Logic, Interaction and Collective Agency

Lecture 3

ESSLLI’10, Copenhagen

Eric Pacuit
TiLPS, Tilburg University
ai.stanford.edu/~epacuit

Olivier Roy
University of Groningen
philos.rug.nl/~olivier

August 17, 2010
Plan for Today

1. Intro: group/team preferences, frames and identification.
2. Unreliable Team Interaction (I).
4. Unreliable Team Interaction (II).
Team preferences and team reasoning.
The Main Question(s)

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- When there is **scope for cooperation**, what does it mean to say that the agents are **rational**?
  - ✓ **Analytical** question.
Team Preferences and team reasoning

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When there is scope for cooperation, what does it mean to say that the agents are rational?

✓ Analytical question.

As such the question is under-specified.

- One needs to specify the context of interaction (or of the game). This includes:
  - Information of the agents about all relevant aspects of interaction.
  - Additional group- or team-related aspects of the game.
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  - ✓ **Analytical question.**

- As such the question is **under-specified**.
  - One needs to specify the **context of interaction** (or of the game). This includes:
    - Information of the agents about all relevant aspects of interaction.
    - Additional **group- or team-related** aspects of the game.
What is a team?

   - Information about who’s in and who’s out.
   - Reasoning as group members.
   - Shared goal.
     - Group preference / utilities.

2. Shared commitments.
   - Shared intentions.
   - Sanctions for lapsing?
   - Shared praise[blame] for success[failure]?

3. Common knowledge (beliefs?) of the above?
What is a team?

1. **Group identification.**
   - Information about who’s in and who’s out.
   - **Reasoning as group members.**
   - **Shared goal.**
     - Group preference / utilities.

2. **Shared commitments.**
   - Shared intentions.
   - Sanctions for lapsing?
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3. **Common knowledge (beliefs?) of the above?**
Group- or team- preferences.

Groups or teams may have their own objectives/goals/“preferences”:

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- “Many hands write The Economist, but it speaks with a collective voice.

http://www.economist.com
Group- or team- preferences.

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Team Preferences and team reasoning

Group- or team- preferences.

Groups or teams may have their own objectives/goals/“preferences”:”

► “Many hands write The Economist, but it speaks with a collective voice. Leaders are discussed, often disputed, each week in meetings that are open to all members of the editorial staff. Journalists often co-operate on articles. And some articles are heavily edited. The main reason for anonymity, however, is a belief that what is written is more important than who writes it. As Geoffrey Crowther, editor from 1938 to 1956, put it, anonymity keeps the editor ”not the master but the servant of something far greater than himself. You can call that ancestor-worship if you wish, but it gives to the paper an astonishing momentum of thought and principle.”

http://www.economist.com
Group preferences vs shared preferences.

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Group preferences are often recognized as constitutive of the team. • "We're in the same boat." • Part of what it means to be a team member seems to adopt the team objectives, or at least to reason on the basis of them.
Group preferences vs shared preferences.

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- Preferable individually (?).
Team Preferences and team reasoning

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▶ Preferable individually (\(?\)).
▶ Preferable for the team \{Eric, Olivier\} (\(?\)).
Group preferences vs shared preferences.

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- Where do team preferences come from?
  - Function of the members’ preferences?
    - Preference aggregations impossibilities are looming.
Group preferences vs shared preferences.

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- Where do team preferences come from?
  - Function of the members’ preferences?
    - Preference aggregations impossibilities are looming.
  - Primitives?
  - One recurring requirement: Paretian in the member’s preferences. I.e. If a profile is Pareto-optimal then it is also most preferred for the team.
Individual vs team reasoning

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Individual Reasoning.
(1)

(2)

(C)
Individual vs team reasoning

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Individual Reasoning.

(1) Either I work hard or I work minimally

(2)

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Individual Reasoning.
(1) Either I work hard or I work minimally

(2) I’m better off in all circumstances (*ceteris paribus*) when I work minimally.

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Individual Reasoning.

(1) Either I work hard or I work minimally

(2) I’m better off in all circumstances (*ceteris paribus*) when I work minimally.

(C) I should work minimally.
### Individual vs team reasoning

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Team Reasoning.

1. 

2. 

(C)
Individual vs team reasoning

\[
\begin{array}{c|cc}
 & \text{Hard Work} & \text{Minimal Work} \\
\hline
\text{Hard Work} & 3, 3 (3) & 0, 4 (2) \\
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**Team Reasoning.**

(1) Eric and I are a team.

(2) 

(C)
Team Preferences and team reasoning

Individual vs team reasoning

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Team Reasoning.

(1) Eric and I are a team.

(2) (Work hard, Work hard) is the best option for the team.
Team Preferences and team reasoning

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**Team Reasoning.**

(1) Eric and I are a team.

(2) (Work hard, Work hard) is the best option for the team.

(C) I should work hard.
Individual vs team reasoning

(1) Either I work hard or I work minimally

(2) I’m better off in all circumstances (ceteris paribus) when I work minimally.

(C) I should work minimally.

▶ Individual Reasoning.

(1) Eric and I are a team.

(2) (Work hard, Work hard) is the best option for the team.

(C) I should work hard.

▶ Team Reasoning.
Acting as team member  \Rightarrow \begin{bmatrix} \text{Team identification} & + & \text{Team reasoning} \end{bmatrix}
Acting as team member \( \Rightarrow \) \[
\begin{bmatrix}
\text{Team identification} \\
+ \\
\text{Team reasoning}
\end{bmatrix}
\]

\[\downarrow\]

Adopting the team’s preferences.
Acting as team member ⇒ \[
\begin{bmatrix}
\text{Team identification} & + & \text{Team reasoning}
\end{bmatrix}
\]
⇓

 Adopting the team’s preferences.

Claim: Acting as a group member is different than: 
Acting as team member ⇒ \[
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⇓

Adopting the team’s preferences.

Claim: Acting as a group member is different than:

- Individual action based on individual preferences.
Acting as team member \(\Rightarrow\) \[[\text{Team identification} + \text{Team reasoning}]\]

\[\downarrow\]

**Adopting the team’s preferences.**

**Claim:** Acting as a group member is different than:

- Individual action based on individual preferences.
- Individual action based on group preference.
(Unreliable) Team Interaction and Team Reasoning.
Bacharach (1999, 2006), Sugden (200X)
Step 1: Adding teams, conservatively

Definition
A game in strategic form $TI$ is a tuple $\langle A, S_i, v_i \rangle$ such that:

- $A$ is a finite set of agents.

- $S_i$ is a finite set of actions or strategies for $i$.

- $v_i : \Pi_{i \in A} S_i \rightarrow \mathbb{R}$ is an utility function that assigns to every strategy profile $\sigma \in \Pi_{i \in A} S_i$ the utility of that profile for agent $i$. 
Step 1: Adding teams, conservatively

Definition

A team interaction $TI$ is a tuple $\langle A, M, S_i, v_k \rangle$ such that:

- $A$ is a finite set of agents.
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Step 1: Adding teams, conservatively

Definition

A team interaction $TI$ is a tuple $\langle A, M, S_i, \nu_k \rangle$ such that:

- $A$ is a finite set of agents.
- $M \subseteq 2^A$ is the set of teams.
- $S_i$ is a finite set of actions or strategies for $i$.
- $\nu_i : \prod_{i \in A} S_i \rightarrow \mathbb{R}$ is an utility function that assigns to every strategy profile $\sigma \in \prod_{i \in A} S_i$ the utility of that profile for agent $i$. 
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A team interaction $TI$ is a tuple $\langle A, M, S_i, v_k \rangle$ such that:

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- $v_k : \Pi_{i \in A} S_i \rightarrow \mathbb{R}$ is an utility function that assigns to every strategy profile $\sigma \in \Pi_{i \in A} S_i$ the utility of that profile for the team $k \in M$. 
Few remarks:

1. Team interactions are generalizations of games in strategic form:
   • Given a set of agents $A = \{1, 2, \ldots, i\}$, the team interaction such that $M = \{\{1\}, \{2\}, \ldots, \{i\}\}$ is a game in strategic form.

2. Only individuals take action, but sometimes they act for a team. How:
   • For any team $k \in M$, call $\alpha_k \in \Pi_{i \in k} S_i$ a protocol for $k$, and write $\alpha$ for a protocol for all team $k \in M$. $P = \Pi_{k \in M} \Pi_{i \in k} S_i$ is the set of all protocols.
Few remarks:

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Few remarks:

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2. Only individuals take action, but sometime they act for a team. How:
   - For any team $k \in M$, call $\alpha^k \in \Pi_{i \in k} S_i$ a protocol for $k$, and write $\alpha$ for a protocol for all team $k \in M$. $\Psi = \Pi_{k \in M} \Pi_{i \in K} S_i$ is the set of all protocols.
Step 2: Types and Uncertainty

Definition

A type space for a team interaction $TI$ is a tuple:

$$T = \langle S, \{ T_i \in A, \Omega \} \rangle$$

- $T_i = \{ k \in M : i \in k \}$ is a set of types for player $i$.
- $S$ a set of signal, the uncertainty domain.
Definition
A type space for a team interaction \( TI \) is a tuple:

\[
\mathcal{T} = \langle S, \{ T_i \}_{i \in A}, \Omega \rangle
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- \( T_i = \{ k \in M : i \in k \} \) is a set of types for player \( i \).
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Step 2: Types and Uncertainty

Definition

A type space for a team interaction $T_I$ is a tuple:

$$\mathcal{T} = \langle S, \{ T_i \}_{i \in \mathcal{A}}, \Omega \rangle$$

- $T_i = \{ k \in M : i \in k \}$ is a set of types for player $i$.
- $S$ a set of signal, the uncertainty domain.
- A state is a tuple:

$$(s, t) = (s, (t_1, \ldots, t_n))$$
Step 2: Types and Uncertainty

Definition
A type space for a team interaction $TI$ is a tuple:

$$T = \langle S, \{ T_i \}_{i \in A}, \Omega \rangle$$

- $T_i = \{ k \in M : i \in k \}$ is a set of types for player $i$.
- $S$ a set of signal, the uncertainty domain.
- $\Omega$ is a probability distribution on the set of states.
  - A Common Prior.
Terminology and Remarks:

- A state is a tuple:

\[(s, t) = (s, (t_1, \ldots, t_n))\]

At each state, each agent belong to one and only one team.
Unreliable Team Interaction

Terminology and Remarks:

▶ A state is a tuple:

\[(s, t) = (s, (t_1, \ldots, t_n))\]

▶ An unreliable team interaction (UTI) a pair \(\langle TI, T \rangle\) such that

\(TI\) is a team interaction and \(T\) is a type space for it.
Unreliable Team Interaction

Terminology and Remarks:

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- An protocol for team \(k\) in an UTI is an function which gives,
  for each member \(i\) of \(k\) and each signal and action in \(A_i\).
Unreliable Team Interaction

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- An **protocol** for team \(k\) in an UTI is a function which gives,
  for each member \(i\) of \(k\) and each signal and action in \(A_i\).

- **Conditioning** gives the functions \(\lambda_i : T_i \rightarrow \Delta(S \times T_{-i})\):
  \[
  \lambda_i(t_i)(s, t) = \frac{\Omega((s, t) \cap t_i)}{\Omega(t_i)}
  \]
Ex Ante Expected Value and Equilibrium

\[ \text{EV}_k(\alpha) = \sum_{(s, t)} \Omega(s, t) v_k(\alpha_{t_1}(s), ..., \alpha_{t_n}(s)) \]

The protocol \( \alpha \) is an Ex Ante UTI-equilibrium iff, for all \( k \in M \), \( \alpha \in \arg\max_{\beta \in P} \beta_k(\alpha_k, \alpha_{\neg k}) \)
Ex Ante Expected Value and Equilibrium

Given an type space $T$ with Team Authority, for a team interaction $TI$, the *ex ante* expected value of protocol $\alpha$ for team $k$:

$$EV^k(\alpha) = \sum_{(s,t)} \Omega(s, t) v^k(\alpha^t_1(s), ..., \alpha^t_n(s))$$
Given an type space $\mathcal{T}$ with Team Authority, for a team interaction $TI$, the \textit{ex ante} expected value of protocol $\alpha$ for team $k$:

$$EV^k(\alpha) = \sum_{(s,t)} \Omega(s,t) v^k(\alpha_{t_1}^1(s), ..., \alpha_{t_n}^n(s))$$

The protocol $\alpha$ is a \textit{ex ante} UTI-equilibrium iff, for all $k \in M$,

$$\alpha \in \text{argmax}_{\beta \in \mathcal{P}}(EV^k(\beta^k, \alpha^{-k}))$$
Teams and utilities:
Unreliable Team Interaction

UTI, an example

Teams and utilities:

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- Teams ($M$). Either:
  - we decide alone: $I_O = \{\text{Olivier}\}$, $I_E = \{\text{Eric}\}$;
  - or as a team $C = \{\text{Olivier, Eric}\}$. 
Unreliable Team Interaction

UTI, an example

Teams and utilities:

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Teams (M). Either:
- we decide alone: \( I_O = \{ Olivier \}, I_E = \{ Eric \} \);
- or as a team \( C = \{ Olivier, Eric \} \).

Utilities for the (non-singleton) team is the average individual payoffs.
Unreliable Team Interaction

UTI, an example

Type space:
Unreliable Team Interaction

UTI, an example

Type space:

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<td>$I_O$</td>
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<td>2/9</td>
<td></td>
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Unreliable Team Interaction

UTI, an example

Type space:

\[
\begin{array}{c|cc}
\Omega & I_E & C \\
\hline
I_O & \frac{4}{9} & \frac{2}{9} \\
C & \frac{2}{9} & \frac{1}{9} \\
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\]

- *A priori* there is a \(\frac{1}{3}\) chance for each agent to act as a team member.
UTI, an example

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▶ *A priori* there is a 1/3 change for each agent to act as a team member.

▶ The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.
UTI, an example

The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.
Unreliable Team Interaction

UTI, an example

The protocol $\alpha = (M, M, HH)$ is an ex-ante equilibrium.

$$EV^C(\alpha) = \sum_{T} \Omega(t) v^C(\alpha_{Olivier}^{t}, \alpha_{Eric}^{t})$$
The protocol $\alpha = (M, M, HH)$ is a \textit{ex-ante} equilibrium.

$$EV_{C}(\alpha) = \Omega(I_E, I_E)\nu^{C}(\alpha^{t}_{Olivier}, \alpha^{t}_{Eric}) + \Omega(C, I_E)\nu^{C}(\alpha^{t}_{Olivier}, \alpha^{t}_{Eric}) + \Omega(I_0, C)\nu^{C}(\alpha^{t}_{Olivier}, \alpha^{t}_{Eric}) + \Omega(C, C)\nu^{C}(\alpha^{t}_{Olivier}, \alpha^{t}_{Eric})$$
The protocol $\alpha = (M, M, HH)$ is an \textit{ex-ante} equilibrium.

$$EV^C(\alpha) = \Omega(I_E, I_E)\nu^C(M, \alpha_{Eric}^{t_{Eric}}) +$$

$$\Omega(C, I_E)\nu^C(\alpha_{Olivier}^{t_{Olivier}}, \alpha_{Eric}^{t_{Eric}}) +$$

$$\Omega(I_0, C)\nu^C(\alpha_{Olivier}^{t_{Olivier}}, \alpha_{Eric}^{t_{Eric}}) +$$

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\[
EV^C(\alpha) = \Omega(I_E, I_E) v^C(M, \alpha^t_{Eric}) + \\
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Unreliable Team Interaction

UTI, an example

The protocol $\alpha = (M, M, HH)$ is a \textit{ex-ante} equilibrium.

$$EV^C(\alpha) = \Omega(I^E, I^E)\nu^C(M, M) + \Omega(C, I^E)\nu^C(H, M) + \Omega(I_0, C)\nu^C(M, H) + \Omega(C, C)\nu^C(H, M)$$

▶ Team Authority:

If $t_i = k$ then $\alpha_i = \alpha^k_i(s)$
The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.

$$EV^C(\alpha) = \Omega(I_E, I_E)1 + \Omega(C, I_E)2 + \Omega(I_0, C)2 + \Omega(C, C)3$$
UTI, an example

The protocol $\alpha = (M, M, HH)$ is an ex-ante equilibrium.

$$EV^C(\alpha) = (4/9)1 + (2/9)2 + (2/9)2 + (1/9)3$$
The protocol $\alpha = (M, M, HH)$ is an *ex-ante* equilibrium.

\[ EV^C(\alpha) = 1.66 \]
\[ EV^C(L, L, ML) = 1.33 \]
\[ EV^C(L, L, LM) = 1.33 \]
\[ EV^C(L, L, LL) = 1 \]
UTI, an example

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$\triangleright$ MM maximizes $EV^C$ given $L, L$. 
The protocol $\alpha = (M, M, HH)$ is an ex-ante equilibrium.

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The protocol $\alpha = (M, M, HH)$ is a \textit{ex-ante} equilibrium.

- MM maximizes $EV^C$ given $L, L$.
- For either Olivier or Eric, $L$ is the only EV-maximizer.
  - A strategy $S_i$ of an \textit{individual} is strictly dominated in a game $G$ \textit{iff} it is strictly dominated in an $TI$ extending $G$ such that $\{i\} \in M$.
  $\Rightarrow$ A consequence of \textit{Team Authority}. 

Frames and Variable Frame Theory
A short digression.
Being part of a given team $\approx$ seeing the interactive situation through a specific frame.
Framing effect

*Logicophilia*, a virulent virus, threatens 600 participants of ESSLLI’10.

[Adapted from Tversky and Kahneman (1981)]
Framing effect

*Logicophilia*, a virulent virus, threatens 600 participants of ESSLLI’10.

1. You must choose between two prevention programs, resulting in:
   - **A**: 200 participants will be saved for sure.
   - **B**: 33 % chance of saving all of them, otherwise no one will be saved.

[Adapted from Tversky and Kahneman (1981)]
Framing effect

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72 % of the participants choose A over B.

[Adapted from Tversky and Kahneman (1981)]
Framing effect

*Logicophilia*, a virulent virus, threatens 600 participants of ESSLLI’10.

2. You must choose between two prevention programs, resulting in:
   - **A’**: 400 will not be saved, for sure.
   - **B’**: 33 % chance of saving all of them, otherwise no one will be saved.

   [Adapted from Tversky and Kahneman (1981)]
Logicophilia, a virulent virus, threatens 600 participants of ESSLLI’10.

2. You must choose between two prevention programs, resulting in:
   A’: 400 will not be saved, for sure.
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78 % of the participants choose B’ over A’.

[Adapted from Tversky and Kahneman (1981)]
Framing effect

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[Adapted from Tversky and Kahneman (1981)]
### Framing effect

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<td>⇒ 72% of the participants choose A over B.</td>
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- Standard decision- and game theory are **extensional** (see e.g. Schick ’97):
  - Choosing A and \( A \leftrightarrow B \) imply Choosing A.
Frames and Variable Frame Theory

Framing effect

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To a lesser extend, this is also true of the epistemic formalism that we have been using:
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- Note: this is different from logical omniscience.
**Framing effect**

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  - To a lesser extend, this is also true of the epistemic formalism that we have been using:
    - “Believing” $A$ and $\vdash A \leftrightarrow B$ imply “Believing” $B$.
- Decision problems in the logicophilia case to be an *intensional* context.
Variable Frame Theory through an example.

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
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<th>$x_4$</th>
</tr>
</thead>
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▶ Set of nondescript actions.
Variable Frame Theory through an example.

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- Set of **nondescript** actions.
- Utility functions defined for each player on the nondescript action profiles.
Variable Frame Theory through an example.

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- Set of **nondescript** actions.
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- **Frames** are predicates of actions profiles.
Variable Frame Theory through an example.

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- Set of non-descript actions.
- Utility functions defined for each player on the non-descript action profiles.
- **Frames** are predicates of actions profiles. E.g.

\[
F_{Team} = \{ H_T, M_T, \} \text{ with } H_T = \{ x_1, y_1, \} \text{ and } M_T = \{ x_2, y_2, \} .
\]
Frames and Variable Frame Theory

Variable Frame Theory through an example.

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- Set of **nondescript** actions.
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- **Frames** are predicates of actions profiles. E.g.
  
  $F_{Team} = \{H_T, M_T\}$ \quad $F_{Ind} = \{M_I, H_I\}$.
Variable Frame Theory through an example.

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- Set of **nondescriptive** actions.
- Utility functions defined for each player on the nondescriptive action profiles.
- **Frames** are predicates of actions profiles.
- Each frame $F$ occurs with a certain probability $v(F)$. E.g. If $v(F_{Team}) = 1$, i.e. common knowledge of team frame.
Variable Frame Theory through an example.

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Sugden [2005]: team reasoning under CK of team membership.
Team reasoning and pro-group I-mode.
Question: What is, if any, the difference between team or group agency and individual agency with group preferences?
Some terminology

- Acting as team member
  \[ \Rightarrow \text{Adopting the team’s preferences} + \text{Team reasoning} \]
Some terminology

- **Team agency / We-Mode**

  
  \[ \Rightarrow \begin{array}{l}
  \text{Adopting the team’s preferences + Team reasoning}
  \end{array} \]

From [Tuolema 1995, Forthcoming]
Some terminology

- **Team agency / We-Mode**
  \[\Rightarrow \text{Adopting the team’s preferences} + \text{Team reasoning}\]

- **Pro-Group I mode**
  \[\Rightarrow \text{Having the team’s preferences}\]

From [Tuolema 1995, Forthcoming]
Some terminology

- **Team agency / We-Mode**
  
  ⇒ \[
  \text{Adopting the team’s preferences} + \text{Team reasoning}
  \]

  - *We* write the paper together.

- **Pro-Group I mode**
  
  ⇒ \[
  \text{Having the team’s preferences}
  \]

  - *I* write the paper with *Eric*.

From [Tuolema 1995, Forthcoming]
Some terminology

▶ Team agency / We-Mode

\[ \Rightarrow \begin{bmatrix} \text{Adopting the team’s preferences + Team reasoning} \\ \bullet \text{We write the paper together.} \end{bmatrix} \]

▶ Pro-Group I mode \Rightarrow \begin{bmatrix} \text{Having the team’s preferences} \\ \bullet \text{I write the paper with Eric.} \end{bmatrix}

▶ Question: What is the specific import of team reasoning?

From [Tuolema 1995, Forthcoming]
Some terminology

- **Team agency / We-Mode**
  - Adopting the team’s preferences + Team reasoning
  - We write the paper together.

- **Pro-Group I mode**
  - Having the team’s preferences
  - I write the paper with Eric.

- **Question:** Can we reduce UTI to Bayesian Games, i.e. uncertainty about the payoffs?

  From [Tuolema 1995, Forthcoming]
Bayesian Games

Informally: structures to reasons about games with incomplete information, i.e. where there is uncertainty about the structure of the game.

Bayesian Games

Informally: structures to reasons about games with \textit{incomplete information}, i.e. where there is \textit{uncertainty} about the structure of the game.

▶ Each players can be of certain \textit{types}.

Informally: structures to reasons about games with incomplete information, i.e. where there is uncertainty about the structure of the game.

- Each players can be of certain types.
- Payoffs are dependent from strategy choice and types.

Bayesian Games

Informally: structures to reasons about games with incomplete information, i.e. where there is uncertainty about the structure of the game.

- Each players can be of certain types.
- Payoffs are dependent from strategy choice and types.
- Historical note: predates the use of types for imperfect and higher-order information! See [Brandenburger'10] and the talk today/tomorrow.

Bayesian Games

Formally:
Bayesian Games

Formally:

Definition

A Bayesian Game \( \mathcal{B} \) is a tuple \( \langle \mathcal{A}, S_i, T_i, v_i, \lambda_i \rangle \) such that:

- \( \mathcal{A} \) is a finite set of agents.
- \( S_i \) is a finite set of actions for \( i \). We write \( S \) for the set \( \prod_{i \in \mathcal{A}} S_i \) of all action profiles.
- \( T_i \) is a finite set of types for \( i \).
- A strategy \( \sigma_i : T_i \rightarrow A_i \) is a function assigning to each type of \( i \) an action in \( A_i \).
- \( v_i : (S \times T_i) \rightarrow \mathbb{R} \) is an utility function given that she is of type \( t_i \).
- \( \lambda_i : T_i \rightarrow \Delta(T_{-i}) \).
Bayesian Games

Formally:

Definition
A Bayesian Game $\mathcal{B}$ is a tuple $\langle \mathcal{A}, S_i, T_i, v_i, \lambda_i \rangle$ such that:

- $\mathcal{A}$ is a finite set of agents.
- $S_i$ is a finite set of actions for $i$. We write $S$ for the set $\Pi_{i \in \mathcal{A}} S_i$ of all action profiles.
- $T_i$ is a finite set of types for $i$.
- A strategy $\sigma_i : T_i \rightarrow A_i$ is a function assigning to each type of $i$ an action in $A_i$.
- $v_i : (S \times T_i) \rightarrow \mathbb{R}$ is an utility function given that she is of type $t_i$.
- $\Omega$ is a common prior over $(T)$. 
The \textbf{ex ante expected value} of profile $\sigma$ for player $i$ is defined as:

$$EV_i(\sigma) = \sum_t \Omega(t) v_i((\sigma_i(t_i), \sigma_{-i}(t_{-i})), t_i)$$

A \textbf{Bayesian equilibrium} is a strategy profiles $\sigma$ such that, for all $i$,

$$\sigma_i \in \arg\max_{\sigma'_i}(EV_i(\sigma'_i, \sigma_{-i}))$$
The ex ante expected value of profile $\sigma$ for player $i$ is defined as:

$$EV_i(\sigma) = \sum_{t_i} \Omega(t_i) \left( \sum_{t_{-i}} \Omega(t_{-i}/t_i) v_i((\sigma_i(t_i), \sigma_{-i}(t_{-i})), t_i) \right)$$

A Bayesian equilibrium is a strategy profiles $\sigma$ such that, for all $i$,

$$\sigma_i \in \arg\max_{\sigma'_i} (EV_i(\sigma'_i, \sigma_{-i}))$$
The **ex ante expected value** of profile $\sigma$ for player $i$ is defined as:

$$EV_i(\sigma) = \sum_{t_i} \Omega(t_i) \left( EV_i(\sigma/t_i) \right)$$

A **Bayesian equilibrium** is a strategy profiles $\sigma$ such that, for all $i$,

$$\sigma_i \in \arg\max_{\sigma'_i} (EV_i(\sigma'_i, \sigma_{-i}))$$
Let \( \langle TI, T \rangle \) be an unreliable team interaction with no external uncertainty. The Bayesian Game \( B_{UTI} \) based on \( \langle TI, T \rangle \) is defined as follow:

- \( \mathcal{A} \) is the set of individuals in \( TI \).
- \( S_i \) is the same as in \( TI \).
- \( T_i \) the same as in \( T \), i.e. types for \( i \):
  - When \( t_i = k \) we say that \( i \) is a benefactor for \( k \in M \).
- \( v_i(s, t_i) = v^{t_i}(s) \). (Ignoring states of uncertainty for now).
Definition
Let $\alpha$ be a protocol in a given $UTI$, and $\sigma$ a strategy profile in $B_{UTI}$. Then $\sigma$ agrees with $\alpha$ whenever, for all $t_i \in T_i$:

$$\alpha_{i}^{t_i} = \sigma_i(t_i)$$
If $\alpha$ is an UTI equilibrium, then there is a Bayesian Equilibrium $\sigma$ in $B_{UTI}$ that agrees with $\alpha$.

Proof.
Sketch:
1. If $\sigma$ agrees with $\alpha$, maximization of $EV_i((\sigma'_i, \sigma_{-i})/t_i)$ is equivalent to maximizing $EV^k((a_i, \alpha_{-i})/t_i)$ because:
   - For $s = \sigma(t_i, t_{-i})$; $v_i(s, t_i) = v^k(s)$ and;
If $\alpha$ is an UTI equilibrium, then there is a Bayesian Equilibrium $\sigma$ in $\mathcal{B}_{UTI}$ that agrees with $\alpha$.

**Proof.**

Sketch:

1. If $\sigma$ agrees with $\alpha$, maximization of $EV_i((\sigma'_i, \sigma_{-i})/t_i)$ is equivalent to maximizing $EV^k((a_i, \alpha_{-i})/t_i)$ because:
   - For $s = \sigma(t_i, t_{-i})$; $v_i(s, t_i) = v^k(s)$ and;
   - Strategy-wise, for all $j$, $\alpha^j = \sigma_j(t_j)$.
If $\alpha$ is an UTI equilibrium, then there is a Bayesian Equilibrium $\sigma$ in $\mathcal{B}_{UTI}$ that agrees with $\alpha$.

**Proof.**

Sketch:

1. If $\sigma$ agrees with $\alpha$, maximization of $EV_i((\sigma'_i, \sigma_{-i})/t_i)$ is equivalent to maximizing $EV^k((a_i, \alpha_{-i})/t_i)$ because:
   - For $s = \sigma(t_i, t_{-i})$; $v_i(s, t_i) = v^k(s)$ and;
   - Strategy-wise, for all $j$, $\alpha^j_i = \sigma_j(t_j)$.

2. If $\alpha$ is an UTI-equilibrium, then for all $i$, $\alpha^k_i$ maximizes $EV^k((\alpha^t_i, \alpha_{-i})/t_i)$.

$\square$
If $\alpha$ is an UTI equilibrium, then there is a Bayesian Equilibrium $\sigma$ in $B_{UTI}$ that agrees with $\alpha$.

$$\text{UTI-equilibria} \subseteq \text{Bayesian equilibria in } B_{UTI}.$$
If $\alpha$ is an UTI equilibrium, then there is a Bayesian Equilibrium $\sigma$ in $B_{UTI}$ that agrees with $\alpha$.

$\text{UTI-equilibria} \subsetneq \text{Bayesian equilibria in } B_{UTI}$. 
**UTI-equilibria ⊈ Bayesian equilibria**

<table>
<thead>
<tr>
<th>Hard Work</th>
<th>Minimal Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 3 ( (3) )</td>
<td>0, 4 ( (2) )</td>
</tr>
<tr>
<td>4, 0 ( (2) )</td>
<td>1, 1 ( (1) )</td>
</tr>
</tbody>
</table>

**Preliminary observation:**
- Let \( w \) be the probability that \( t_i = C \) for either player in a type space \( T \) for this TI. If \((M, M, HH)\) is an UTI-equilibrium then \( w \geq 1/3 \).

See [Bacharach, 1999] for details.
UTI-equilibria $\subsetneq$ Bayesian equilibria

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Preliminary observation:

- Let $w$ be the probability that $t_i = C$ for either player in a type space $T$ for this TI. If $(M, M, HH)$ is an UTI-equilibrium then $w \geq \frac{1}{3}$.

See [Bacharach, 1999] for details.
UTI-equilibria ⊆ Bayesian equilibria

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See [Bacharach, 1999] for details.
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- Let $T$ be a type space for this game such that $w = 1/6$. 
UTI-equilibria ⊈ Bayesian equilibria

Let $T$ be a type space for this game such that $w = 1/6$. The strategy profile $\sigma$ which agrees with $(M, M, HH)$ is an equilibria in the Bayesian Game for this UTI.
UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

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UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

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$$EV_E((M, H), (M, H)) = \sum_t \Omega(t) v_E((M, H)(t_E), (M, H)(t_O)), t_E)$$
**UTI-equilibria ⊆ Bayesian equilibria**

The Bayesian Game:

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$$EV_E(((M, H), (M, H))) =$$

$$\Omega(I_O, I_E)v_E(((M, M), I_E) + \Omega(I_O, C_E)v_E(((M, H), C_E) + \Omega(C_O, I_E)v_E(((H, M), I_E) + \Omega(C_O, C_E)v_E(((H, H), C_E)$$
UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

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$EV_E((M, H), (M, H)) =$

$$
\Omega(I_O, I_E)v_E((M, M), I_E) + \Omega(I_O, C_E)v_E((M, H), C_E) + \\
\Omega(C_O, I_E)v_E((H, M), I_E) + \Omega(C_O, C_E)v_E((H, H), C_E)
$$
UTI-equilibria $\subseteq$ Bayesian equilibria

The Bayesian Game:

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$$EV_E((M, H), (M, H)) =$$
$$\Omega(I_O, I_E)(1) + \Omega(I_O, C_E)v_E((M, H), C_E) +$$
$$\Omega(C_O, I_E)v_E((H, M), I_E) + \Omega(C_O, C_E)v_E((H, H), C_E)$$
UTI-equilibria $\subseteq$ Bayesian equilibria

The Bayesian Game:

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\[
EV_E((M, H), (M, H)) = \\
\Omega(I_0, I_E)(1) + \Omega(I_0, C_E)(2) + \\
\Omega(C_0, I_E)_{\nu_E}((H, M), I_E) + \Omega(C_0, C_E)_{\nu_E}((H, H), C_E)
\]
UTI-equilibria ⊊ Bayesian equilibria

The Bayesian Game:

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</tbody>
</table>

\[
EV_E((M, H), (M, H)) = \Omega(I_O, I_E)(1) + \Omega(I_O, C_E)(2) + \Omega(C_O, I_E)(4) + \Omega(C_O, C_E)v_E((H, H), C_E)
\]
**UTI-equilibria \( \subsetneq \) Bayesian equilibria**

The Bayesian Game:

<table>
<thead>
<tr>
<th>( I_0, I_E ) (0.69)</th>
<th>H</th>
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<tbody>
<tr>
<td>H</td>
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\[
EV_E((M,H),(M,H)) = 0.69(1) + 0.14(2) + 0.14(4) + 0.03(3)
\]
Team Reasoning and Pro-Group I-Mode

UTI-equilibria $\subseteq$ Bayesian equilibria

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$E V_E((M, H), (M, H)) = 1.62$
UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

$\begin{array}{|c|c|c|}
\hline
I_0, I_E (0.69) & H & M \\
\hline
H & 3, 3 & 0, 4 \\
M & 4, 0 & 1, 1 \\
\hline
\end{array}$

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$EV_E((M, H), (M, H)) = 1.62$

$EV_E((H, H), (M, H)) = .75$

$EV_E((M, M), (M, H)) = 1.56$

$EV_E((H, M), (M, H)) = 1.3$
Some Remarks

\[(L, L, HH) \text{ UTI-equ. only if } w \geq 1/3.\]

\[((M, H), (M, H)) \text{ Bayesian equ. even for } 1/6 \leq w < 1/3.\]
Some Remarks

\[(L, L, HH) \text{ UTI-equ. only if } w \geq \frac{1}{3}.\]

\[((M, H), (M, H)) \text{ Bayesian equ. even for } \frac{1}{6} \leq w < \frac{1}{3}.\]

- What’s going on in the zone \(\frac{1}{6} \leq w < \frac{1}{3}\)?
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\(((M, H), (M, H))\) Bayesian eq. even for \( 1/6 \leq w < 1/3 \).

▶ What’s going on in the zone \( 1/6 \leq w < 1/3 \)?
  • The profile \(((M, H), (M, H))\) is a sub-optimal Bayesian equilibrium for the team if \( 1/6 \leq w < 1/3 \).
    ▶ \(((L, L), (L, L))\) is also an Bayesian equilibria, which as a better expected value for the team.
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    - \(((L, L), (L, L))\) is also an Bayesian equilibria, which as a better expected value for the team.
  - Individual benefactors (Pro-group I-mode decision makers) who don’t team reason have no way to exclude this sub-optimal equilibrium.
  - Team Reasoning is the missing ingredient.
Recap and Conclusion

UTI-equilibria $\subseteq$ Bayesian equilibria in $\mathcal{B}_{UTI}$...
Recap and Conclusion

UTI-equilibria $\subseteq$ Bayesian equilibria in $\mathcal{B}_{UTI}$...BUT: an UTI equilibrium is a Nash equilibrium in the strategic game $G_{UTI}$ constructed as follow:

- Each team in the UTI is a separate agent in $G_{UTI}$.
- The actions of each “agent” in $G_{UTI}$ are the protocols in UTI.
- The utility for “agent” $k$ of a profile $\alpha$ in $G_{UTI}$ is the expected value of $\alpha$ in the UTI.
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$\Rightarrow$ UTI can be seen as games between teams.
Recap and Conclusion

UTI-equilibria \( \subsetneq \) Bayesian equilibria in \( B_{UTI} \).
- UTI-equilibrium *refines* Bayesian equilibrium.
Recap and Conclusion

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  - In social psychology: see [Hindriks, 2010] for a review.
- *Ex interim* rationality in UTI?
  - Still open.
Coming up next

▶ Other modes of shared attitudes: correlations.