Constraint Propagation

(Where a better exploitation of the constraints further reduces the need to make decisions)

R&N: Chap. 5 + Chap. 24, p. 881-884

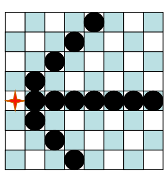
Constraint Propagation …

… is the process of determining how the constraints and the possible values of one variable affect the possible values of other variables

It is an important form of "least-commitment" reasoning

Forward checking is only on simple form of constraint propagation

When a pair ($X \leftarrow v$) is added to assignment *A* do:
For each variable *Y* not in *A* do:
For every constraint *C* relating *Y* to variables in *A* do:
Remove all values from *Y*'s domain that do not satisfy *C*



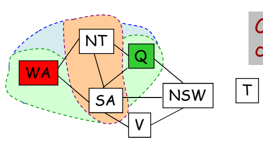
- n = number of variables
- d = size of initial domains
- s = maximum number of constraints involving a given variable ($s \leq n-1$)
- Forward checking takes O(nsd) time

Forward Checking in Map Coloring

Empty set: the current assignment {(WA ← R), (Q ← G), (V ← B)} does not lead to a solution

WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB
R	~~R~~GB	RGB	RGB	RGB	~~R~~GB	RGB
R	~~G~~B	G	R~~G~~B	RGB	~~G~~B	RGB
R	B	G	R~~B~~	B	~~B~~	RGB

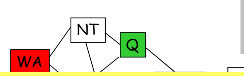
Forward Checking in Map Coloring



Contradiction that forward checking did not detect

WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB
R	~~R~~GB	RGB	RGB	RGB	~~R~~GB	RGB
R	~~G~~B	G	R~~G~~B	RGB	~~G~~B	RGB
R	B	G	R~~B~~	B	~~B~~	RGB

Forward Checking in Map Coloring



Contradiction that forward checking did not detect

Detecting this contradiction requires a more powerful constraint propagation technique

WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB
R	~~R~~GB	RGB	RGB	RGB	~~R~~GB	RGB
R	~~G~~B	G	R~~G~~B	RGB	~~G~~B	RGB
R	B	G	R~~B~~	B	~~B~~	RGB

Constraint Propagation for Binary Constraints

REMOVE-VALUES(X,Y)

1. removed ← false
2. For every value v in the domain of Y do
   - If there is no value u in the domain of X such that the constraint on (X,Y) is satisfied then
     a. Remove v from Y's domain
     b. removed ← true
3. Return removed

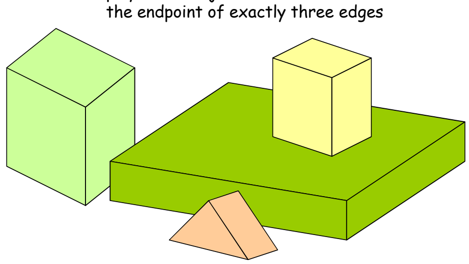
Constraint Propagation for Binary Constraints

AC3

1. Initialize queue Q with all variables (not yet instantiated)
2. While $Q \neq \emptyset$ do
   a. X ← Remove(Q)
   b. For every (not yet instantiated) variable Y related to X by a (binary) constraint do
      - If REMOVE-VALUES(X,Y) then
        i. If Y's domain = ∅ then exit
        ii. Insert(Y,Q)

Edge Labeling

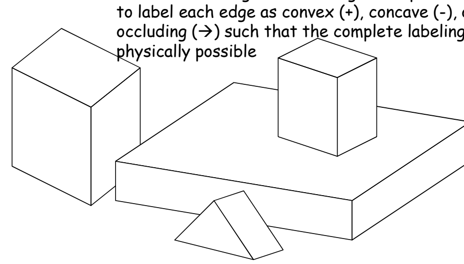
We consider an image of a scene composed of polyhedral objects such that each vertex is the endpoint of exactly three edges

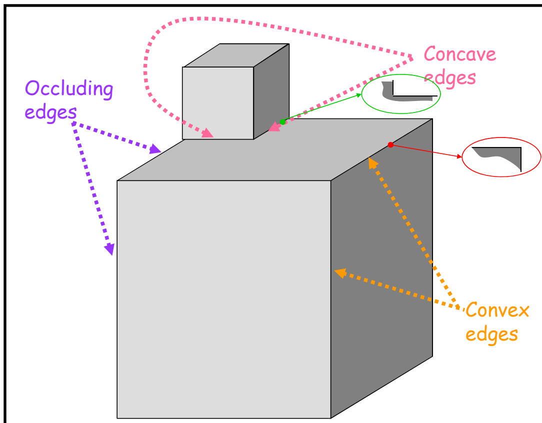


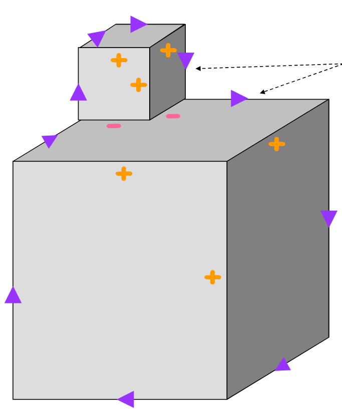
R&N: Chap. 24, pages 881-884

Edge Labeling

An "edge extractor" has accurately extracted all the visible edges in the image. The problem is to label each edge as convex (+), concave (-), or occluding (→) such that the complete labeling is physically possible

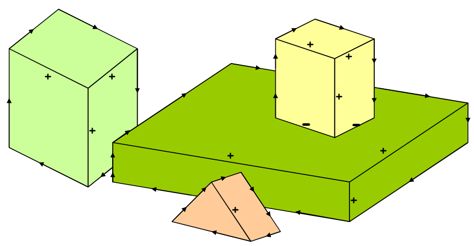






The arrow is oriented such that the object is on the right of the occluding edge

One Possible Edge Labeling



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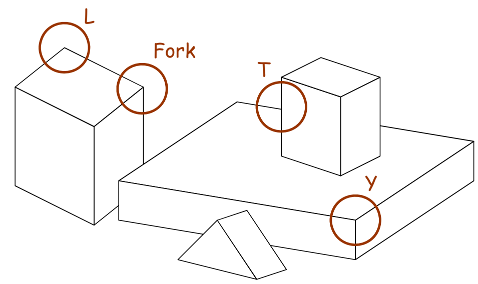
Junction Types



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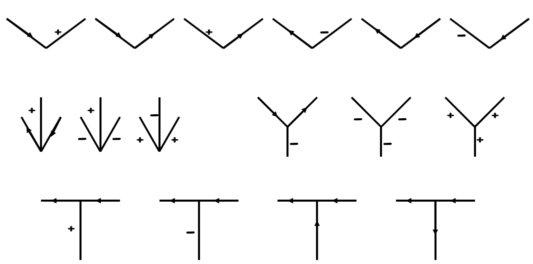
Junction Label Sets



(Waltz, 1975; Mackworth, 1977)

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Edge Labeling as a CSP

- A variable is associated with each junction
- The domain of a variable is the label set associated with the junction type
- Constraints: The values assigned to two adjacent junctions must give the same label to the joining edge

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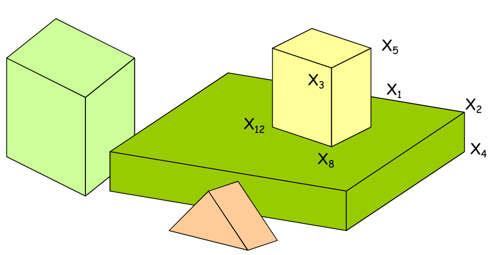
AC3 Applied to Edge Labeling

Q = (X_1, X_2, X_3, ...)



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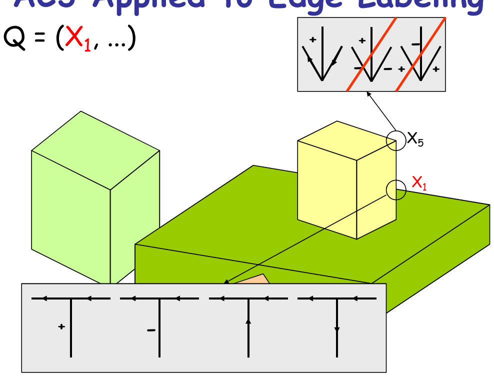
AC3 Applied to Edge Labeling

Q = (X_1, ...)



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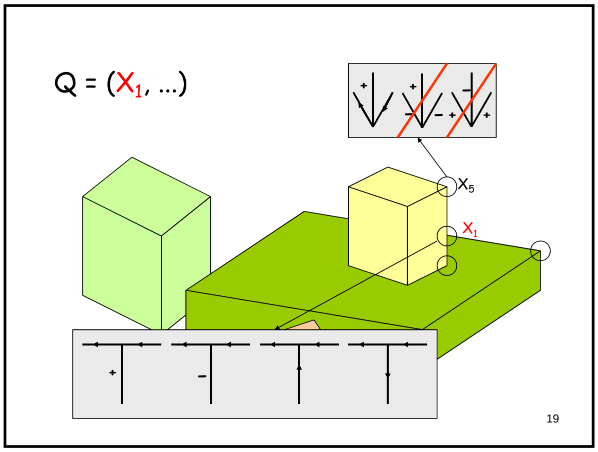
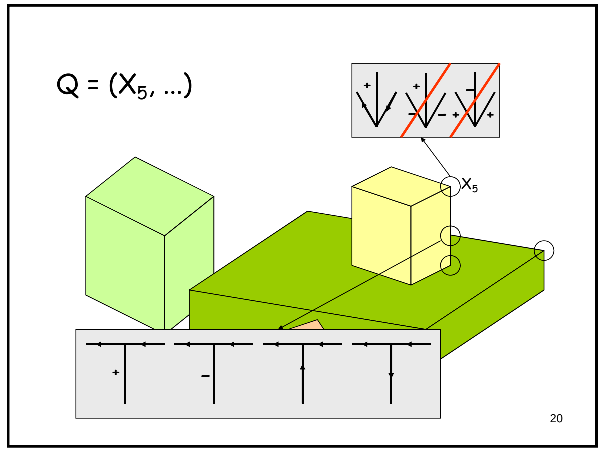
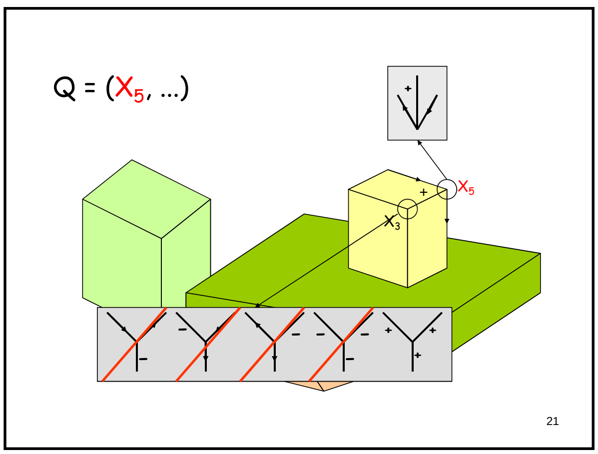
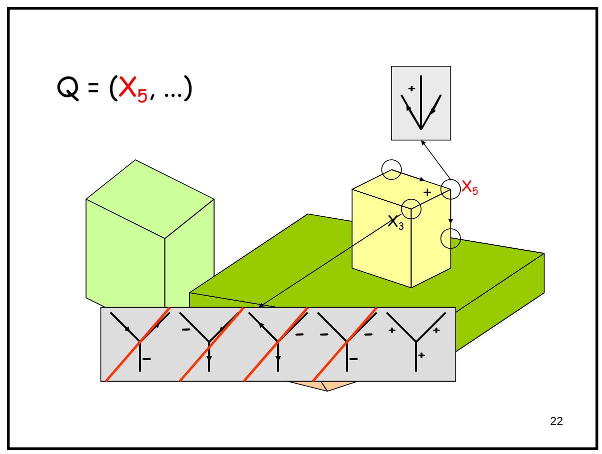
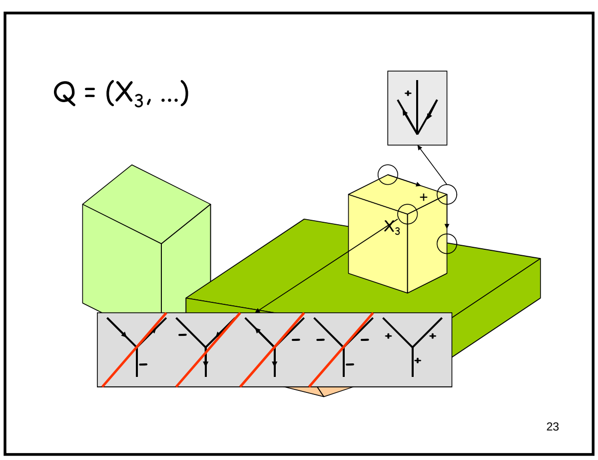
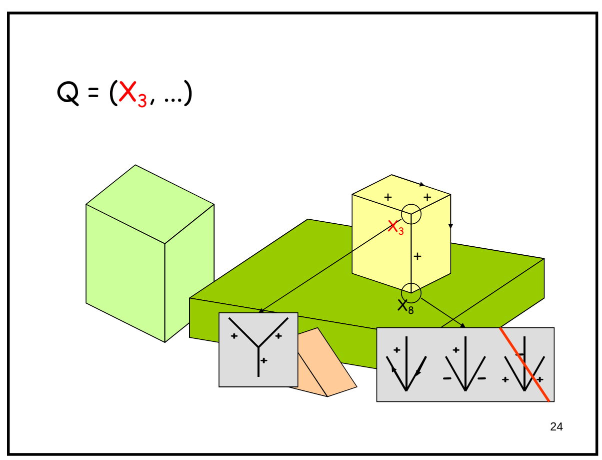

Q = (X1, …)
X5
X1
19


Q = (X5, …)
X5
20

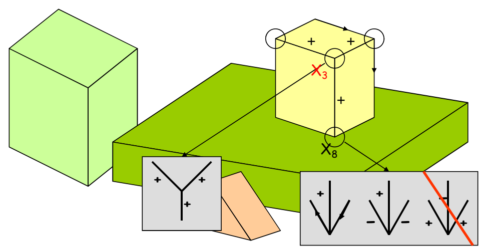

Q = (X5, …)
X5
X3
21


Q = (X5, …)
X5
X3
22


Q = (X3, …)
X3
23


Q = (X3, …)
X3
X8
24

Q = (X_3, …)



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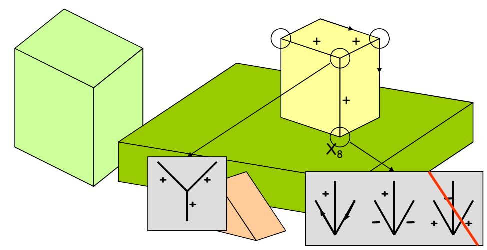
Q = (X_8, …)



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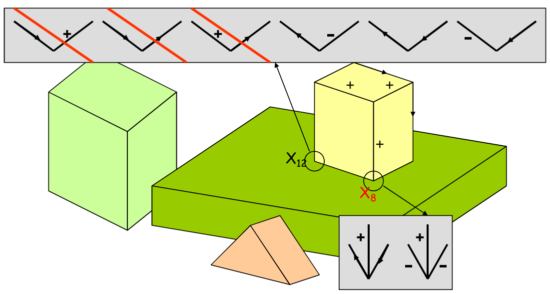
Q = (X_8, …)



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Complexity Analysis of AC3

- n = number of variables
- d = size of initial domains
- s = maximum number of constraints involving a given variable ($s \leq n-1$)
- Each variables is inserted in Q up to d times
- REMOVE-VALUES takes $O(d^2)$ time
- AC3 takes $O(n \times d \times s \times d^2) = O(n \times s \times d^3)$ time
- Usually more expensive than forward checking

AC3
1. Initialize queue Q with all variables (not yet instantiated)
2. While Q ≠ Ø do
   a. X ← Remove(Q)
   b. For every (not yet instantiated) variable Y related to X by a (binary) constraint do
      - If REMOVE-VALUES(X,Y) then
        i. If Y's domain = Ø then exit
        ii. Insert(Y,Q)

REMOVE-VALUES(X,Y)
1. removed ← false
2. For every value v in the domain of Y do
   - If there is no value u in the domain of X such that the constraint on (x,y) is satisfied then
     a. Remove v from Y's domain
     b. removed ← true
3. Return removed

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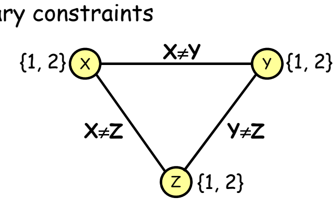
Is AC3 all that we need?

- No !!
- AC3 can't detect all contradictions among binary constraints



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Is AC3 all that we need?

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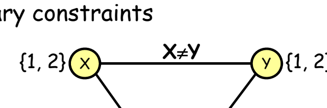
- AC3 can't detect all contradictions among binary constraints



REMOVE-VALUES(X,Y)
1. removed ← false
2. For every value v in the domain of Y do
   - If there is no value u in the domain of X such that the constraint on (X,Y) is satisfied then
     a. Remove v from Y's domain
     b. removed ← true
3. Return removed

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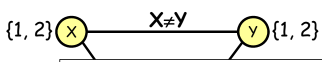
Is AC3 all that we need?

- No !!
- AC3 can't detect all contradictions among binary constraints

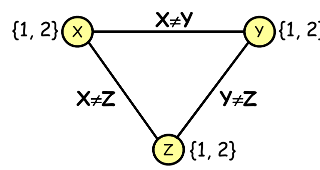


REMOVE-VALUES(X,Y
1. removed ← false
2. For every value v
   - If there is no
     that the const
     a. Remove v
     b. removed
3. Return removed

REMOVE-VALUES(X,Y,Z)
1. removed ← false
2. For every value w in the domain of Z do
   - If there is no pair (u,v) of values in the domains of X and Y verifying the constraint on (X,Y) such that the constraints on (X,Z) and (Y,Z) are satisfied then
     a. Remove w from Z's domain
     b. removed ← true
3. Return removed

Is AC3 all that we need?

- No !!
- AC3 can't detect all contradictions among binary constraints



- Not all constraints are binary

Tradeoff

Generalizing the constraint propagation algorithm increases its time complexity

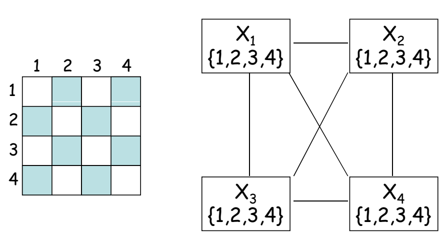
→Tradeoff between time spent in backtracking search and time spent in constraint propagation

A good tradeoff when all or most constraints are binary is often to combine backtracking with forward checking and/or AC3 (with REMOVE-VALUES for two variables)

Modified Backtracking Algorithm with AC3

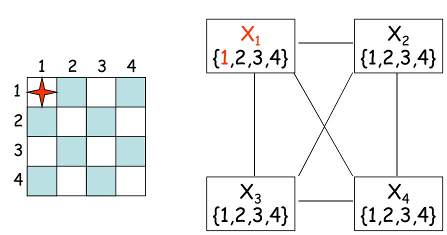
CSP-BACKTRACKING(A, var-domains)
1. If assignment A is complete then return A
2. Run AC3 and update var-domains accordingly
3. If a variable has an empty domain then return failure
4. X ← select a variable not in A
5. D ← select an ordering on the domain of X
6. For each value v in D do
   a. Add (X←v) to A
   b. var-domains ← forward checking(var-domains, X, v, A)
   c. If no variable has an empty domain then
      (i) result ← CSP-BACKTRACKING(A, var-domains)
      (ii) If result ≠ failure then return result
   d. Remove (X←v) from A
7. Return failure

A Complete Example: 4-Queens Problem



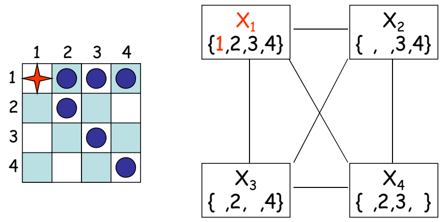
1) The modified backtracking algorithm starts by calling AC3, which removes no value

4-Queens Problem



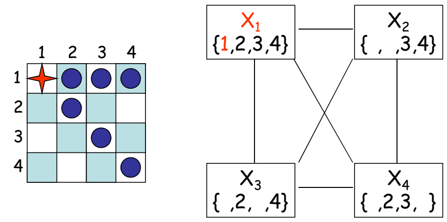
2) The backtracking algorithm then selects a variable and a value for this variable. No heuristic helps in this selection. X_1 and the value 1 are arbitrarily selected

4-Queens Problem



3) The algorithm performs forward checking, which eliminates 2 values in each other variable's domain

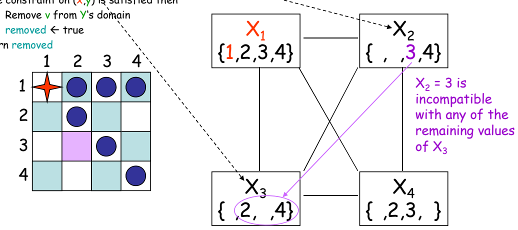
4-Queens Problem



4) The algorithm calls *AC3*

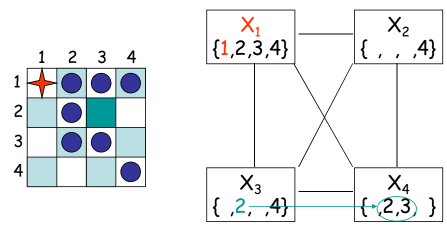
4-Queens Problem

REMOVE-VALUES(X,Y)
1. removed ← false
2. For every value v in the domain of Y do
   - If there is no value u in the domain of X such that the constraint on (x,y) is satisfied then
     a. Remove v from Y's domain
     b. removed ← true
3. Return removed



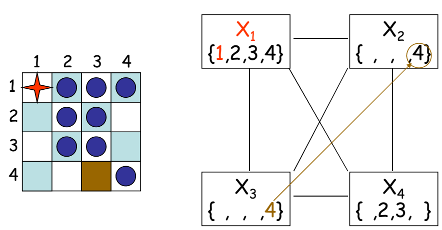
4) The algorithm calls *AC3*, which eliminates 3 from the domain of X_2

4-Queens Problem



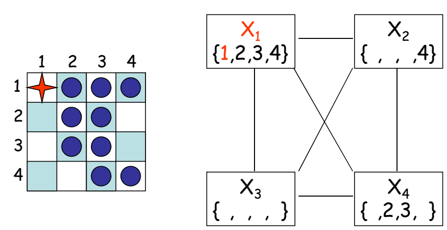
4) The algorithm calls *AC3*, which eliminates 3 from the domain of X_2, and 2 from the domain of X_3

4-Queens Problem



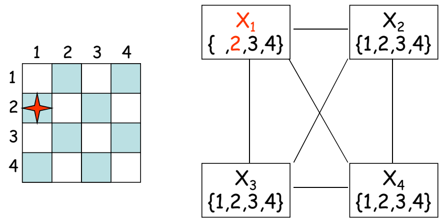
4) The algorithm calls *AC3*, which eliminates 3 from the domain of X_2, and 2 from the domain of X_3, and 4 from the domain of X_3

4-Queens Problem



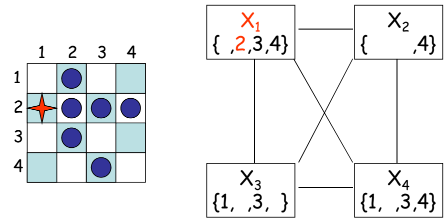
5) The domain of X_3 is **empty** → backtracking

4-Queens Problem



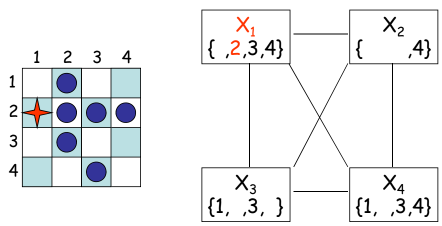
6) The algorithm removes 1 from X_1's domain and assign 2 to X_1

4-Queens Problem



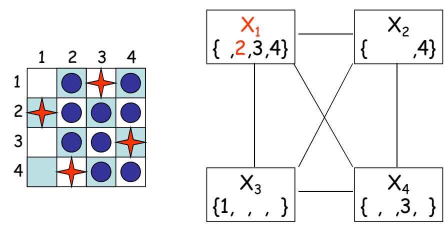
7) The algorithm performs forward checking

4-Queens Problem



8) The algorithm calls AC3

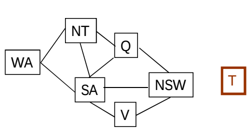
4-Queens Problem



8) The algorithm calls AC3, which reduces the domains of X_3 and X_4 to a single value

Exploiting the Structure of CSP

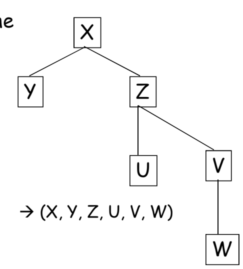
If the constraint graph contains several components, then solve one independent CSP per component



Exploiting the Structure of CSP

If the constraint graph is a tree, then :

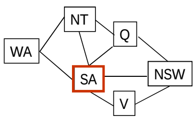
1. Order the variables from the root to the leaves
   → (X_1, X_2, …, X_n)
2. For j = n, n-1, …, 2 call REMOVE-VALUES(X_j, X_i) where X_i is the parent of X_j
3. Assign any valid value to X_1
4. For j = 2, …, n do
   Assign any value to X_j consistent with the value assigned to its parent X_i



→ (X, Y, Z, U, V, W)

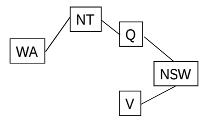
Exploiting the Structure of CSP

Whenever a variable is assigned a value by the backtracking algorithm, propagate this value and remove the variable from the constraint graph



Exploiting the Structure of CSP

Whenever a variable is assigned a value by the backtracking algorithm, propagate this value and remove the variable from the constraint graph



If the graph becomes a tree, then proceed as shown in previous slide