# Car-Like Robot: How to Park a Car?

(Nonholonomic Planning)

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## Types of Path Constraints

- Local constraints:
  e.g., lie in free space
- Differential constraints:
  e.g., have bounded curvature
- Global constraints:
  e.g., have minimal length

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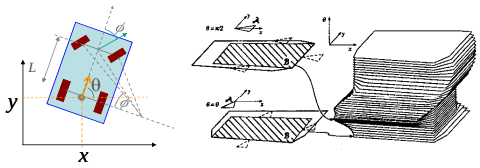
## Car-Like Robot




Configuration space is 3-dimensional: $q = (x, y, \theta)$

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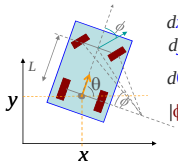
## Example: Car-Like Robot




$dx/dt = v\cos\theta$
$dy/dt = v\sin\theta$
$\Rightarrow dx\sin\theta - dy\cos\theta = 0$

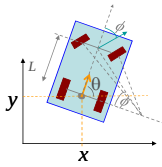
$d\theta/dt = (v/L)\tan\phi$

$|\phi| \le \Phi$

Configuration space is 3-dimensional: $q = (x, y, \theta)$

But control space is 2-dimensional: $(v, \phi)$ with
$|v| = \text{sqrt}[(dx/dt)^2+(dy/dt)^2]$

## Example: Car-Like Robot

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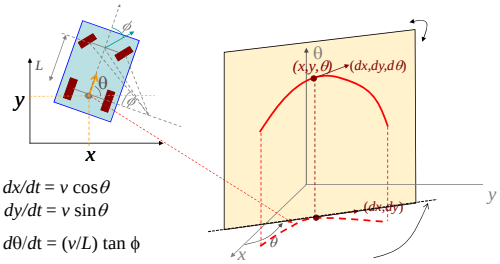
Lower-bounded turning radius

## How Can This Work? Tangent Space/Velocity Space

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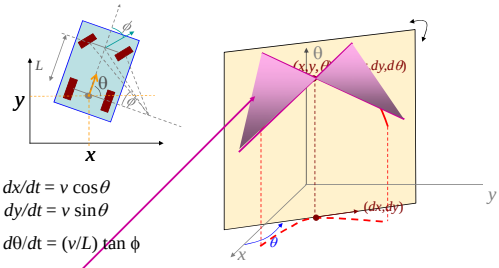


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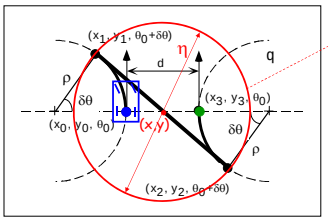
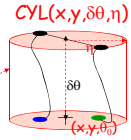
## Type 1 Maneuver



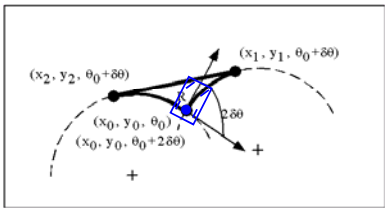




$\eta = 2\rho \tan\delta\theta$
$d = 2\rho(1/\cos\delta\theta - 1) > 0$

When $\delta\theta \rightarrow 0$, so does $d$ and the cylinder becomes arbitrarily small

→ Allows sidewise motion

## Type 2 Maneuver




→ Allows pure rotation

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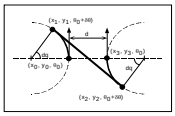
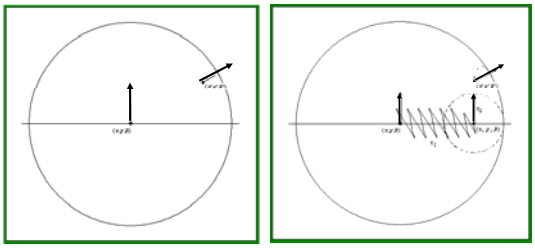
## Combination





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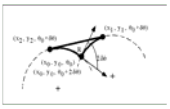
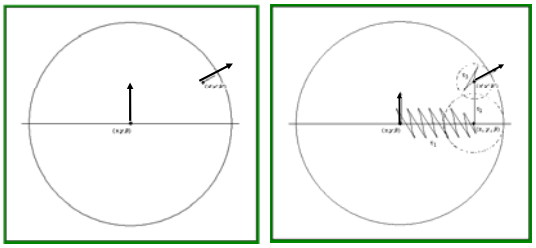
## Combination





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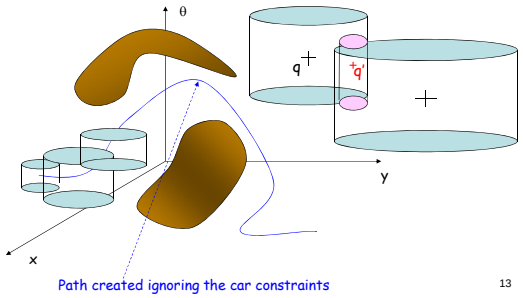
## Coverage of a Path by Cylinders




Path created ignoring the car constraints

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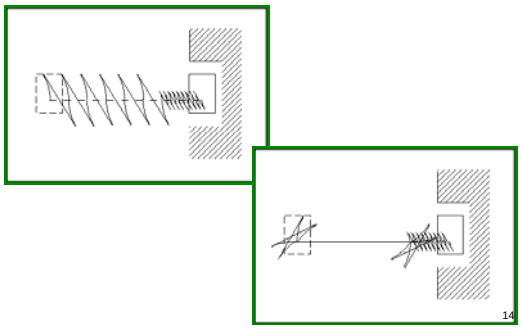
## Path Examples



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## Drawbacks

- Final path can be far from optimal

- Not applicable to car that can only move forward (e.g., think of an airplane)

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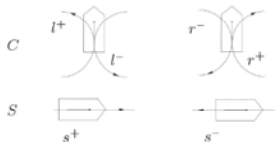
## Reeds and Shepp Paths

notation




C, S elementary path types

$l_a^{+(-)}, r_a^{+(-)}, s_a^{+(-)}$ elementary paths of length a

example: path $l_a^+ l_b^-$  is of type C|C

## Reeds and Shepp Paths

$(I)$ $l_a^+ l_b^- l_c^+$ or $r_a^+ r_b^- r_c^+$ $0\le a\le\pi,\ 0\le b\le\pi,\ 0\le c\le\pi$

$(II)(III)$ $C_a|C_b C_c$ or $C_a C_b|C_c$ $0\le a\le b,\ 0\le c\le b,\ 0\le b\le\pi/2$

$(IV)$ $C_a C_b|C_b C_c$ $0\le a\le b,\ 0\le c\le b, 0<b\le\pi/2$

$(V)$ $C_a|C_b C_b|C_c$ $0\le a\le b,\ 0\le c\le b, 0<b\le\pi/2$

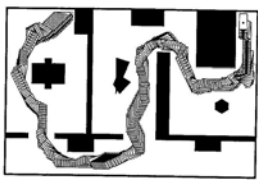
$(VI)$ $C_a|C_{\pi/2} S_b C_{\pi/2}|C_c$ $0\le a\le\pi/2,\ 0\le b,\ 0\le c\le\pi/2$

$(VII)(VIII)$ $C_a|C_{\pi/2} S_b C_c$ or $C_a S_b C_{\pi/2}|C_c$ $0\le a\le\pi,\ 0\le b,\ 0\le c\le\pi/2$

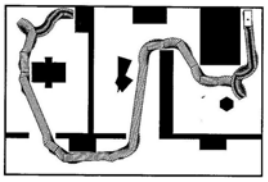
$(IX)$ $C_a S_b C_c$ $0\le a\le\pi/2,\ 0\le b,\ 0\le c\le\pi/2$


Given any two configurations, the shortest RS paths between them is also the shortest path

## Example of Generated Path



Holonomic



Nonholonomic

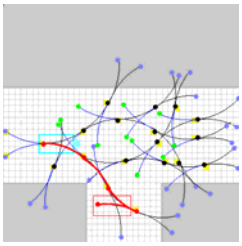
## Other Technique: Control-Based Sampling

$$dx \sin\theta - dy \cos\theta = 0$$

$$dx/dt = v \cos\theta$$
$$dy/dt = v \sin\theta$$
$$d\theta/dt = (v/L) \tan\phi$$

$$|\phi| \leq \Phi$$

1. Select a node m
2. Pick $v$, $\phi$, and dt
3. Integrate motion from m
→new configuration



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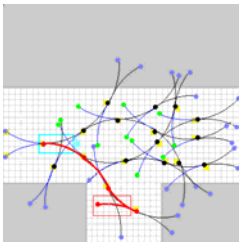
## Other Technique: Control-Based Sampling

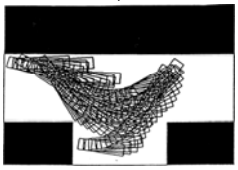
Indexing array:
A 3-D grid is placed over the configuration space. Each milestone falls into one cell of the grid. A maximum number of milestones is allowed in each cell (e.g., 2 or 3).

Asymptotic completeness:
If a path exists, the planner is guaranteed to find one if the resolution of the grid is fine enough.



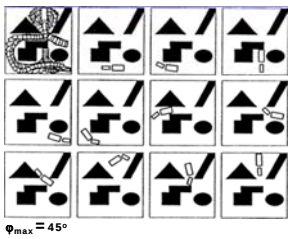
## Computed Paths

Car That Can Only Turn Left



$\varphi_{max} = 45°, \varphi_{min} = 22.5°$

Tractor-trailer



$\varphi_{max} = 45°$

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## Application

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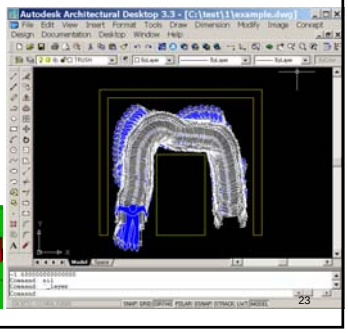
## Architectural Design: Verification of Building Code




C. Han

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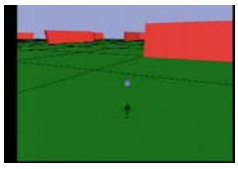
## Other "Similar" Robots/Moving Objects (Nonholonomic)

- Rolling-with-no-sliding contact (friction), e.g.: car, bicycle, roller skate
- Submarine, airplane
- Conservation of angular momentum: satellite robot, under-actuated robot, cat



Why is it useful?
- Fewer actuators: design simplicity, less weight
- Convenience (think about driving a car with 3 controls!)

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