CS121 – Introduction to Artificial Intelligence – Spring 2008

Homework #1
(Search Problems)
Out: 4/9/08 — Due: 4/17/08 (at noon)

How to complete this HW: First copy this file; then type your answers in the file immediately below each question; finally print this file and return it in the drawer marked CS121 of a file cabinet located in the entryway of the Gates building next to office Gates 182 (see red arrow in http://ai.stanford.edu/~latombe/cs121/2008/map.jpg) no later than Thursday 4/17 at noon.

Your name: ……………………………………………………………………………………

Your email address: ……………………………………………………………………………

Note on Honor Code: You must NOT look at previously published solutions of any of these problems in preparing your answers. You may discuss these problems with other students in the class (in fact, you are encouraged to do so) and/or look into other documents (books, web sites), with the exception of published solutions, without taking any written or electronic notes. If you have discussed any of the problems with other students, indicate their name(s) here:
………………………………………………………………………………………………………
Any intentional transgression of these rules will be considered an honor code violation.

General information: Justify your answers, but keep explanations short and to the point. Excessive verbosity will be penalized. If you have any doubt on how to interpret a question, tell us in advance, so that we can help you understand the question, or tell us how you understand it in your returned solution.

Grading:

Problem#	Max. grade	Your grade
I	20	
II	30	
III	30	
IV	20	
Total	100	

I. Jug Problem (20 points)

Consider the following problem::

> "Three jugs are filled with water. Neither has any measuring marker on it. Each jug can be fully emptied in a drain. It can also be poured into another jug until it is empty or the receiving jug is completely filled (pouring stops immediately when one of these two conditions becomes true). No other action is possible. The jugs respectively measure 15, 7, and 3 gallons. One needs to measure out exactly 2 gallons."

1. Express this problem as a search problem, that is, give (1) the general description of a state, (2) the initial state, (3) the goal test, and (4) the successor function.
   [Notes: Do not list all possible states. For the successor function, it is not necessary that you write all successors of every state, but you should make it clear how one may derive the successor states of any given state. We do not ask you to describe a solution of the problem.]

2. Draw the search tree for this search problem down to depth 2 (recall that the root is at depth 0). What is the branching factor at depth 0? At depth 1?

II. 8-Puzzle (30 points)

Consider the 8-puzzle game, a 3×3 board of 9 tiles (Figure 1). Eight tiles are numbered 1 through 8. The 9^{th} tile is the empty tile. Each legal move consists of switching the positions of the empty tile and one of its adjacent tiles. For instance, in the state of the 8-puzzle shown in Figure 1, the empty tile can be switched with either tile 2, or tile 7.

8	2	
3	4	7
5	1	6

Figure 1: A state of the 8-puzzle game

1. Assume that the goal state *G* is as shown below, where all the tiles are sorted from left to right and top to bottom, with the empty tile at the bottom right:

1	2	3
4	5	6
7	8	

   What criterion can we use to determine whether a given initial state *I* can be transformed into *G* by a sequence of legal moves of the empty tile? [Hint: Adapt the criterion of slides 18-19 of class #2.]

2. Consider the search problem where the goal test GOAL? (see slide 11 of class #2) only requires the empty tile to be located at the center of the board. This goal test says nothing about the location of the other tiles.
    a. How many states of the 8-puzzle satisfy the goal test?
    b. Given an initial state of the 8-puzzle, how many goal states can be reached?

III. Four-Peg Towers-of-Hanoi Puzzle (30 points)

The four-peg version of the Towers-of-Hanoi puzzle (Figure 2) is as follows. Four pegs – A, B, C, and D – can each hold n rings at a time. There are n rings R_1, R_2, …, R_n, such that R_i is smaller than R_j for any $i < j$. A *legal* placement of the rings on the pegs requires that (1) whenever any two rings appear on the same peg, the smaller one is above the larger one and (2) all n rings must be on pegs. In the start placement, all rings are on peg A (the figure below shows the start state for n = 4). In the goal placement, all rings are on peg D. A *move* consists of moving a single ring from one peg to another. It is *legal* if it produces a legal placement. The goal of the puzzle is to find a sequence of legal moves that achieves the goal placement from the start placement.

1. Formulate this problem as a search problem, that is:
    a. Define a representation for each state. [We don't ask you to list all states! Only define a representation that is general enough to describe every state.]
    b. Give the initial and goal states in this representation.
    c. Define the successor function. [Here, we ask you to describe how one can compute the successors of a state.]
2. What is the total number of legal states?
3. How many successors can a state have at most?

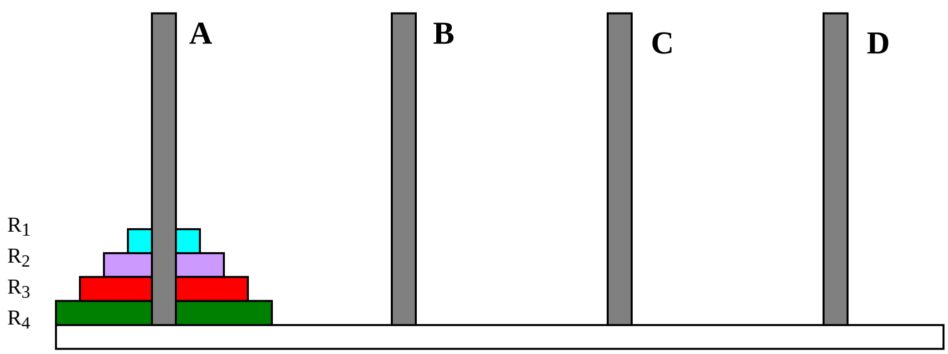

Figure 2: Towers-of-Hanoi puzzle (start placement)

IV. *n*-Queen Problem (20 points)

The goal of the 8-queen problem is to place 8 queens on a chessboard, such that no two queens attack each other. The n-queen problem generalizes this problem to an arbitrary number, $n > 1$, of queens and an $n \times n$ board.

One way to formulate the n-queen problem as a search problem is the following (this corresponds to formulation #2 given in the slides shown in class):

- The states are all the arrangements of k queens in the k leftmost columns of the board with no two queens attacking each other, where k = 0, 1, 2, ..., n,

- The initial state is the state with 0 queens on the board,
- Each of the successors of a state is obtained by adding one queen in a square that is not attacked by any queen already in the board, in the leftmost empty column,
- The goal test is "all *n* queens are on the board".

1. Explain why the number of states in the state space is at least $(n!)^{1/3}$.

2. In class you were told that the exact number of states for the 8-queen problem is 2,057. Compare this number to $(8!)^{1/3}$. Are the two numbers very different? If yes, why?