\text{then } \left[ p \in F(A_1, \ldots, A_n) \ \text{iff} \ q \in F(A_1^*, \ldots, A_n^*) \right].
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**Monotonicity:** For any \( p \in X \) and all \((A_1, \ldots, A_i, \ldots, A_n)\) and \((A_1, \ldots, A_i^*, \ldots, A_n)\) in the domain of \( F \),

\[
\text{if } \left[ p \not\in A_i, \ p \in A_i^* \ \text{and} \ p \in F(A_1, \ldots, A_i, \ldots, A_n) \right]
\text{then } \left[ p \in F(A_1, \ldots, A_i^*, \ldots, A_n) \right].
\]
Anonymity: If \((A_1, \ldots, A_n)\) and \((A_1^*, \ldots, A_n^*)\) are permutations of each other, then

\[
F(A_1, \ldots, A_n) = F(A_1^*, \ldots, A_n^*)
\]

Non-dictatorship: There exists no \(i \in N\) such that, for any profile \((A_1, \ldots, A_n)\), \(F(A_1, \ldots, A_n) = A_i\).
Theorem (List and Pettit, 2001) If $X \subseteq \{a, b, a \land b\}$, there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.
**Baseline Result**

**Theorem (List and Pettit, 2001)** If \( X \subseteq \{ a, b, a \land b \} \), there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.

See personal.lse.ac.uk/LIST/doctrinalparadox.htm for many generalizations!
Whether or not judgment aggregation gives rise to serious impossibility results depends on how the propositions in the agenda are *interconnected*.
Introduction

Agenda Richness

Whether or not judgment aggregation gives rise to serious impossibility results depends on how the propositions in the agenda are interconnected.

**Definition** An agenda $X$ is **minimally connected** if

1. it has a minimal inconsistent subset $Y \subseteq X$ with $|Y| \geq 3$
2. it has a minimal inconsistent subset $Y \subseteq X$ such that $Y - Z \cup \{\neg z \mid z \in Z\}$ is consistent

for some subset $Z \subseteq Y$ of even size.
Theorem (Dietrich and List, 2007) For a minimally connected agenda $X$, an aggregation rule $F$ satisfies universal domain, collective rationality, systematicity and the unanimity principle iff it is a dictatorship.
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Pause for proof
Characterization Result

**Even-Number-Negation Property:** The agenda $X$ has a minimal inconsistent subset $Y \subseteq X$ such that $Y - Z \cup \{\neg z \mid z \in Z\}$ is consistent for some subset $Z \subseteq Y$ of even size.

**Median Property:** All minimally inconsistent subsets of the agenda $X$ contain exactly two propositions.
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Median Property: All minimally inconsistent subsets of the agenda \( X \) contain exactly two propositions.

Theorem (Dietrich and List, 2007) There exists regular, systematic and non-dictatorial aggregation rules on the agenda \( X \) iff \( X \) satisfies the median property or violates the even-number-negation property.

regular means collectively rational, universal domain and unanimity
Theorem (Nehring and Puppe 2006) There exists regular, monotonic, systematic and non-dictatorial aggregation rules on the agenda $X$ iff $X$ has the median property.
Characterization Result

**Total Blockedness:** Say $p$ conditionally entails $q$ if $p \neq \neg q$ and there is a minimally inconsistent subset $Y \subseteq X$ such that $p, \neg q \in Y$. 

Theorem (Nehring and Puppe 2006) There exists regular, monotonic, independent and non-dictatorial aggregation rules on the agenda $X$ iff $X$ is not totally blocked.
**Total Blockedness**: Say $p$ conditionally entails $q$ if $p \neq \neg q$ and there is a minimally inconsistent subset $Y \subseteq X$ such that $p, \neg q \in Y$. ($q$ can be deduced from $p$ using propositions in $X$)

$X$ is **totally blocked** if for any pair $p, q \in X$ there is a sequence $p = p_1, \ldots, p_m = q$ where each $p_{i-1}$ conditionally entails $p_i$. 
**Characterization Result**

**Total Blockedness:** Say $p$ conditionally entails $q$ if $p \neq \neg q$ and there is a minimally inconsistent subset $Y \subseteq X$ such that $p, \neg q \in Y$. ($q$ can be deduced from $p$ using propositions in $X$)

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**Theorem (Nehring and Puppe 2006)** There exists regular, monotonic, independent and non-dictatorial aggregation rules on the agenda $X$ iff $X$ is not totally blocked.
Many Variants!

Sen’s Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley’s Lover*, consider
Sen’s Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley’s Lover*, consider

\[ l: \text{Lewd reads the book}; \]
\[ p: \text{Prude reads the book}; \]
\[ l \rightarrow p: \text{If Lewd reads the book, then so does Prude.} \]
Introduction

Sen’s Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude’s moral outlook being corrupted).

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.
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Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.
Sen’s Liberal Paradox

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere.

   \[ l \quad p \quad l \rightarrow p \]

2. Unanimous desires of all individuals must be respected.

   So, society must be inconsistent!
### Sen’s Liberal Paradox

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**Sen’s Liberal Paradox**

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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere. $l$ is Lewd’s case, $p$ is Prude’s case.
Introduction

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2. Unanimous desires of all individuals must be respected.
Sen’s Liberal Paradox

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere. 
   
   $l$ is Lewd’s case, $p$ is Prude’s case

2. Unanimous desires of all individuals must be respected.

   So, society must be inconsistent!
Individual Rights

Call an individual $i$ decisive on a set $Y \subseteq X$ if any proposition in $Y$ is collectively accepted iff it is accepted by $i$, formally for each $(A_1, \ldots, A_n)$

$$F(A_1, \ldots, A_n) \cap Y = A_i \cap Y$$

**Minimal Rights** There exist (at least) two individuals who are each decisive on (at least) on proposition-negation pair \{p, \neg p\} $\subseteq X$. 
Indivdual Rights: Impossibility Theorem

**Theorem** If (and only if) the agenda is connected, there exists no aggregation function that satisfies universal domain, minimal rights and the unanimity principle.

Conclusions

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▶ Bayesian,...
Thank You!

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