Dynamic Doxastic Probability Logic

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1 Introduction

In this paper we will propose a dynamic doxastic probability logic which is inspired by two already existing systems and tries to meet them somewhere in between. On the one hand you have the original simple Kooi system of public announcement logic with hard public information only, and on the other hand you have the very rich and powerful BGK system of van Benthem, Gerbandy and Kooi [5].
Public announcement logic is built on a static logic which models the knowledge of agents by their information states. This system is made dynamic by the public announcement operator. The idea is that only truthful information can be announced and that the announcement is public (reaches everyone of the group).
Along the same line there is the dynamic doxastic logic, which except for knowledge, also models beliefs of agents. Beliefs are modeled by an ordering relation which specifies which world is more plausible than another world. The update modalities in this logic consist of updates in the plausibility order. Note that this logic can only express updates that make certain worlds top worlds, since the language of beliefs only allows one to speak about the top worlds. Changing the order among worlds without one of them becoming top worlds is not expressible here.
Extensions of this kind of dynamic epistemic logics with probabilistic information have been investigated by several researchers ([3], [4] and [5]). In this paper we will focus on the work of van Benthem, Gerbrandy and Kooi [5], which is a subsuming of the other two systems. The BGK system is a complete probability update logic. As static language they take a simple instance of a system described by Halpern and Tutle [2] and further developed by Fagin and Halpern [1] (of which we will use a simple instance in this paper). Then they use probabilistic update models as update modality

in the language. This is a very powerful and expressive update modality, but therefore also, relative to the update modalities discussed above, a very complicated update modality. The BGK system doesn't just extend the logics described above with probabilities, but also with event models used for updating.

The aim of this paper is to give a probabilistic epistemic update logic, which uses some of the same techniques as the BGK system to get probabilities into the language, but with the probabilistic update modalities more in the line of public announcement logic. So we will not use action models to update the information state of an agent, but we will just use expressions to update.

A great advantage of such a system is that it is able to model a lot more situations than the non-probabilistic logics described above. Such as modeling some kind of memory by taken into account how certain you was about φ before someone announced φ. But also that if a lot of people tell you that φ is the case you become more and more certain that φ is the case. Taking a probabilistic update modality also allows you to update with more or less confidence. If you're observing something you can be less certain about what you saw when it was far away. Or if someone telling you something you can take into account how reliable you consider information to be if it is uttered by this specific person. All these arguments are also raised by van Benthem, Gerbrandy and Kooi [5] for motivating their system. Actually the system proposed there is more powerful then the system proposed in this paper, but the system proposed here is simpler and the update modalities are more in line with well known systems as public announcement logic. This can have its advantage in situations were you don't need such a powerful system as GBK, but you do need more expressive power than a dynamic epistemic logic without probabilities can give you. In this paper two examples will be provided for showing that there are situation in which such an intermediate system gives one enough expressive power but a non-probabilistic logic will not. Especially games where you know the possible outcomes is such a situation, since you then don't need the possibility to expand the possible worlds you're considering.

In the next section we will describe the static logic with probabilities which will be used as a building block for the dynamic logic with probabilities, described in section 3. In section 4 we give a real live example where we can use the proposed logic to describe a person's information state. Then in section 5 we give an example of how this logic can be used to choose the best move in a game for two players. In the end there will be some discussion about the relation with previous work followed by a conclusion of the findings in this paper.

2 Static Doxastic Probability Logic

The static part of the doxastic probability Logic will be basically the same as the static part for epistemic probability logic as described by van Benthem, Gerbrandy and Kooi [5]. The difference will be that the epistemic relation is not explicit in the model for the doxastic probability logic described here and instead of the logical operator $P_i(\varphi)m = x$ a new name for the same operator $B_{xi}(\varphi)$ will be introduced. The reason for this change is to make the meaning of the operator more intuitive (agent i believes that φ is the case with a certainty of x).

Definition 1 (Doxastic Probability Models) *Given a set of agents Ag and a set of propositional variables At. A Doxastic Probability Model is a structure $M = (S, P, V)$ such that:*

- *S is a finite non-empty set of states,*
- *$P : Ag \rightarrow (S \rightarrow (S \rightarrow [0, 1]))$ assigns a probability function over S to each agent $i \in Ag$ and each state $s \in S$ (the probability assigned to t by the probability function assigned to i at s is denoted as $P_i(s)(t)$),*
- *V assigns a set of states to each propositional variable.*

We leave the epistemic relation out of the model, since you can infer this relation from the probability function (only for the equivalence class of the actual world). Given that some state s is the actual world in some model M, we have that $P_i(s)(t) > 0$ iff agent i can not distinguish between s and t (he does not know whether the actual world is s or t) and $P_i(s)(t) = 0$ otherwise. Note that if $P_i(s)(t) > 0$ and $P_i(s)(t') > 0$ agent i can also not distinguish between t and t'. And similar if $P_i(s)(t) > 0$ and $P_i(s)(t') = 0$ agent i can distinguish between t and t'. This implies that if for agent i some world w get probability 0, agent i *knows* that we are not in that world w [1].

Definition 2 (Static Doxastic Probability Language) *Given a set of agents Ag and a set of propositional variables At. The Static Doxastic Probability Language is defined by:*

[1] It may seems a bit un-intuitively that to belief something for 100 percent means that you know that it is the case, but later on will be clear why it will be interpreted this way.

$$\begin{array}{ll} \varphi ::= & p|\neg\varphi|(\varphi\wedge\varphi)|B_{Xi}\varphi \\ X ::= & [0,1] \end{array}$$

where $p \in At$ and $i \in Ag$.

Definition 3 (Static Doxastic Probability Semantics)

$$\begin{array}{lll} M,s \models p & \textit{iff} & s \in V(p) \\ M,s \models \neg\varphi & \textit{iff} & M,s \not\models \varphi \\ M,s \models \varphi \wedge \psi & \textit{iff} & M,s \models \varphi \textit{ and } M,s \models \psi \\ M,s \models B_{xi}\varphi & \textit{iff} & \left(\sum_{\{t \in S | M,t \models \varphi\}} P_i(s)(t) \right) = x \end{array}$$

The belief operator defined here gets a certain probability (a number between 0 and 1). You can read $B_{xi}\varphi$ as agent i beliefs for $x \cdot 100$ percent that φ is the case.

We will assume that the probability function of the model will be such that

$$\left(\sum_{t \in S} P_i(s)(t) \right) = 1$$

This means that we will not allow for a probability distribution that assign a chance of 0 to every possible world. It also implies together with the semantics for B_{xi} the following axiom:

$$B_{xi}\varphi \leftrightarrow B_{(1-x)_i}\neg\varphi$$

We will also assume, as we have already stated in a more informal way earlier and for reasons which we will discuss later on in the paper, that if an agent beliefs φ with a certainty of 100 percent, that he knows that φ. This means that

$$\left(\sum_{\{t \in S | M,t \models \varphi\}} P_i(s)(t) \right) = 1 \textbf{ implies that } M,s \models \varphi$$

Thus also

$$\left(\sum_{\{t \in S | M,t \models \varphi\}} P_i(s)(t) \right) = 0 \textbf{ implies that } M,s \not\models \varphi$$

3 Dynamic Doxastic Probability Logic

The simplest update policy is the policy in public announcement logic. In public announcement logic only truthful information can be announced. An

agent is updating his information state after φ is announced such that the only worlds he is still considering possible are φ worlds.
Other explored update policies are those of radical upgrade and conservative revision in dynamic doxastic logic. Upgrading your belief radically with φ means that all φ worlds become top worlds in your belief state. Thus in the plausibility ordering all φ worlds are more plausible than $\neg\varphi$ worlds. In a conservative revision of φ only the top of the φ worlds become top worlds in your belief state and for all other worlds the plausibility order stays the same.
In the van Benthem, Gerbrandy and Kooi system the upgrade policy is defined by their product update model. The new probabilities are calculated by the arithmetic product of the prior probabilities for the world s' (the old $P_i(s)(t)$), the probability that action e actually occurs in s', and the probability that agent i assigns to e (the probability that e occurred). To obtain a proper probability measure the compute product value is normalized.

In this paper we will propose an update modality likewise as in public announcement, but we will combine it with probabilities in a similar way as is done in the BGK system.

Definition 4 (Probability update) *The probability update $\uparrow_x \chi$ changes the current probability function $P_i(s)(t)$ in M, s to a new model $M \uparrow_x \chi$ as follows:*

- $S \uparrow_x \chi = S$
- **If** $M, t \models \chi$ *then*

$$P_i \uparrow_x \chi(s)(t) = \frac{P_i(s)(t) \cdot x}{\sum_{\{t \in S \mid t \models \chi\}} P_i(s)(t) \cdot x + \sum_{\{t \in S \mid t \not\models \chi\}} P_i(s)(t) \cdot (1-x)}$$

  If $M, t \not\models \chi$ *then*

$$P_i \uparrow_x \chi(s)(t) = -\frac{P_i(s)(t) \cdot (1-x)}{\sum_{\{t \in S \mid t \models \chi\}} P_i(s)(t) \cdot x + \sum_{\{t \in S \mid t \not\models \chi\}} P_i(s)(t) \cdot (1-x)}$$

- $V \uparrow_x \chi = V$

The probability update used here will as well update the χ worlds with probability x as also update the $\neg\chi$ worlds with probability $1-x$. This means that when you upgrade χ with a probability of 1, then you downgrade $\neg\chi$ at the same time with a probability of 0. Thus after normalizing all $\neg\chi$ worlds will have probability 0. When a certain world has the probability 0 there is no update possible such that it will get a higher probability again. That is the reason why in this system $B_0\varphi$ means that you know that φ is not the case and therefore also know that φ is the case ($B_1\varphi$).
The update function for calculating the probability will always return a value between 0 and 1 and summed together the probabilities of all possible worlds will always lead to a probability 1, unless the divisor of the update function becomes 0. We made the assumption that all possible worlds always sum up to 1 so we have to make sure that the divisor never becomes 0. Note that the only way for making the divisor 0 (assuming that before updating the probabilities of all possible worlds sum up to 1) is when you update a formula with probability 1 (or 0) of which you already knew that it could not have been the case (that it was the case). Therefore we can not allow for updates with probability 1 of something that is not true at the current state (equally we can not allow for updating a true statement with probability 0. We take a matching probability update modality in the dynamic doxastic probability language which takes that into account:

$$M, s \models [\uparrow_x \chi]\varphi \textit{ iff if } \left(\sum_{\{t \in S\}} P_i \uparrow_x \chi(s)(t) \right) = 1 \textit{ then } M \uparrow_x \chi, s \models \varphi$$

Next we will give the reduction axioms, which together with a complete axiom system for the static doxastic probability logic, will fully axiomatize the dynamic doxastic probability logic.
The reduction axioms for the logical connectives are the same as in public announcement logic and dynamic doxastic logic.

Definition 5 (Reduction axioms for the logical connectives)

$$\begin{array}{lll} [\uparrow_x \chi]p & \leftrightarrow & p \quad \textit{for all atomic propositions } p \\ [\uparrow_x \chi]\neg\varphi & \leftrightarrow & \neg[\uparrow_x \chi]\varphi \\ [\uparrow_x \chi]\varphi \wedge \psi & \leftrightarrow & [\uparrow_x \chi]\varphi \wedge [\uparrow_x \chi]\psi \end{array}$$

A problem arises when one wants to come up with a reduction axiom for probability belief. It is not possible to make a reduction axiom for $[\uparrow_x \chi]B_z\varphi$ only in terms of x, χ, z and φ. You know that the probability after the update of φ is z, but if there are possible worlds which are $\varphi \wedge \chi$ worlds and there

are also worlds which are $\varphi \wedge \neg\chi$ worlds you can not tell from only this knowledge how probable the $\varphi \wedge \chi$ worlds are and how probable the $\varphi \wedge \neg\chi$ worlds are. So then one can not find out which part of z was updated with x and which part was updated with $1 - x$.
So to make in principle infinitely (if there are only n updates possible, there are always formulas of length $l > n$ which are not reducible) many repetitions of updates possible we will need to keep track of the probability function. Therefore we define a new notion of probability belief.

Definition 6 (Dependent Probability Belief)

$$M, s \models B_{xi}{}^{P}\varphi \textit{ iff } \left(\sum_{\{t \in S | M, t \models \varphi\}} P(t) \right) = x$$

where P is a function that assigns values $[0, 1]$ to worlds $t \in S$.

Notice that the amount of worlds will not be changed by any logical operator of the language, only the probabilities assigned to it can be changed. The adjusted language of the logic will now be as follows:

Definition 7 (Dynamic Doxastic Probability Language) *The Dynamic Doxastic Probability Language is defined by:*

$$\begin{array}{rl} \varphi ::= & p|\neg\varphi|(\varphi \wedge \varphi)|B_{Xi}{}^{P}\varphi|[A]\varphi \\ A ::= & \uparrow_x \varphi \\ X ::= & [0, 1] \\ P ::= & S \to [0, 1] \end{array}$$

where $p \in At$ and $i \in Ag$.

This language is a real extension from the static language proposed since $B_{xi}\varphi \leftrightarrow B_{xi}{}^{P}$ where for every $t \in S$ we have that $M, s \models P(t) = y$ iff $P_i(s)(t) = y$.

Definition 8 (Reduction axiom for dependent probability belief)

$$\begin{array}{rl} [\uparrow_x \chi]B_{zi}{}^{P}\varphi \leftrightarrow & (x = 1 \wedge \chi \to B_{zi}{}^{P'}\varphi) \vee \\ & (x = 0 \wedge \neg\chi \to B_{zi}{}^{P'}(\varphi)) \vee \\ & (x \neq 1 \wedge z \neq 0 \wedge B_{zi}{}^{P'}\varphi) \end{array}$$

where for any world $t \in S$ *such that* $t \models \chi$

$$P'(t) = \frac{P(t) \cdot x}{\sum\limits_{\{t \in S | t \models \chi\}} P(t) \cdot x + \sum\limits_{\{t \in S | t \not\models \chi\}} P(t) \cdot (1-x)}$$

and all other worlds $t \in S$ *such that* $t \not\models \chi$

$$P'(t) = \frac{P(t) \cdot (1-x)}{\sum\limits_{\{t \in S | t \models \chi\}} P(t) \cdot x + \sum\limits_{\{t \in S | t \not\models \chi\}} P(t) \cdot (1-x)}$$

And where for any world $t \in S$ *such that* $t \models \chi$

$$P(t) = \frac{P'(t) \cdot (1-x)}{\sum\limits_{\{t \in S | t \models \chi\}} P'(t) \cdot (1-x) + \sum\limits_{\{t \in S | t \not\models \chi\}} P'(t) \cdot x}$$

and all other worlds $t \in S$ *such that* $t \not\models \chi$

$$P(t) = \frac{P'(t) \cdot x}{\sum\limits_{\{t \in S | t \models \chi\}} P'(t) \cdot (1-x) + \sum\limits_{\{t \in S | t \not\models \chi\}} P'(t) \cdot x}$$

The interesting case is of course when ${B_{zi}}^P = B_{zi}$. It is easy to see that when the probability with which is updated is between $\langle 0, 1\rangle$ that the two formulas say exactly the same. The condition for updating φ with a probability of 1 is that $M, s \models \varphi$ and for updating φ with a probability of 0 that $M, s \not\models \varphi$. This are exactly the conditions stated in the reduction rule for dependent probability belief.

4 Example from real live

In the Netherlands you learn very early in your live that one should never crush a cockroach with ones foot. The reason for this is that the females wear their eggs on their back and when you stamp on them they will spread their eggs. One hears this all the time, so it is not surprisingly we believe this with high confidence.
A Dutch friend (agent i) once spent her summer in China, leaving the Netherlands with a strong belief that one never should crush a cockroach to death (see figure 1). She went to China together with a friend of hers,

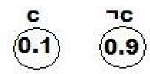

Figure 1: Belief state of i

visiting the Chinese family of that friend of hers. Then on one day she was working together with the Chinese family in the kitchen, when they saw a cockroach running on the kitchen floor. Just when the mother was about to crush the cockroach with her feet, agent *i* screamed 'NO!'. She explained to the bewildered family that the reason was not that she wanted to spare the animal, but that in no time their kitchen would swarm of cockroaches if one crushed the cockroach. They looked at her and started to laugh out loud (see figure 2). She didn't feel as confident anymore that one should not

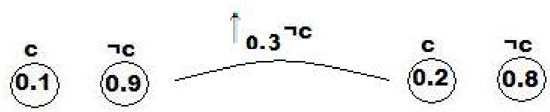

Figure 2: Revision of belief state of i

crush cockroaches. People always become somewhat more insecure when they're being laughed at. But still, people had told her all her life that one shouldn't crush cockroaches on their own kitchen floor. So she became somewhat more insecure but she still truly believed it. So to convince the others and get her confidence back she started to look on the internet for evidence for her belief. Unfortunately with the whole family waiting for the evidence she could not find a single page supporting her belief (see figure 3). She ended up not knowing what to belief anymore.

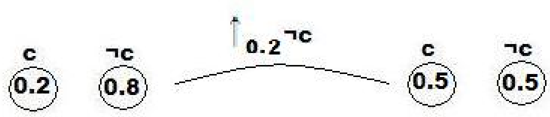

Figure 3: Further revision of belief state of i

5 Dynamic Doxastic Probability Logic and Games

Suppose a game that is characterize by the game tree as in the figure 4, where R means that it is a rational move and $\neg R$ means that it is an irrational move. Player E in the beginning has no idea what the other players

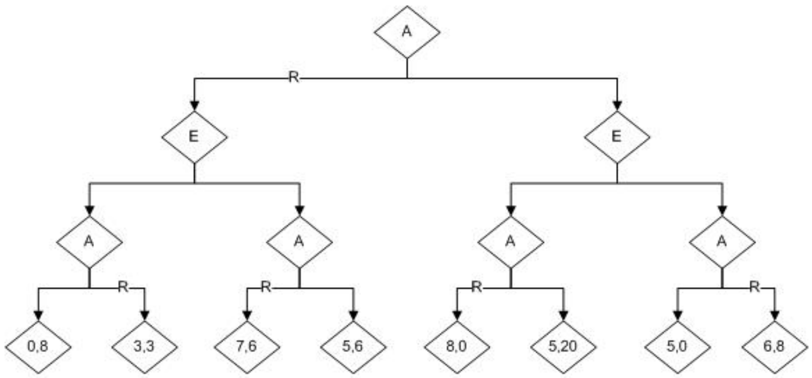

Figure 4: Game tree

move will be. So assigns to every move the probability of 0.5 (there are only 2 possible moves). Then since he is playing the game with a rational person he updates his information state with the fact that he thinks his opponent will play rational: $[\uparrow_{0.8} R_A]$. Thus al rational moves get probability 0.8 and all irrational moves get probability 0.2. Then you can calculate the expected value for E when A is on turn by taking for every possible move m the product of the probability that player A plays move m, times the expected value of move m, added together. And calculating the expected value for E when E is on turn by taking the highest expected value (thereby assuming that you yourself will always play the move with the highest expected value). The expected values at every move, after update are displayed in figure 5. Then the game begins and player A makes an irrational move. This means that E has to update his information state with the fact that A certainly not always plays rational. Thus E updates his information state with $\uparrow_{0.3} R_A$. This means that all rational moves now get probability $\frac{0.24}{0.38}$ and all irrational moves get probability $\frac{0.14}{0.38}$. Picture 6 shows the new expected values. This simple example of a game shows that even though player E still believes that agent A is rational, he can better play different from what the original dynamic doxastic logic will say. Since the original dynamic doxastic logic only takes into account what the player believes, but the doxastic proba-

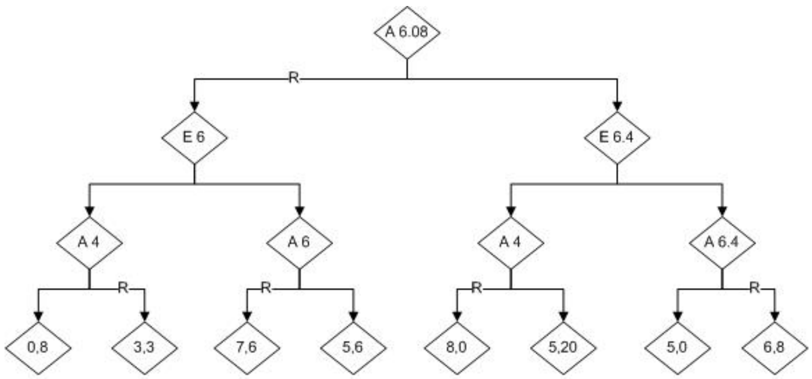

Figure 5: Expected value after update with $[\uparrow_{0.8} R_A]$

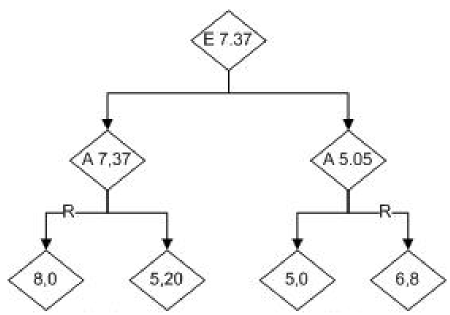

Figure 6: Expected value after update with $[\uparrow_{0.3} R_A]$

bility logic also takes into account how certain he is about his beliefs. And where doxastic logic only takes into account that the one outcome of the game is better than the other, the doxastic probability logic also takes into account how much better the one outcome of the game is compared to the other.

There can be a lot more said about games in combination with probability logic of belief states as is done in this example.
For example we will get for free using this logic the fact that if the opponent keeps playing irrational moves that after a few moves we don't belief anymore that our opponent is playing rational. After only one 'mistake' you most of the time do not immediately belief that the opponent is not playing rational, but you do become less certain about your belief that your oppo-

nent is playing rational. On the other hand if your opponent only makes rational moves you will become more and more certain that your opponent is playing rational.
Except for breaking down your belief that the opponent will play rational, this logic also makes it possible to learn your opponents strategy without having prior beliefs about such a strategy. For instance when your opponent plays move g you can also store which move he made by updating all g moves with a probability somewhat higher than 0.5. When your opponent keeps on playing move g, you will believe with more and more certainty that your opponent will play a g move next time.
In the example we didn't take into account that player E also beliefs that player A has a certain probability distribution and that A too can update this probability distribution according to the moves that E makes. So that Es belief about As belief will change if he's making a certain move. But this can of course also be modelled by this logic.

6 Relation to Previous work

The simple Kooi system, public announcement logic, is totally captured by the logic proposed in this paper. Namely knowledge from public announcement logic, $K\varphi$, is captured by $B_1\varphi$ (and $B_0\neg\varphi$) [2]. And the update modality $[!\varphi]\psi$ is captured by $[\uparrow_1 \varphi]\psi$ (and $[\uparrow_0 \neg\varphi]$). So the dynamic doxastic probability logic proposed in this paper is as least as expressive as public announcement logic.

Doxastic logic has a belief operator different from the doxastic probability logic proposed here. In doxastic logic agent i beliefs φ if and only if the top worlds (the worlds which agent i beliefs most) are φ worlds. This operator is not expressible in the language we use in this paper. But the language could be extended with such an operator which is expressible in the model used in this paper, i.e. agent i beliefs φ if and only if the most probable worlds according to is belief state are φ worlds. The same holds for safe belief which can be represented in this framework by an operator $B^+\varphi$ which is true if and only if φ is true in the current world and all worlds with an higher probability as the current world.
The update modalities used in dynamic doxastic logic, as putting all φ worlds up $[\Uparrow \varphi]$ or putting the best φ worlds up $[\uparrow \varphi]$, are not expressible

[2]We write $B_x\varphi$ for ${B_x}^P$ where for every $t \in S$ we have that $M, s \models P(t) = y$ iff $P(s)(t) = y$

in the logic described here. Since for imitating these modalities one needs a radicle update rule such as the Jeffrey Rule, in which one would replace ones original thoughts by the update. However in the update modality proposed in this paper we take into account the previous probability that you assigned to a certain world in similar way as in the BGK system. We do this to imitate some sort of memory, by pushing the update propositions only somewhat in the right direction. The update modality as described here does therefore (as in the BGK system) not satisfy the Jeffrey Rule. For a detailed discussion see [5]. None the less if one would want an update modality which replaces the old probability by a new one, it is straight forward to adjust the system (changing the update rules) accordingly.

We have showed that dynamic doxastic probability logic implies public announcement logic and that after adjustments it could also implies dynamic doxastic logic. An interesting question would then be whether the BGK system implies the dynamic doxastic probability logic. To answer this question there are certainly some adjustments needed which do not all seem to be trivial. In some way the operator from dynamic doxastic probability logic ${B_{xi}}^P$ should be made compatible with the operator from the BGK system $\alpha_1 \cdot P_i(\varphi_1) + ... + \alpha_n \cdot P_i(\varphi_n) \geq \beta$ to get the reduction axioms right.
Concerning the update modality there is also some work to do, since in the dynamic doxastic probability logic there is a constraint that the probabilities from all possible worlds together sum up to 1 and in the BGK system it is possible to have a model in which this sum would be 0.
In the calculation for the new probabilities after update are some other differences; in dynamic doxastic probability logic we have that after updating φ (with more than 50%) both φ worlds get a push forwards and $\neg\varphi$ worlds get a push backwards. As in BGK only the φ worlds will get pushed. And the probability distribution is different since in BGK it is $\frac{1}{3}$ prior probabilities, $\frac{1}{3}$ occurrence probabilities and $\frac{1}{3}$ observation probabilities. But in dynamic doxastic probability logic this is $\frac{1}{2}$ prior probabilities and $\frac{1}{2}$ occurrence and observation probabilities. In both systems it is not hard to overcome these calculation differences.
If all these differences can be made compatible the BGK system would indeed imply the dynamic doxastic probability logic in expressive power. Even though it requires more research to work the differences out, intuitively it should be the case.

7 Conclusion

The dynamic doxastic probability logic presented in this paper is more expressive and powerful as earlier developped epistemic logic as public announcement logic and dynamic doxastic logic. Since this logic doesn't only make two possible worlds comparable by means of a plausibility order, but it states how plausible or probable a possible world is.
The dynamic doxastic probability is less powerful and expressive than the BGK system. Since in the BGK system it is possible to extend the possible worlds considered, by adding extra propositions to the worlds (by means of action models). Leaving out the possibility to extend the possible worlds makes that the dynamic doxastic probability logic is less complicated than the BGK system. Hence easier to use and a lower computational complexity.
Two examples in this paper show that this logic is able to take memory into account. Also that it can distinguish between the (un)certainty with which a possible world is believed and it can take into account the hight of the costs/gain by playing a certain move (in game theory).
To conclude this paper describes a logic which is a nice and useful intermediate of two already existent logics. Which has the expressive power to describe the information state of an agent; concluding his knowledge, his beliefs (plausibility ordering) and how strong this believes are.

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