CS26N
Motion Planning for Robots, Digital Actors and Other Moving Objects
(Winter 2012)

Homework #2
Posted: February 8 – Due: February 15

How to complete this HW: First copy this file; then type your answers in the file immediately below each question; finally print this file and return it to the Wednesday class on the day indicated above. If you need to draw anything, you can do it by hand.

Your name: ………………………………………………………………………………………

Your email address: …………………………………………………………………………

Note: You may discuss this problem set with other students in the class (in fact, you are encouraged to do so).

Grading:

Question#	Max. grade	Your grade
I	20	
II	30 (10 + 10 +10)	
III	50 (15+15+10+10)	
Total	100	

Problem I: Configuration Space or a Moving Bar

Consider a bar of length 2, whose center can only translate along the *x*-axis (see Figure 1). The bar can also rotate freely around its center. The only two obstacles to the bar are two vertical walls located at $x = 0$ and $x = 4$.

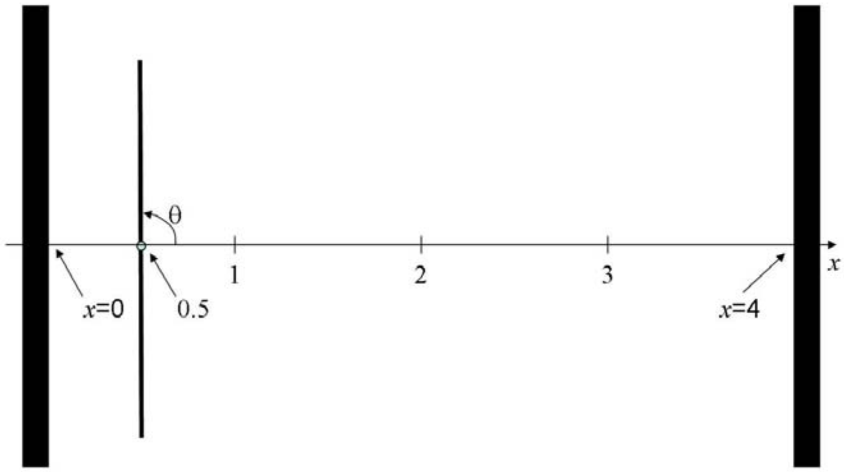
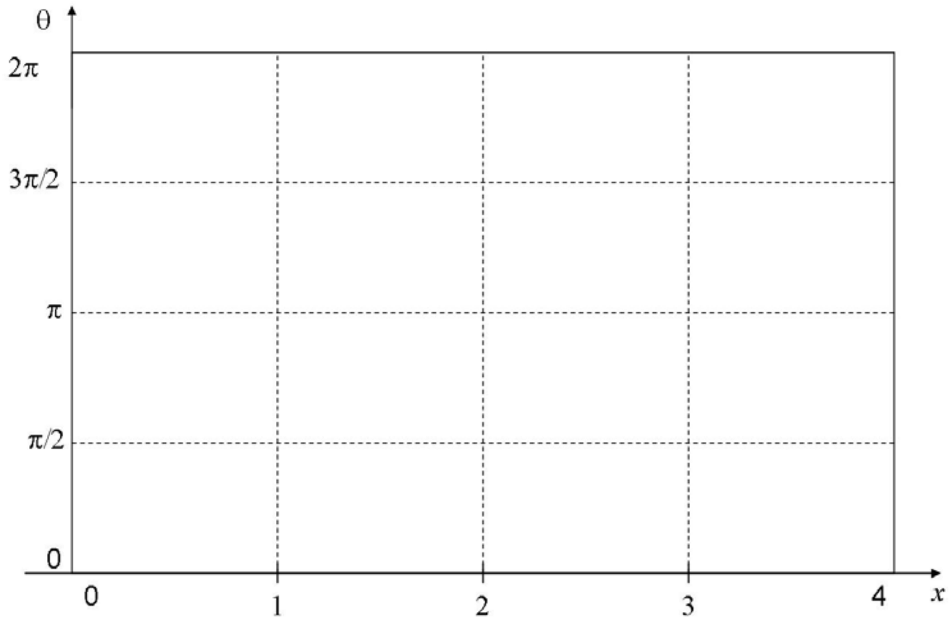


Figure 1: Bar translating and rotating among obstacles

Let a configuration of the bar be defined by (x,θ), where $0 \leq x \leq 4$ is the position of the bar's center along the horizontal *x*-axis and $\theta \in [0,2\pi]$ is the angle made by the bar with the *x*-axis (as shown in the figure). So, the configuration space is the rectangle $[0,4]\times[0,2\pi]$ shown below:



Draw the forbidden regions due to the obstacles on the above picture. You don't have to be very accurate in your drawing, but ensure that your regions have the correct relationships to the grid lines. Mark by a thick dot the point representing the configuration of the bar shown in Figure 1.

Problem II: Configuration Space of a Telescopic Arm

Consider the planar robotic arm shown in Figure 2. It has two joints: a telescopic one embedded in the first link and a revolute one between the first and second link. The first link always points along the x axis. Its length can vary between 1 (minimal length) and 3 (maximal length). In Figure 1 this lonk is extended to length 3. The second link is a thin bar of fixed length 3. Due to mechanical stops, the second joint cannot rotate past vertical in either direction, hence the angle θ shown in the figures varies between $-\pi/2$ and $+\pi/2$. There are two 3×3 square obstacles in the workspace, whose left edges are at $x = 4$ and that are separated by a distance of 2 (i.e., each one is at distance 1 from the y axis). The Start and Finish placements of the second link are shown in red and green lines, respectively. In both cases, the telescopic link is fully extended.

1. The 4 drawings in Figure 3 present four copies of the configuration space $[1,3]\times[-\pi,+\pi]$ of this robot. Which drawing depicts the correct map: (i), (ii), (iii), or (iv)? [Simple observations should allow you to eliminate three drawings.]

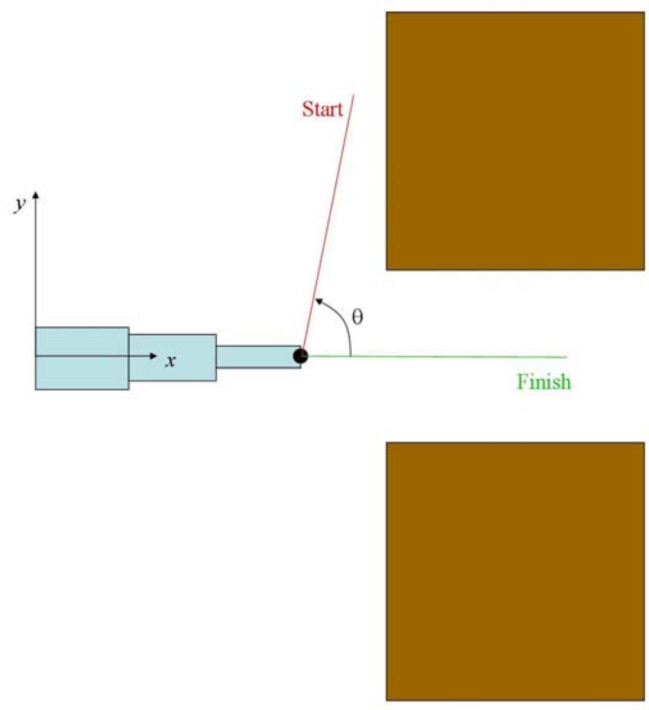


Figure 2: Robot arm and obstacles (Problem II)

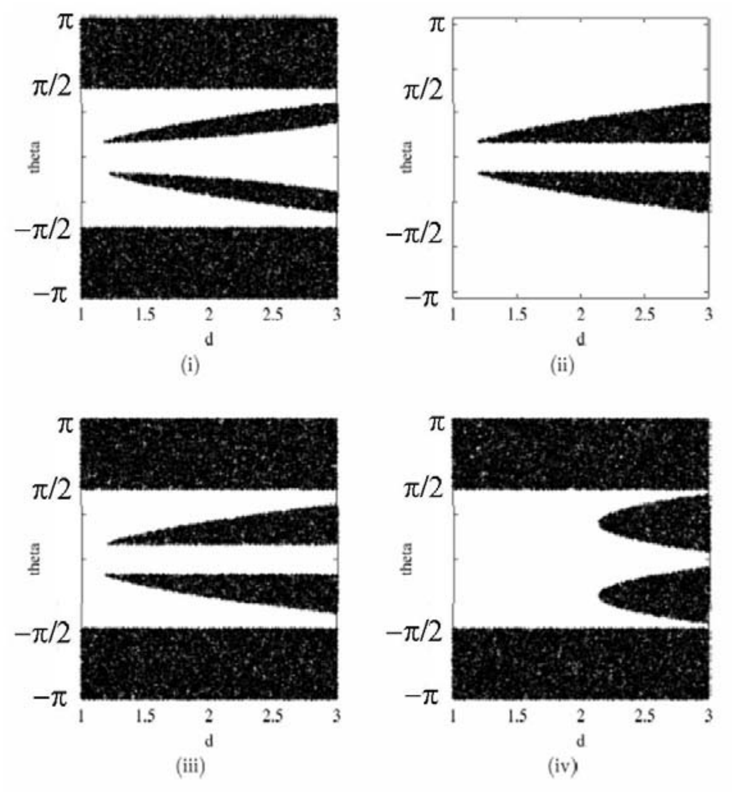


Figure 3: Candidate configuration spaces (Problem I)

2. On the drawing you have selected in Figure 3, mark the start configuration with a plus (+) and the goal configuration with a bold bullet (•). Draw a collision-free path for the robot to move from the initial configuration to the goal configuration.

3. Suppose that the robot has no position sensing due to a malfunction, but perfect touch binary sensing to detect contact with obstacles and motion limits of the joints. Suppose further that each of the two joint can move at constant, but unknown velocity, and that the robot can also perfectly measure elapsed time. Describe a motion plan to reach Finish from Start.

Problem III: Multi-Step Motion Planning

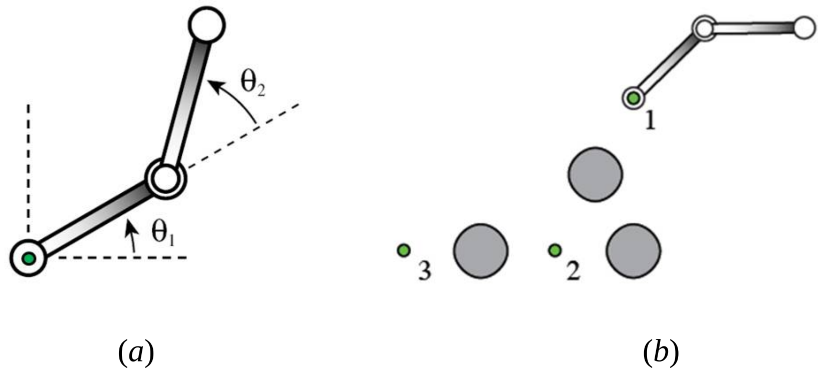


Figure 4: The robot *R* considered in Problem III and its workspace

In this problem we consider the planar robot *R* depicted in Figure 4(*a*). It consists of two identical rods connected by a revolute joint (in the middle). The environment of the robot is shown in Figure 4(*b*). It contains three obstacles and three pegs. The obstacles are the 3 large discs shown gray. The pegs are the 3 small circles labeled 1 to 3. The pegs are not obstacles for the robot. In Figure 4(b) the robot *R* is attached to peg 1 at one of its extremities. One of the two rods is "above" the other, so that it can cross over the other without collision.

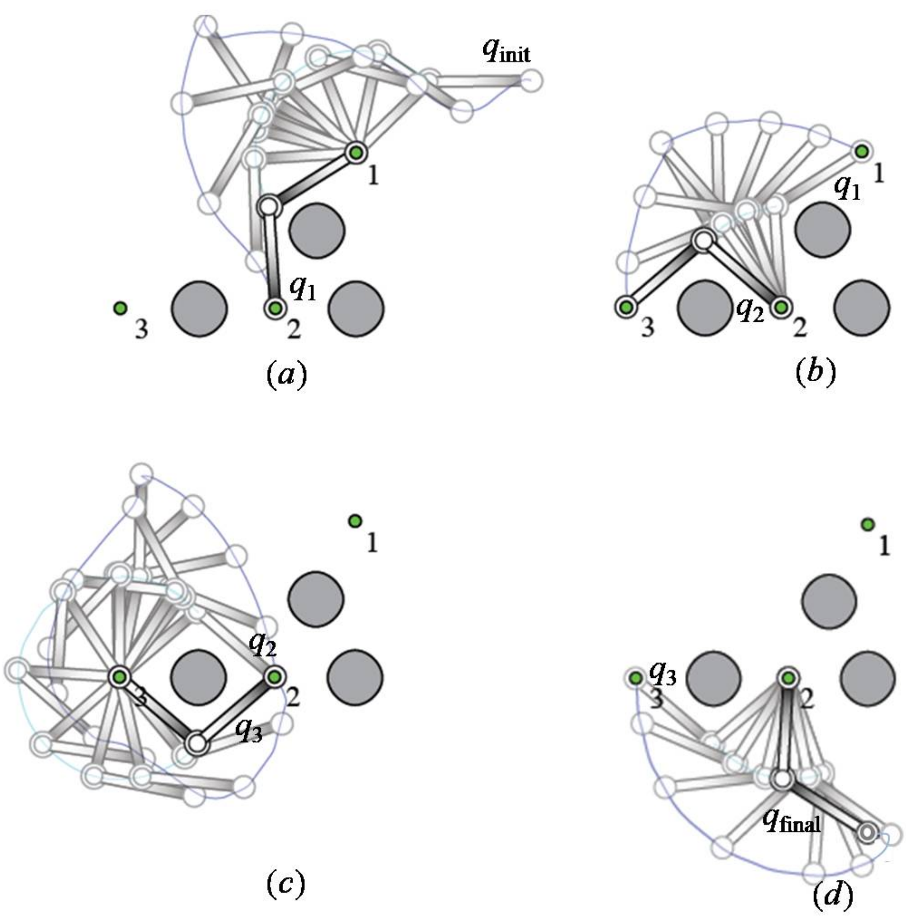
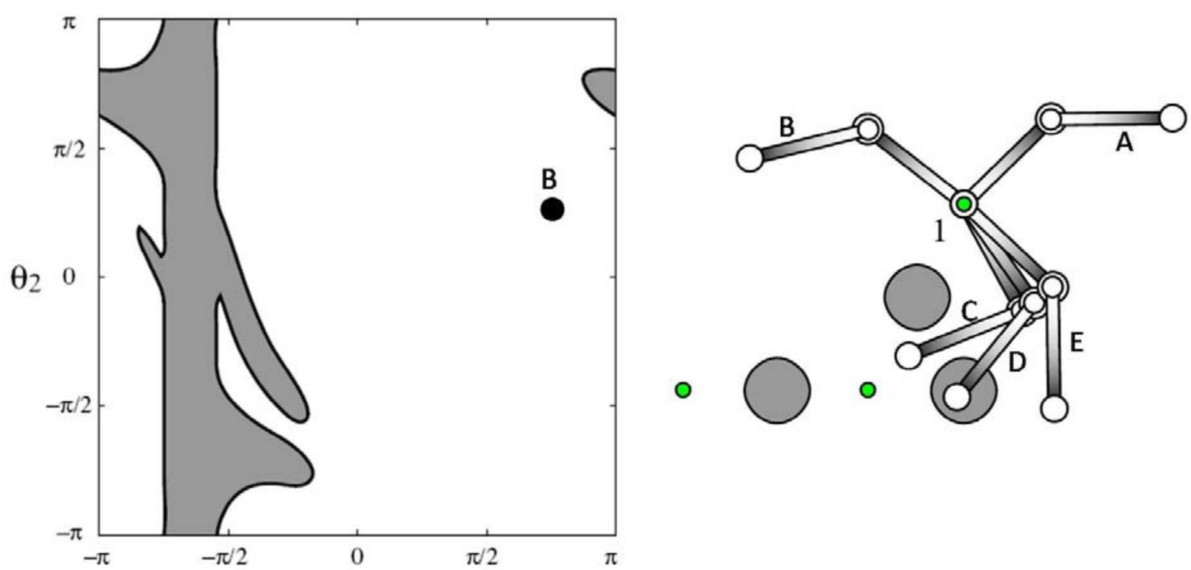


Figure 5: A possible motion sequence of robot *R*

At any one time R must have one of its two extremities attached at a peg position around which it can rotate. For instance, in Figure 4(*b*), one end of R is attached at peg 1. In this case, the rod attached to peg 1 can rotate around peg 1. The other rod can simultaneously rotate around the revolute joint in the middle of R. One such motion is shown in Figure 5(*a*). In this motion, R moves from the configuration q_{init} shown in Figure 4(*b*) to a configuration q_1 where it reaches peg 2 with its other extremity. At the end of this motion, R can attach itself to peg 2 and detach from peg 1 (though one extremity remains at peg 1). Then, R can perform the motion in Figure 5(*b*) from configuration q_1 to configuration q_2 where it reaches peg 3 with its moving extremity. There R can attach itself to peg 3 and detach from peg 2. Figure 5(*c*) and (*d*) show two more motions of R, from q_2 to q_3, and from q_3 to q_{final}. In each figure, the configurations along the motion are shown in shaded graphics, except the last one. The continuous thin line shows the path traced out by the moving extremity of R.

1. To move R must be attached to a single peg. The configuration of R is then defined by two angles (θ_1, θ_2), where θ_1 is the angle made by the rod attached to the peg with the horizontal axis and θ_2 is the angle made by the second rod with the first rod (see Figure 4(*a*) for an illustration). Each of the two angles can vary between $-\pi$ and $+\pi$. Angles are counted positively in the counterclockwise direction and negatively in the other direction. If an angle reaches $-\pi$ (resp. $+\pi$) in one direction, it switches to $-\pi$ (resp. $-\pi$), meaning that the robot is not limited in its rotations, except possibly by the obstacles.



(*a*) (*b*)

Figure 6: Configuration space of R at peg 1

(*i*) Figure 6(*a*) shows the configuration space of R when it is attached to peg 1. The gray region is the region where the robot collides with an obstacle. Represent each of the 5 configurations labeled A, C, D, and E in Figure 6(*b*) by a bold point in Figure 6(*a*). Besides each of the 5 point indicate clearly the corresponding configuration (A, C, D, or E). To help you I have already mapped configuration labeled B in this space.

(*ii*) The collision-free region for the robot is shown white in Figure 6(*a*). A connected component *X* of this region is such that any two configurations in X can be connected by a collision-free path. How many connected components does the collision-free region shown in Figure 6(*a*) contain? [*Hint: Remember that* $(\theta_1, -\pi)$ *and* $(\theta_1, +\pi)$ *are the same configuration; so are* $(-\pi, \theta_2)$ *and* $(-\pi, \theta_2)$.]

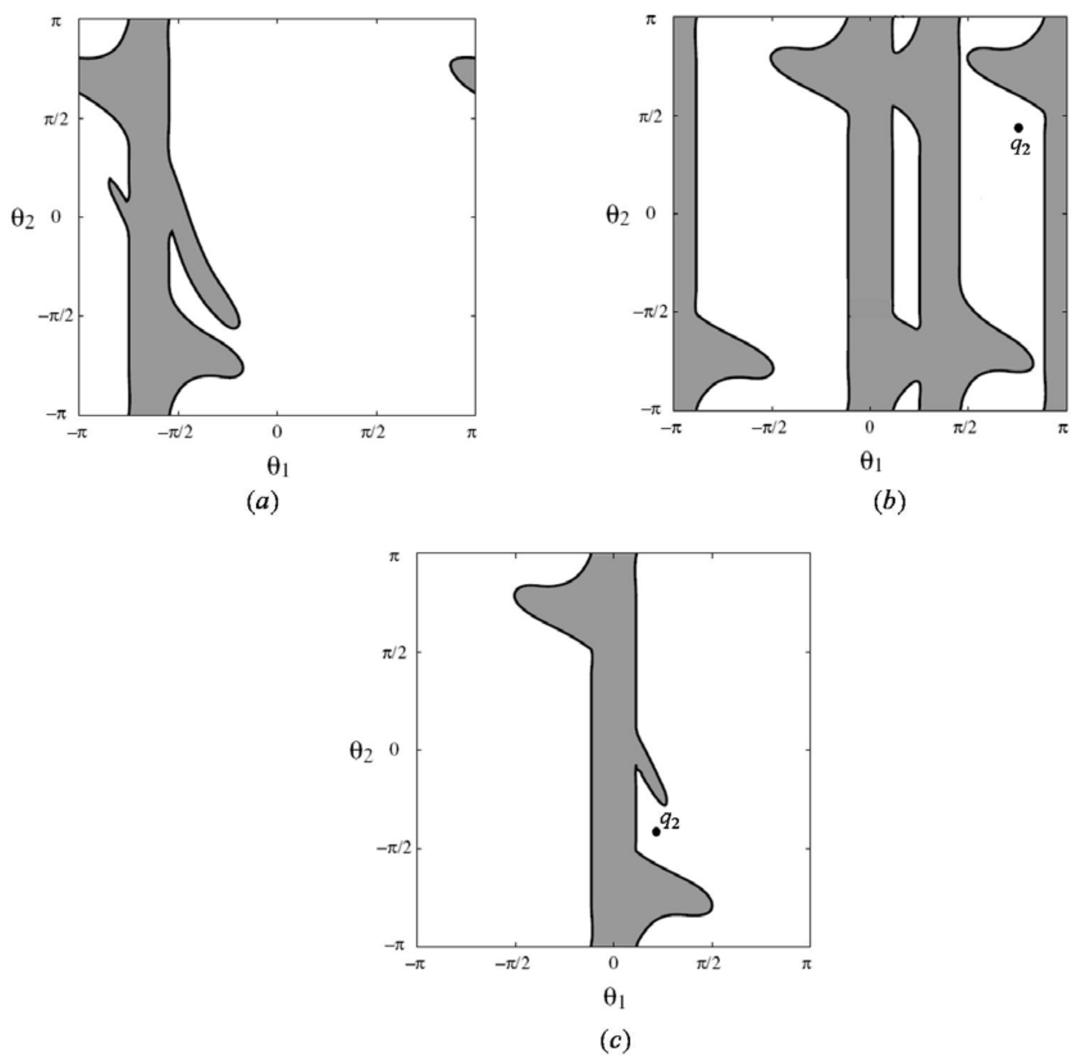


Figure 7: The configuration spaces of *R* when it is attached to peg 1 (Figure (*a*)), to peg 2 (*b*), and to peg 3 (*c*)

2. Figures 7(*a*), (*b*), and (*c*) show the configuration spaces of *R* when it is attached to pegs 1, 2, and 3, respectively. In these configuration spaces draw (approximately) the

successive paths followed by *R* when it performs the motion sequence shown in the 4 drawings of Figure 5. To do this, proceed as follows:

(*i*) Let q_{init} be the initial configuration in Figure 5(*a*), also shown in Figure 4(*b*). Let q_1, q_2, and q_3 be the configurations reached by the motions shown in Figure 5(*a*), (*b*), and (*c*), respectively. Let q_{final} be the configuration reached by the motion shown in Figure 5(*d*). Mark each of these configurations by a bold point in Figure 7. Indicate the name of the configuration beside each point. [*Hints: Every configuration, except* q_{init}*, maps to two points in two different spaces. In addition, when the robot switches between two pegs (i.e., detach from one peg to attach at the other peg, the fixed extremity changes also, so does the definition of* (θ_1, θ_2)*. To help you, in Figure 7 I have shown* q_2 *in the two spaces where R is attached to peg 2 and to peg 3.*]

(ii) Now, in Figure 7, connect q_{initial} to q_1 by a path in the appropriate space, q_1 to q_2 in the appropriate space, ..., and q_3 to q_{final} in the appropriate space. [*Hints: Some of the paths may cut the lines* $\theta_1 = \pm\pi$ *and* $\theta_2 = \pm\pi$*, and so may consist of multiple curve segments. Note that two out of the 4 paths are in the same space.*] Your paths need not be accurate (in fact, this would be quasi-impossible to do by hand), but they must cross the lines $\theta_1 = \pm\pi$ and $\theta_2 = \pm\pi$ whenever needed and only when needed (the positions of the crossing points do not have to be accurate either).

3. How many connected components does the collision-free region shown in Figure 7(*b*) contain?

4. The Probabilistic Roadmap (PRM) approach was presented in class for a robot with one configuration space. But here *R* has several configuration spaces, one for each peg where it can attach itself. Describe briefly how you would extend the PRM approach to automatically plan a collision-free path of *R* between two given configurations. [*Note: I do not ask you to re-describe the PRM approach presented in class. I only ask you to describe how you would extend the approach for R.*]