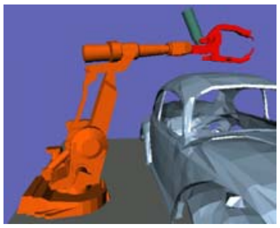
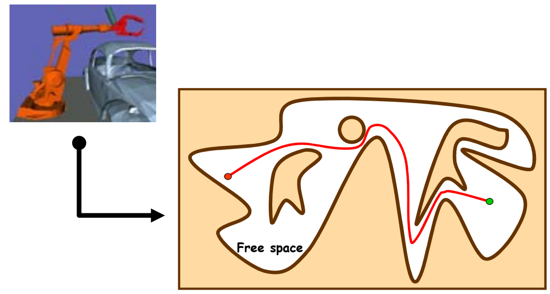
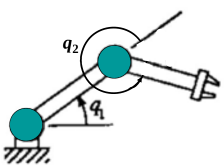
Configuration Space of an Articulated Robot

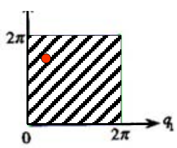


Idea: Reduce the Robot to a Point → Configuration Space



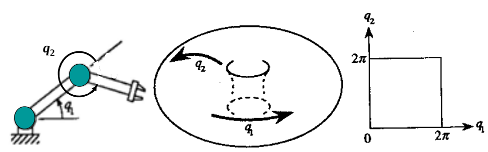
Two-Revolute-Joint Robot

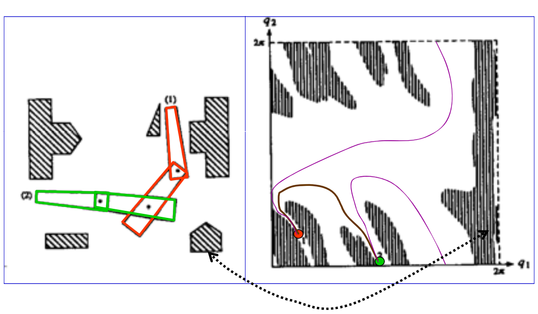


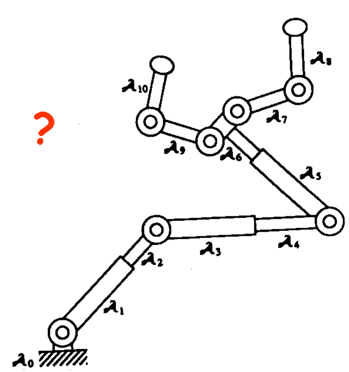


- A configuration of a robot is a list of non-redundant parameters that fully specify the position and orientation of each of its bodies
- In this robot, one possible choice is: (q_1, q_2)

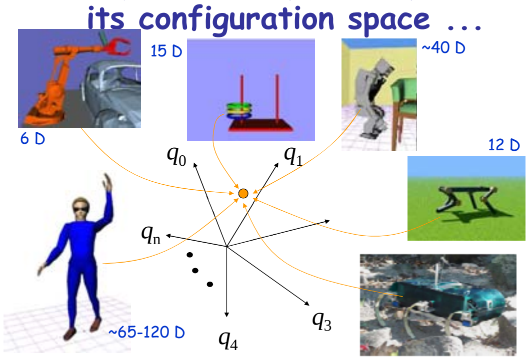
  The configuration space (C-space) has 2 dimensions







Every robot maps to a point in its configuration space ...



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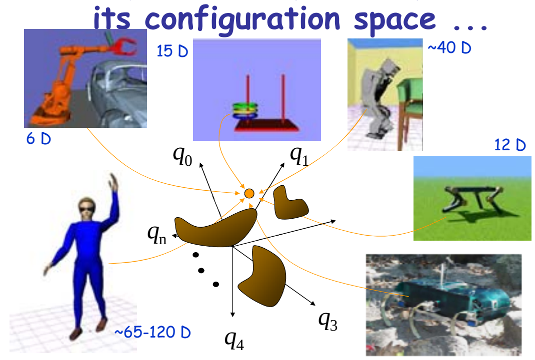
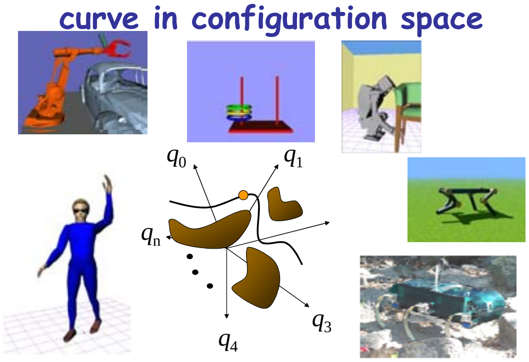


... and every robot path is a curve in configuration space



Issues!!

- Dimensionality of configuration space
- Geometric complexity of free region

→ Plan in configuration space, but compute in workspace

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Probabilistic Roadmaps (Sampling-Based Planning)

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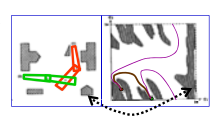
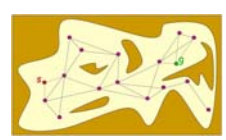
Rationale of Probabilistic Roadmap (PRM) Planners

- The cost of computing an exact representation of the configuration space of a multi-joint articulated object is often prohibitive.



- But very fast algorithms exist that can check if an articulated object at a given configuration collides with obstacles.
- A PRM planner computes an extremely simplified representation of F in the form of a network of "local" paths connecting configurations sampled at random in F according to some probability measure



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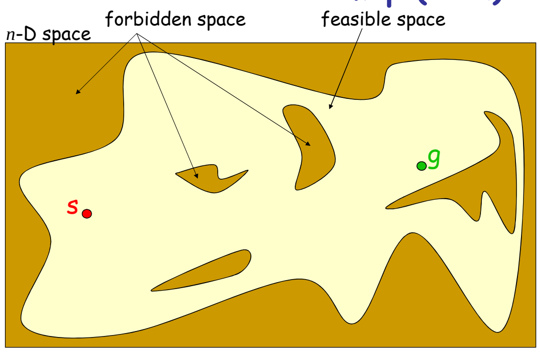
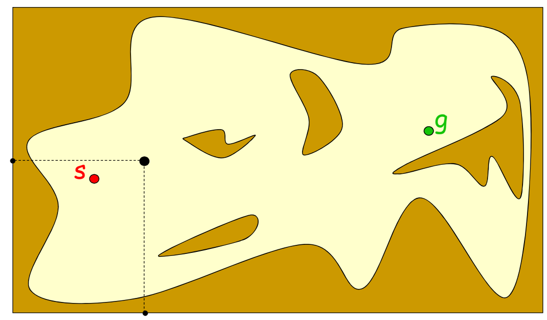
Probabilistic Roadmap (PRM)



Probabilistic Roadmap (PRM)

Configurations are sampled by picking coordinates at random



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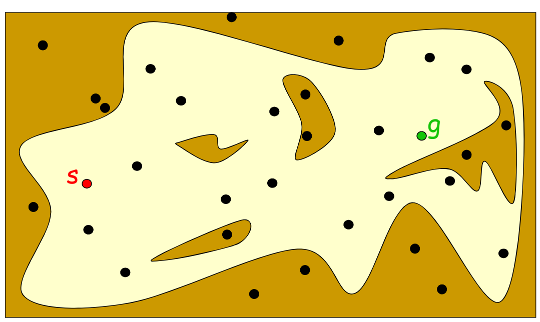
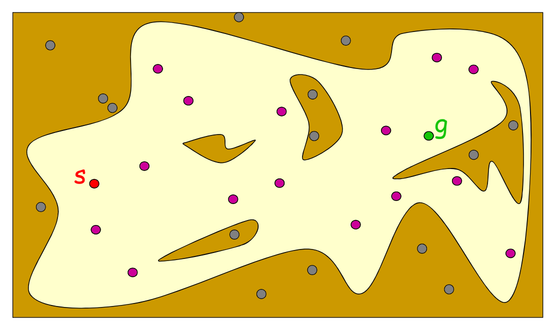


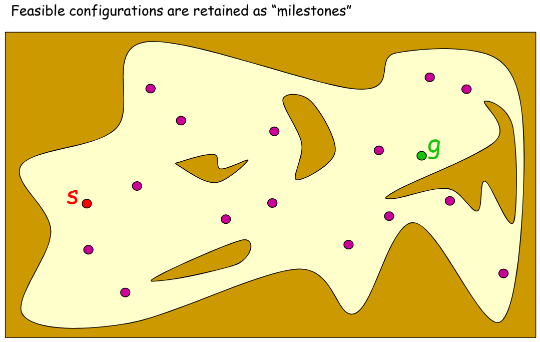
Probabilistic Roadmap (PRM)

Sampled configurations are tested for feasibility



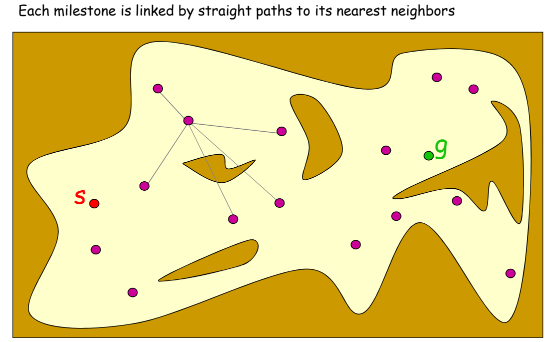
Probabilistic Roadmap (PRM)

Feasible configurations are retained as "milestones"



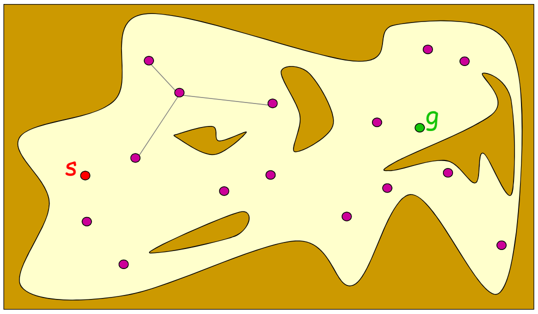
Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



Probabilistic Roadmap (PRM)

The feasible links are retained to form the PRM



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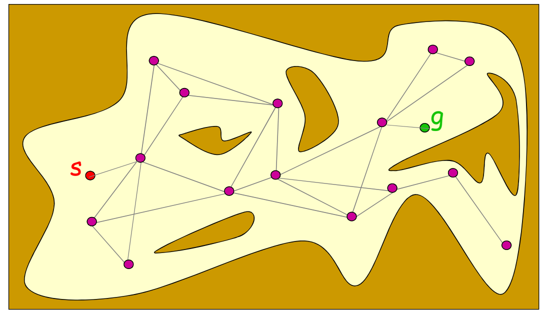
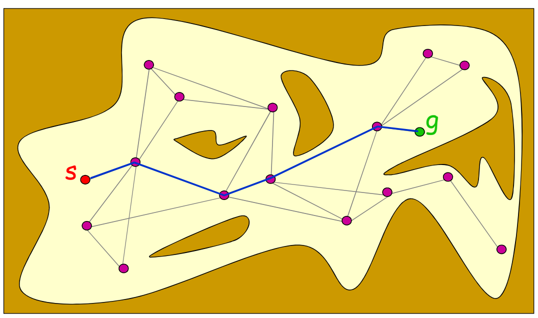


Probabilistic Roadmap (PRM)

The PRM is built until s and g are connected



Procedure BasicPRM(s,g,N)

1. Initialize the roadmap R with two nodes, s and g
2. Repeat:
   - Sampling strategy
   a. Sample a configuration q from C with probability measure π
   b. If $q \in F$ then add q as a new node of R
   c. For some nodes v in R such that $v \neq q$ do
      - Connection strategy
      If path(q,v) $\in$ F then add (q,v) as a new edge of R

   until s and g are in the same connected component of R or R contains N+2 nodes
3. If s and g are in the same connected component of R then
   Return a path between them
4. Else
   Return NoPath
   This answer may occasionally be incorrect

PRM planners work well in practice. Why?

- Why are they probabilistic?
- What does their success tell us?
- How important is the probabilistic sampling measure π?
- How important is the randomness of the sampling source?

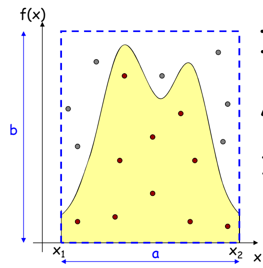
Why is PRM planning probabilistic?

- A PRM planner ignores the exact shape of F. So, it acts like a robot building a map of an unknown environment with limited sensors
- The probabilistic sampling measure π reflects this uncertainty. The goal is to minimize the expected number of remaining iterations to connect s and g, whenever they lie in the same component of F

So ...

- PRM planning trades the cost of computing F exactly against the cost of dealing with uncertainty, by incrementally sampling milestones and connecting them in order to "learn" the connectivity of F
- This choice is beneficial only if a small roadmap has high probability to represent F well enough to answer planning queries correctly and such a small roadmap has high probability to be generated
- **Under which conditions is this the case?**

Monte Carlo Integration



$$I = \int_{x_1}^{x_2} f(x)dx$$

$$A = a \times b$$

$$I \approx \frac{\#brown}{\#brown+\#gray} \times A$$

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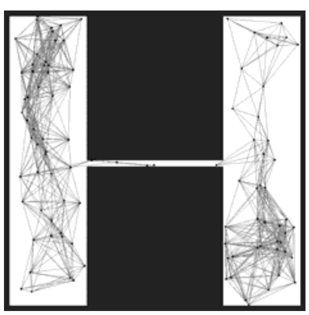
Connectivity Issue



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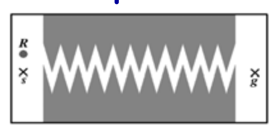
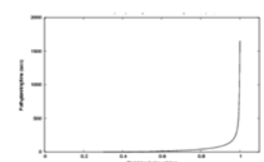
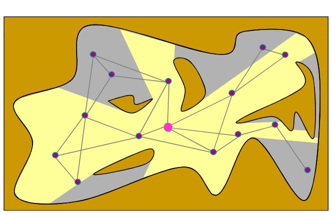
Experiment



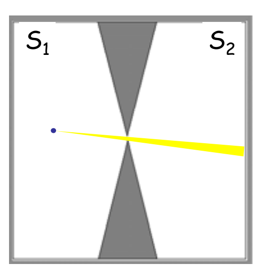


Visibility in F

Two configurations q and q' see each other if **path**(q,q') ∈ F



Connectivity Issue



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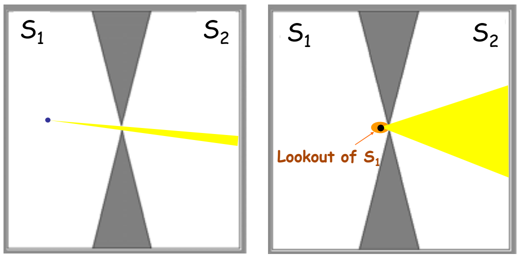
Connectivity Issue



F is expansive if each one of its subsets X has a "large" lookout

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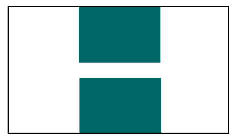
- Expansiveness only depends on volumetric ratios
- It is not directly related to the dimensionality of the configuration space



In 2-D the expansiveness of the free space can be made arbitrarily poor

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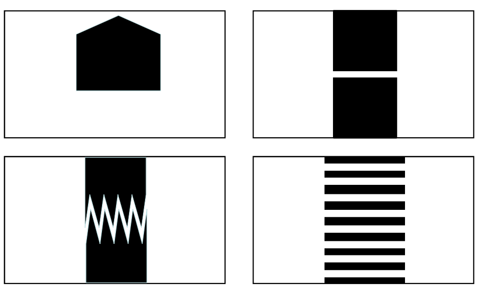
Which Ones are Most Difficult?



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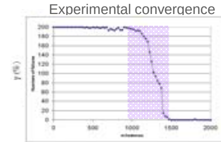
Probabilistic Completeness of PRM Planning

Theorem 1

Let F be (ε,α,β)-expansive, and s and g be two configurations in the same component of F.
BasicPRM(s,g,N) with uniform sampling returns a path between s and g with probability converging to 1 at an exponential rate as N increases

$\gamma = \Pr(\text{Failure}) \leq \left(\frac{c_1}{\varepsilon\alpha}\right)\exp\left(c_2\varepsilon\alpha\left(-N+\frac{c_3}{\beta}\right)\right)$

Experimental convergence



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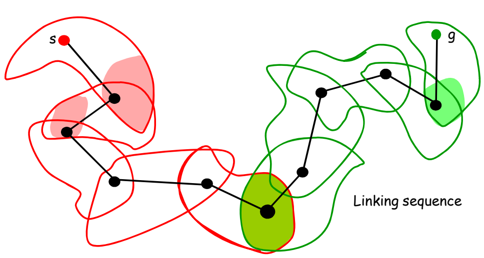
Intuition

If F is favorably expansive, then it is easy to capture its connectivity by a small network of sampled configurations



Probabilistic Completeness of PRM Planning

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Theorem 2

For any $\varepsilon > 0$, any $N > 0$, and any g in (0,1], there exists α_0 and β_0 such that if F is not (ε,α,β)-expansive for $\alpha > \alpha_0$ and $\beta > \beta_0$, then there exists s and γ in the same component of F such that **BasicPRM**(s,g,N) fails to return a path with probability greater than γ.

Probabilistic Completeness of PRM Planning

Theorem 1

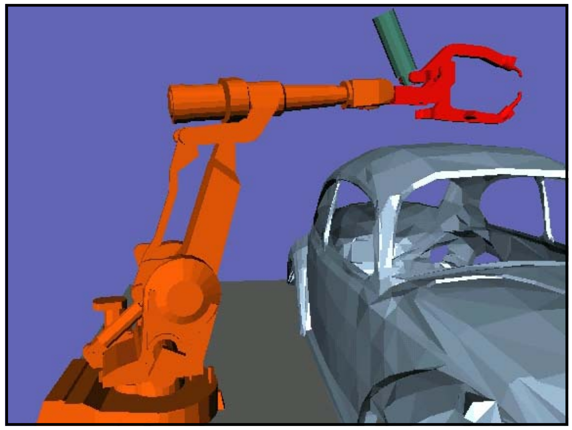
Let F be (ε,α,β)-expansive, and s and g be two configurations in the same component of F. **BasicPRM**(s,g,N) with uniform sampling returns a path be…

ex…

In general, a PRM planner is unable to detect that no path exists

Theorem 2

For any $\varepsilon > 0$, any $N > 0$, and any g in (0,1], there exists α_0 and β_0 such that if F is not (ε,α,β)-expansive for $\alpha > \alpha_0$ and $\beta > \beta_0$, then there exists s and γ in the same component of F such that **BasicPRM**(s,g,N) fails to return a path with probability greater than γ.



What does the empirical success of PRM planning tell us?

It tells us that F has often good visibility properties despite its overwhelming geometric complexity

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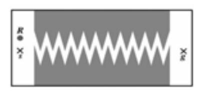
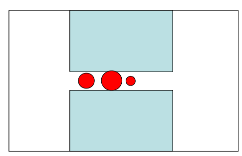
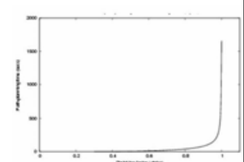
In retrospect, is this property surprising?

- Not really! Narrow passages are unstable features under small random perturbations of the robot/workspace geometry







Most narrow passages in F are intentional …

… but it is not easy to intentionally create complex narrow passages in F

Alpha puzzle

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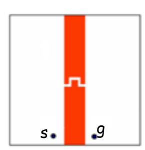
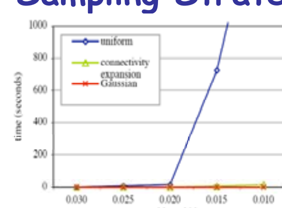
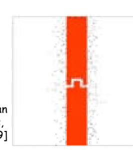
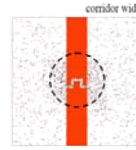
Impact of Sampling Strategy





Gaussian [Boor, Overmars, van der Stappen, 1999]





Connectivity expansion [Kavraki, 1994]

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Key Topics for Future Lectures

- Sampling/connection strategies
- Fast collision checking

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