The Surprise Examination Paradox in Dynamic Epistemic Logic

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Introduction

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An important reason why this is so is that most philosophers pick out a preferred way of formulating the paradox and then they come up with a solution for that particular formulation. It has proven to be quite simple to come up with a reluctant formulation of the paradox for each solution. (e.g. Ayer (1973) to Quine (1953), Sorensen (1988) to Wright and Sudbury (1977))

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An important reason why this is so is that most philosophers pick out a preferred way of formulating the paradox and then they come up with a solution for that particular formulation. It has proven to be quite simple to come up with a reluctant formulation of the paradox for each solution. (e.g. Ayer (1973) to Quine (1953), Sorensen (1988) to Wright and Sudbury (1977))
Recently SEP has made its way in the dynamic epistemic logic literature: J. Gerbrandy (2007) and A. Baltag (2009,2010)

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In the end I will raise a problem that I believe is widespread in the literature on SEP regarding what surprise is.

The Paradoxical Scenario

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Given the teacher’s announcement a student will reason in the following manner:

- Assume that by Friday I will not have received an exam. Since there has to be an exam on one of the five days, it will have to be on Friday. However, I will then know the exam will be on Friday and I will not be surprised. Therefore Friday cannot be the day of the exam.

The Paradoxical Scenario (ctd.)

- Assume then that by Thursday I will not have received an exam. Since there has to be an exam on one of the five days and cannot be on Friday (by the previous argument), it has to be on Thursday. However, I will then know the exam will be on Thursday and I will not be surprised. Therefore Thursday cannot be the day of the exam.

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- Assume then that by Wednesday I will not have received an exam. Since there has to be an exam on one of the five days and it cannot be on Thursday or Friday (by the previous arguments), it has to be on Wednesday. However, I will then know the exam will be on Wednesday and I will not be surprised. Therefore Wednesday cannot be the day of the exam.

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However, an exam comes on Wednesday and the student will indeed be surprised. **What went wrong?**

Formalizing the scenario

$$
\begin{tikzcd}
we \arrow[loop above] \arrow[r, leftrightarrow] & \mathbf{th} \arrow[loop above] \arrow[r, leftrightarrow] & fr \arrow[loop above]
\end{tikzcd},
$$

$$exam = \bigvee_{i \in \{we, th, fr\}} i$$

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$$exam = \bigvee_{i \in \{we, th, fr\}} i$$

$$surprise_{Gerbrandy} = (we \wedge \neg K we) \vee (th \wedge [!\neg we]\neg K th) \vee$$
$$(fr \wedge [!\neg we][!\neg th]\neg K fr) \vee K\bot$$

$$surprise_{Baltag} = \bigwedge_{we \leq i \leq fr} (i \rightarrow [!(\bigwedge_{we \leq j < fr} \neg j)]\neg B i)$$

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The students reason that there can be no exam the following day, since if it were, it would not come as a surprise.

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The students reason that there can be no exam the following day, since if it were, it would not come as a surprise. However, the students do get an exam the next day, and are indeed surprised.

Gerbrandy's solution and tomorrow's surprise exam

The *Surprise* Examination	Tomorrow's *Surprise* Examination
$K_{students} exam$	$K_{students} exam$
Teacher announces $!surprise$	Teacher announces $!surprise$
$[!surprise] Kwe \vee th$	$[!surprise] K\perp$
$[!surprise] \neg surprise$	$[!surprise] surprise$
NO PROBLEM!	PROBLEM!

The Surprise *Examination* Paradox

In the kind of school where exams always come as a surprise and the number of exams students may receive during a n-day semester varies from 0 to n (the evaluation of the students is not made in terms of performance in exams), a teacher announces to his class: "Next week, there will be an exam (and only one!)."

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$$\begin{tikzcd} we \arrow[loop above] \arrow[r, leftrightarrow] & \mathbf{th} \arrow[loop above] \arrow[r, leftrightarrow] & fr \arrow[loop above] \arrow[r, leftrightarrow] & \neg we \wedge \neg th \wedge \neg fr \arrow[loop above] \end{tikzcd}$$

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The reasoning is just the same as in the scenario Baltag uses.

FAGM norm: $K\neg\varphi \vee [*\varphi]B(BEFORE\varphi)$
MAGM norm: $[*\varphi]\neg K\neg(BEFORE\varphi) \Rightarrow B(BEFORE\varphi)$

The *Surprise* Examination	The Surprise *Examination*
$K_{students}exam$	$K_{students}NEXTsurprise$
$Bsurprise \Rightarrow \neg Kexam$	$Bexam \Rightarrow \neg Ksurprise$
Teacher announces $*NEXTsurprise$	Teacher announces $*exam$
$[!NEXTsurprise]FALSE$	$[!exam]FALSE$
$[\Uparrow NEXTsurprise]FALSE$	$[\Uparrow exam]FALSE$
$[\uparrow NEXTsurprise]FALSE$	$[\uparrow exam]FALSE$
$[*_{FAGM}(NEXTsurprise)]FALSE$	$[*_{FAGM}exam]FALSE$
$\langle *_{MAGM}(NEXTsurprise)\rangle K\neg surprise$	$\langle *_{MAGM}exam\rangle K\neg exam$
Only one such upgrade: T	Only one such upgrade: $!^{-}$
$K_{students}\neg surprise$	$K_{students}\neg exam$
NO PROBLEM!	PROBLEM!

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$$surprise_{Baltag} = \bigwedge_{we \leq i \leq fr} (i \rightarrow [!(\bigwedge_{we \leq j < fr} \neg j)]\neg Bi)$$

But does this really capture the intuitive notion of surprise?

Surprise

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But does this really capture the intuitive notion of surprise? Consider the following two scenarios:

I don't believe that Inception is playing in Copenhagen, but I consider it possible. I go to the cinema in Copenhagen and I learn that it is actually playing. ($\varphi \wedge \neg B\varphi \wedge \neg K\neg\varphi$)
I believe that Inception is not playing in Copenhagen (say because I believe that the cinemas in Copenhagen only show Danish movies and Inception is not Danish). I go to the cinema in Copenhagen and I learn that it is actually playing.($\varphi \wedge B\neg\varphi$)

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I believe that Inception is not playing in Copenhagen (say because I believe that the cinemas in Copenhagen only show Danish movies and Inception is not Danish). I go to the cinema in Copenhagen and I learn that it is actually playing.$(\varphi \wedge B\neg\varphi)$
In which scenario will I be surprised?

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Also, Gerbrandy's idea is supported by their analysis.

$$\begin{aligned} \mathit{MismatchSurprise}(\mathit{exam}, \bot) =_{\mathit{def}} & \mathit{Datum}(\mathit{exam}) \wedge \mathit{Test}(\bot) \\ & \wedge \mathit{Bel}(\mathit{exam} \rightarrow \neg\bot) \end{aligned}$$

Conclusions

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1. The first step to a solution to SEP is to understand what the paradox really is.
2. There are reasons for looking for a new way of defining surprise - which might lead to some conditions on how belief should be defined ($B\perp$)