Motion Planning for a Point Robot (2/2)

1

- Class scribing
- Position paper

2

Planning requires models

- The Bug algorithms are "reactive motion strategies"; they are not motion planners
- To plan its actions, a robot needs a (possibly imperfect) **predictive model** of its actions, so that it can choose among several possible courses of action

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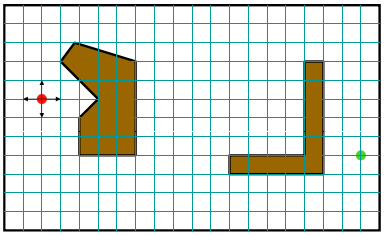
Point Robot on a Grid



Assumptions:
- The robot perfectly controls its actions
- It has an accurate geometric model of the environment (i.e., the obstacles)

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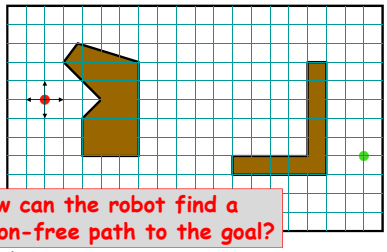
Point Robot on a Grid



How can the robot find a collision-free path to the goal?

Assumptions:
- The robot perfectly controls its actions
- It has an accurate geometric model of the environment (i.e., the obstacles)

5

8-Puzzle Game

8	2	
3	4	7
5	1	6

Initial state

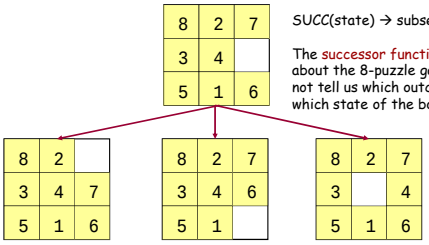
1	2	3
4	5	6
7	8	

Goal state

State: Any arrangement of 8 numbered tiles and an empty tile on a 3x3 board

6

8-Puzzle: Successor Function



SUCC(state) → subset of states

The successor function is knowledge about the 8-puzzle game, but it does not tell us which outcome to use, nor to which state of the board to apply it.

Search is about the exploration of alternatives

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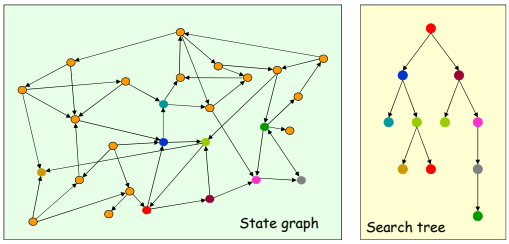
State Graph and Search Tree



Note that some states may be visited multiple times

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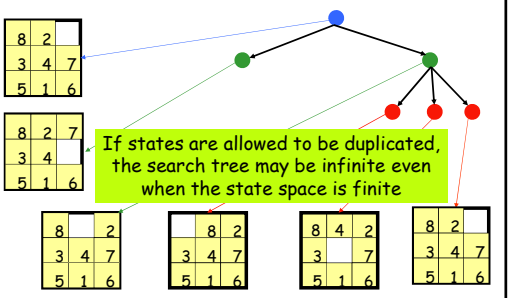
Search Nodes and States



If states are allowed to be duplicated, the search tree may be infinite even when the state space is finite

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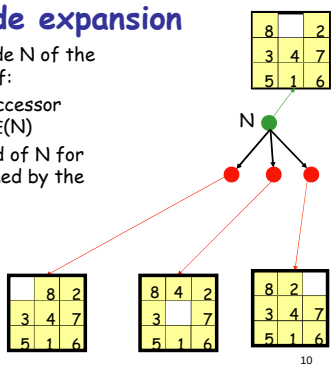
Node expansion

The expansion of a node N of the search tree consists of:

1) Evaluating the successor function on STATE(N)
2) Generating a child of N for each state returned by the function



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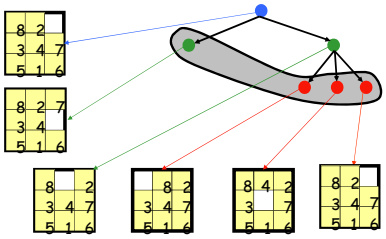
Fringe of Search Tree

- The fringe is the set of all search nodes that haven't been expanded yet



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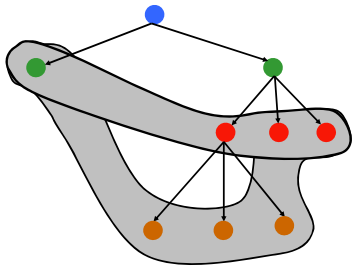

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Search Strategy

- The fringe is the set of all search nodes that haven't been expanded yet
- The fringe is implemented as a priority queue FRINGE
  - INSERT(node,FRINGE)
  - REMOVE(FRINGE)
- The ordering of the nodes in FRINGE defines the search strategy

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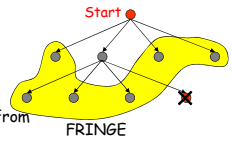
Search Algorithm

SEARCH(Start, Finish)

1. INSERT(Start,FRINGE)
2. Repeat:
   a. If FRINGE is empty then return failure
   b. q ← REMOVE(FRINGE)
   c. If q = Finish then return a path from Start to Finish
   d. For every **new** state q' in SUCCESSORS(q)
      i. Install q' as a child of q in the search tree
      ii. INSERT(q',FRINGE)



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Blind Search Strategies

- Breadth-first
  New positions are inserted at the end of FRINGE What is the effect?
- Depth-first
  New positions are inserted at the beginning of FRINGE What is the effect?

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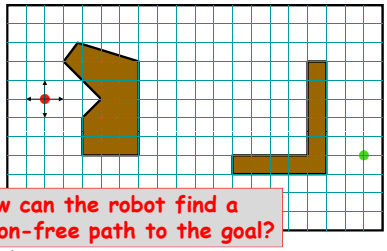
Point Robot on a Grid



How can the robot find a collision-free path to the goal?

Assumptions:

- The robot perfectly controls its actions
- It has an accurate geometric model of the environment (i.e., the obstacles)

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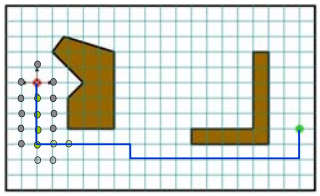
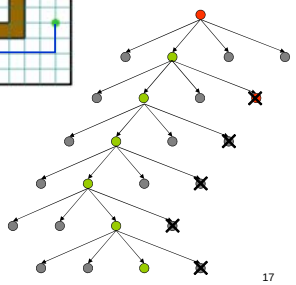


Search tree



Now, the robot can **search** its model for a collision-free path to the goal

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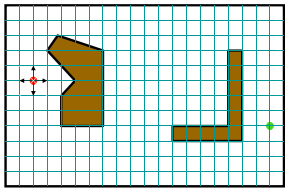

1000×1000 grid → 1,000,000 configurations
In 3-D → 10^9 configurations
In 6-D → 10^{18} configurations!!!

→Need for smart search techniques or sparse discretization

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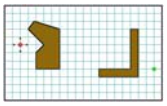
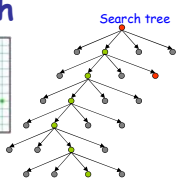
Smart Search





SEARCH(Start$_1$,Finish)
1. INSERT(q_{start},FRINGE)
2. Repeat:
   a. If FRINGE is empty then return failure
   b. q ← REMOVE(FRINGE)
   c. If q = Finish then return a path from Start to Finish
   d. For every **new** state q' in SUCCESSORS(q)
      i. Install q' as a child of q in the search tree
      ii. INSERT(q',FRINGE)

What can smart search be? How can it be done?

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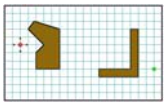
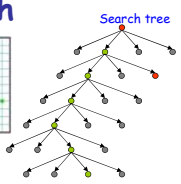
Smart Search





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   c. If q = Finish then return a path from Start to Finish
   d. For every **new** state q' in SUCCESSORS(q)
      i. Install q' as a child of q in the search tree
      ii. INSERT(q',FRINGE)

→ Smart ordering of the configurations in FRINGE
(best-first search)

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Evaluation function

- f : node N → real positive number $f(N)$
- For instance $f(N)$ may be the Euclidean distance (straight-line distance) to the goal
- Sort the fringe by increasing value of f

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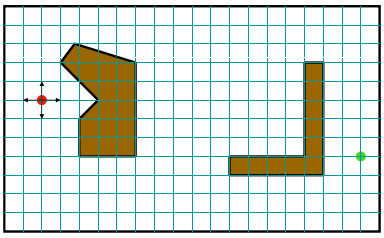
Application

f(N) = Euclidean distance to the goal



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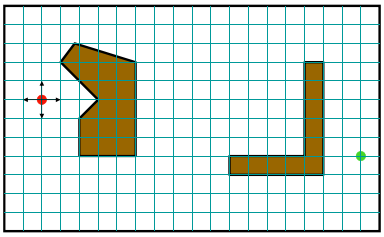
Application

f(N) = Manhattan distance to the goal



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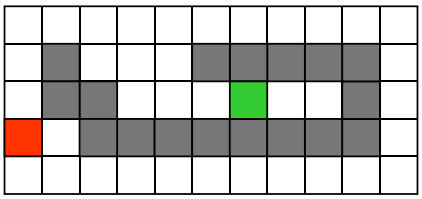
Another Example



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Another Example

f(N) = h(N), with h(N) = Manhattan distance to the goal

8	7	6	5	4	3	2	3	4	5	6
7		5	4	3						5
6			3	2	1	0	1	2		4
7	6									5
8	7	6	5	4	3	2	3	4	5	6

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Another Example

f(N) = Manhattan distance to the goal

8	7	6	5	4	3	2	3	4	5	6
7		5	4	3						5
6			3	2	1	0	1	2		4
7	6									5
8	7	6	5	4	3	2	3	4	5	6

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Another Example

f(N) = g(N)+h(N), with h(N) = Manhattan distance to goal and g(N) = distance traveled to reach N

8+3	7+4	6+3	5+6	4+7	3+8	2+9	3+10	4	5	6
7+2		5+6	4+7	3+8						5
6+1			3	2+9	1+10	0+11	1	2		4
7+0	6+1									5
8+1	7+2	6+3	5+4	4+5	3+6	2+7	3+8	4	5	6

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A* Search

$f(N) = g(N) + h(N)$, where $g(N)$ is the length of the path reaching N and **$0 \le h(N) \le$ length of the shortest path from N to goal** [h is called the **heuristic** function]

FRINGE is a priority queue sorted in increasing order of f

A*(Start,Finish)

1. Insert the initial-node (state = Start) into FRINGE
2. Repeat:
   a. If FRINGE is empty then return failure
   b. Let N be the first node in FRINGE, remove N from FRINGE
   c. Let s be the state of N and mark s as closed
   d. If s = Finish then return the path found between Start and Finish and exit
   e. For every state s' in SUCCESSORS(s)
      i. If s' is closed then do nothing
      ii. Else
         - Create a node N' with state s' as a successor of N
         - If there is a node N" in FRINGE with state s' then
           - If g(N") < g(N') then do nothing, else remove N" from FRINGE and insert N' in FRINGE,
         - Else insert N' in FRINGE

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A* Search

$f(N) = g(N) + h(N)$, where $g(N)$ is the length of the path reaching N and **$0 \le h(N) \le$ length of the shortest path from N to goal** [h is called the **heuristic** function]

FRI

A*(

1.
2.

Both
h(N) = Euclidean distance from N to goal
and
h(N) = Manhattan distance from N to goal
verify $0 \le h(N) \le$ length of the shortest path from N to the goal

e. For every state s' in SUCCESSORS(s)

A* search is guaranteed to return a shortest path if a path exists

- If g(N") < g(N') do nothing, else remove N" from FRINGE and insert N' in FRINGE
- Else insert N' in FRINGE

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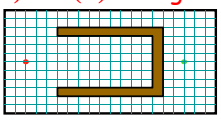
How to Choose h?

- $f(N) = g(N) + h(N)$, where $0 \le h(N) \le$ length $h^*(N)$ of the shortest path from N to goal
- Would $h(N) \equiv 0$ be a good choice?
- Would $h(N) \equiv h^*(N)$ be a good choice?



- Which one is better:
  $h(N)$ = Euclidean distance to goal?
  $h(N)$ = Manhattan distance to goal?

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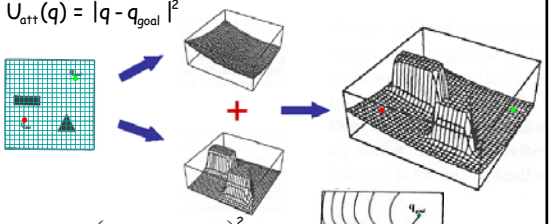
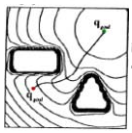
Attractive/Repulsive Potential Fields

$U_{att}(q) = |q - q_{goal}|^2$



$$U_{rep}(q) = \left(\frac{1}{d_{obst}(q)} - \frac{1}{d_{min}} \right)^2$$



Equipotential contours

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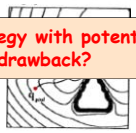
Attractive/Repulsive Potential Fields

$U_{att}(q) = |q - q_{goal}|^2$

Best-first search with potential fields: Sort positions in FRINGE in increasing order of potential

What could be a Bug motion strategy with potential fields? What would be its drawback?

$\left(d_{obst}(q) \quad d_{min} \right)$



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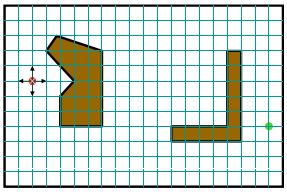

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→Need for smart search techniques
or sparse discretization

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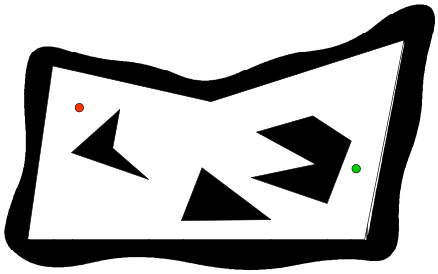
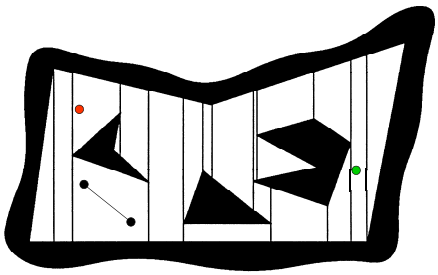
Cell Decomposition



Cell Decomposition



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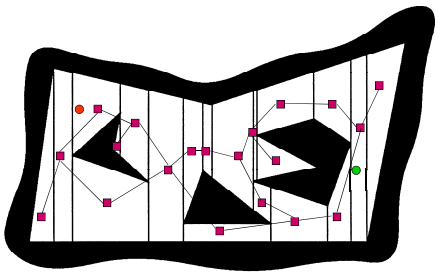

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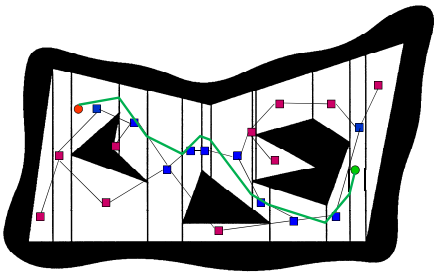

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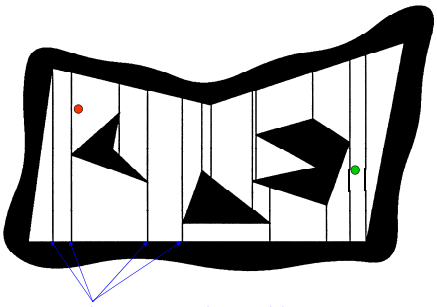
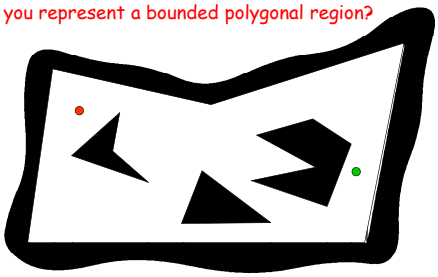


Cell Decomposition

How would you represent a bounded polygonal region?



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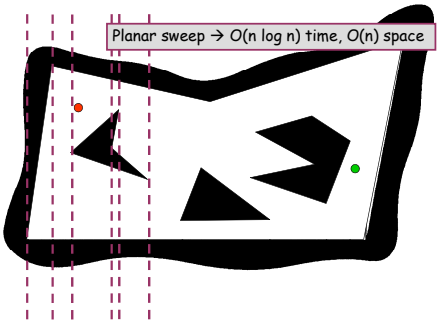

Continuous space

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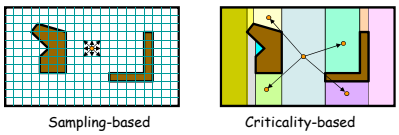
Discretization



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Search

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Tradeoffs

- Cell decomposition: precomputation of data structure
- Path planning: query processing for a given (Start, Finish) pair using this data structure
- Can the precomputation cost be amortized over several queries?
- What if the world changes frequently?

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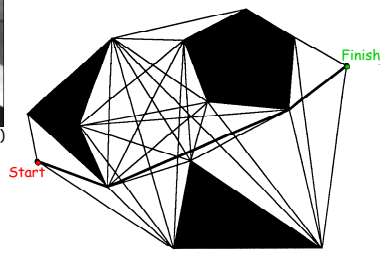
Visibility Graph

SHAKEY (SRI, 1969)



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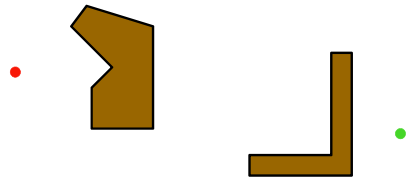
Visibility Graph



Computational complexity?

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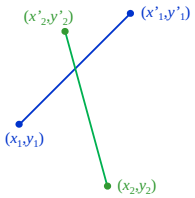
How would you check whether two line segments intersect?



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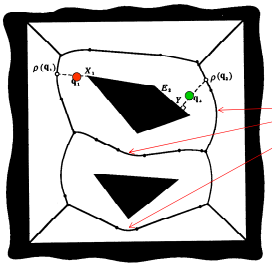
Voronoi Diagram



What is this?

Advantages/drawbacks relative to visibility graph method?

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