

## Multi-Limb Robots on Irregular Terrain

### NASA/JPL's LEMUR Robot

### Free Climbers

Only friction and internal degrees of freedom are used to achieve equilibrium

### Other Climbing Robots

NINJA II  
Hirose et al, 1991

SpinyBot II  
Stanford University  
October 21-22, 2004

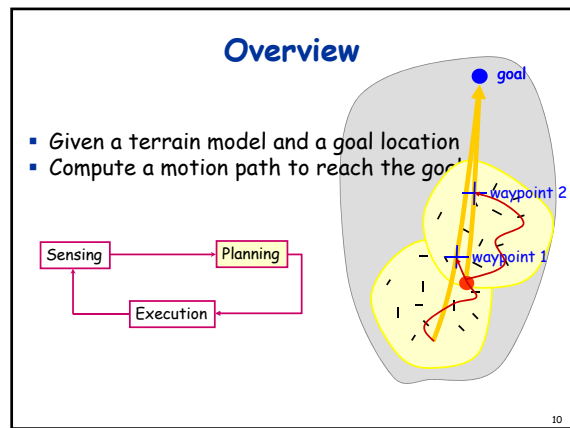
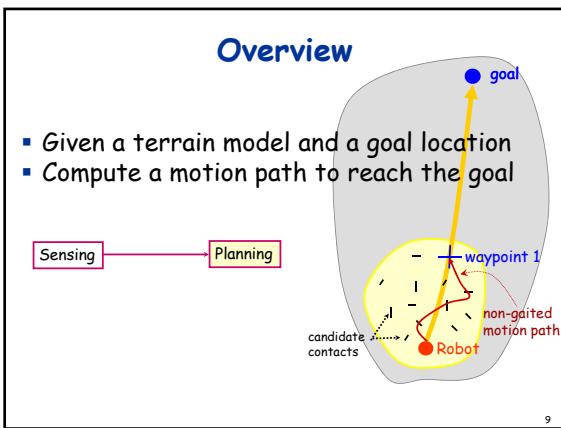
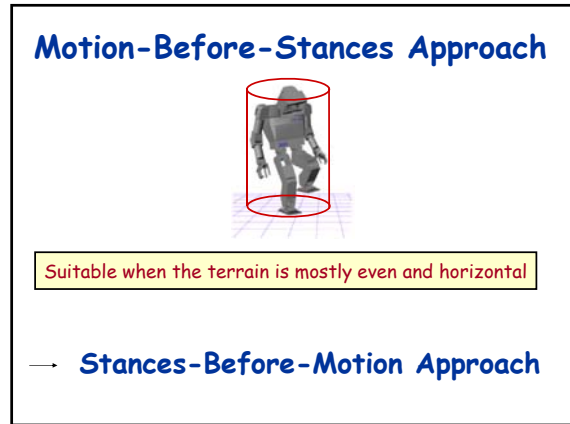
Cutkosky, Stanford, 2004

Yim, PARC, 2002

### Free climbing is a problem-solving activity

- Each step is unique
- Where to make contact?
- Which body posture to take?
- Which forces to exert?
- Decisions at one step may affect the ability to perform future steps

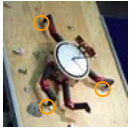
### ATHLETE (NASA/JPL)




### Key Concept: Stance

- Set of fixed robot-environment contacts
- $F_\sigma$ : space of feasible robot configurations at stance  $\sigma$

1. Contacts
2. Quasi-static equilibrium
3. No (self-)collision
4. Torques within bounds



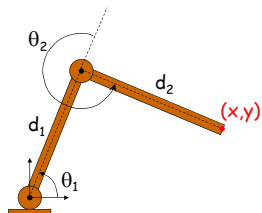
3-stance of LEMUR



Feasible motion at 4-stance

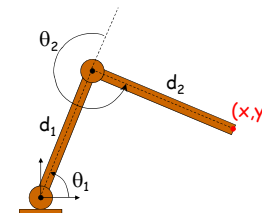


### Forward Kinematics



$$\begin{aligned}
 x &= d_1 \cos \theta_1 + d_2 \cos(\theta_1 + \theta_2) \\
 y &= d_1 \sin \theta_1 + d_2 \sin(\theta_1 + \theta_2)
 \end{aligned}$$

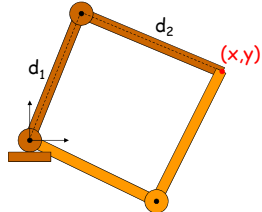
### Inverse Kinematics



$$\theta_2 = \cos^{-1} \left[ \frac{x^2 + y^2 - d_1^2 - d_2^2}{2d_1 d_2} \right]$$

$$\theta_1 = \frac{-x(d_2 \sin \theta_2) + y(d_1 + d_2 \cos \theta_2)}{y(d_2 \sin \theta_2) + x(d_1 + d_2 \cos \theta_2)}$$

### Inverse Kinematics

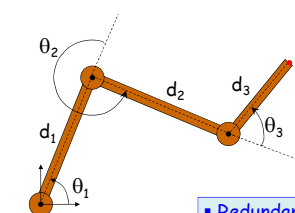


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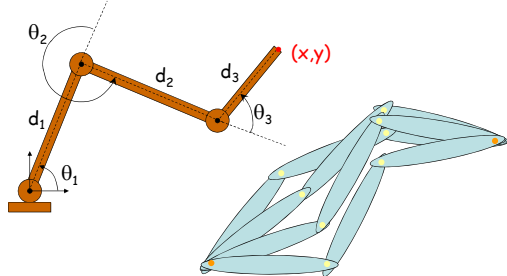
Two solutions

### More Complicated Example

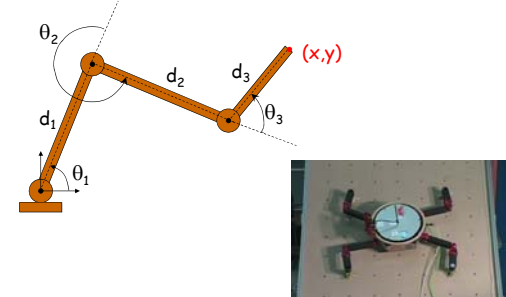


- Redundant linkage
- Infinite number of solutions
- Self-motion space

### More Complicated Example

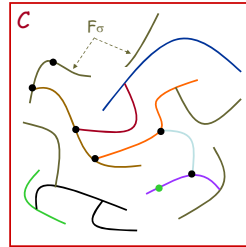


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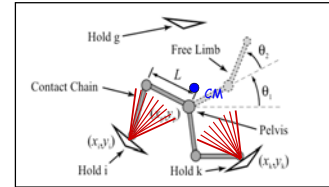


### Challenge

- High-dimensional configuration space  $C$  (11 LEMUR, 42 for ATHLETE, 36 for HRP-2, 16 for Stanford robot)
- Many possible contacts, hence many stances



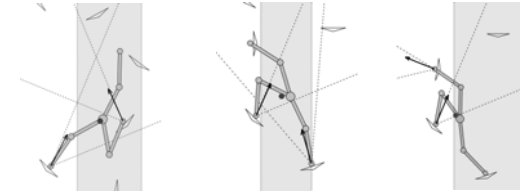
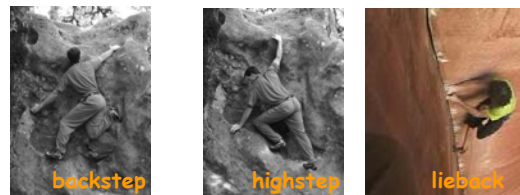
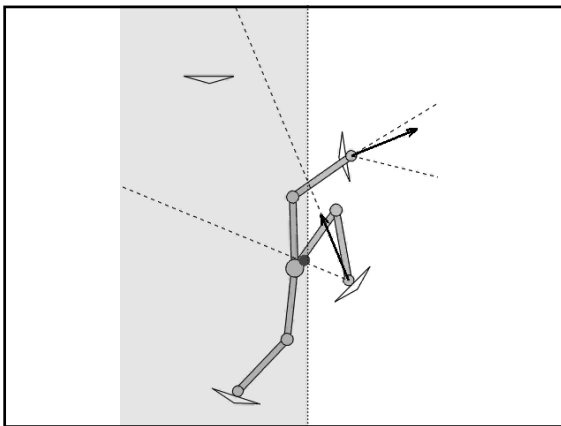
### Equilibrium Constraint



$$\sum_i f_i + mg = 0$$

$$\sum_i p_i \times f_i + CM(q) \times mg = 0$$

$f_i \in \mathcal{FC}_i$  for all  $i$



### Equilibrium Test in 3D

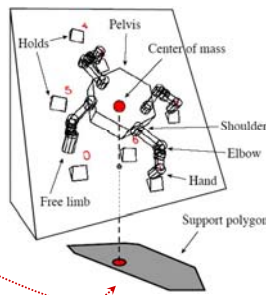
- Assuming infinite torque limits:

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- Center of mass above convex support polygon



### Equilibrium Test

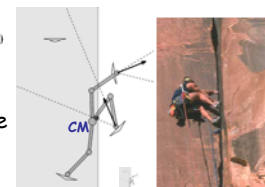
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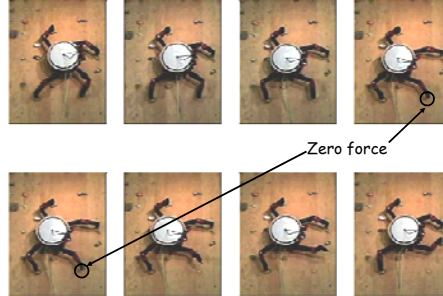
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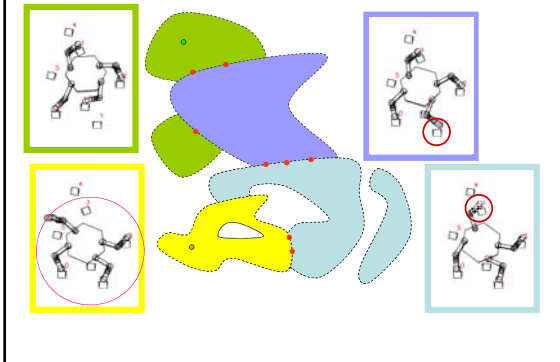
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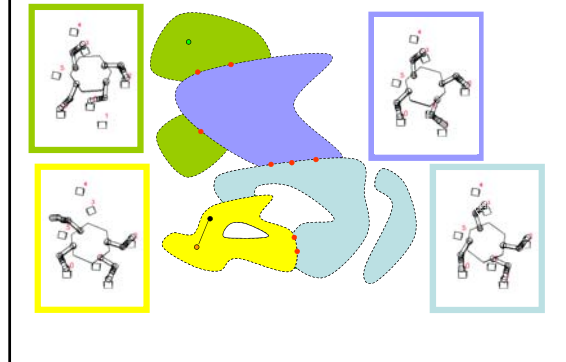
### Transition Configuration



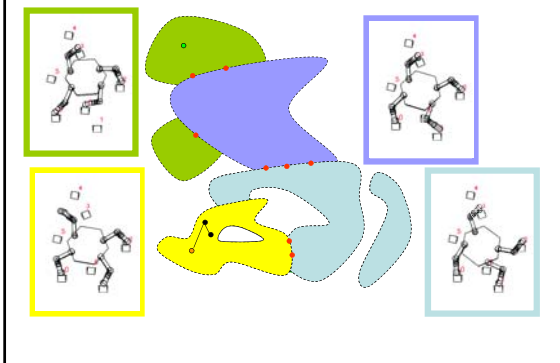
### Lazy Search



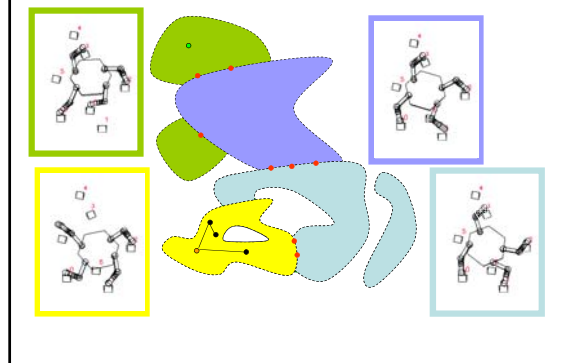
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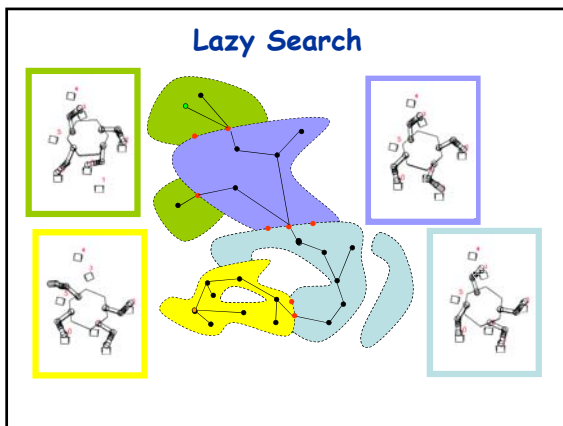
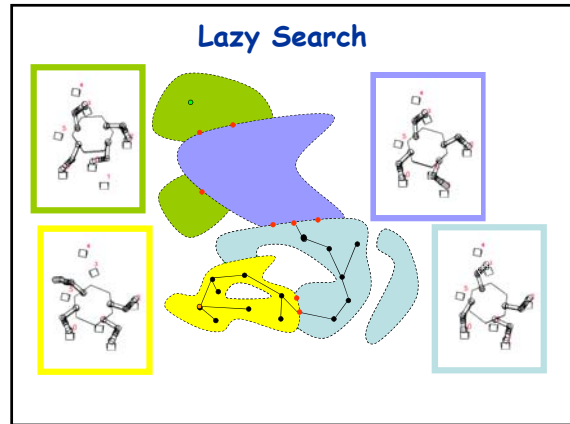
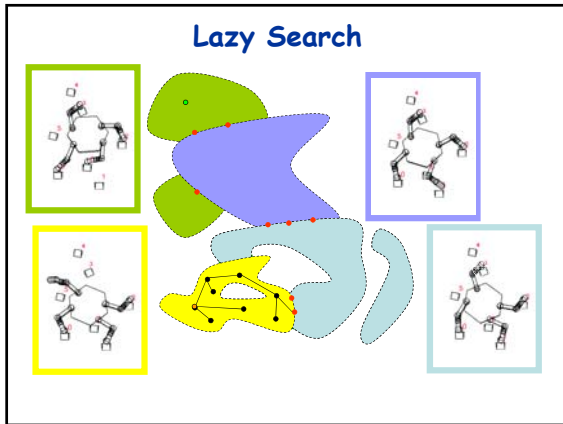


### Lazy Search



### Lazy Search





### Configuration Sampling

1. Sample position/orientation of the chassis at random in restricted area
2. Solve IK for each limb making contact
3. Sample DOFs in free limb at random
4. Test equilibrium constraint

34

