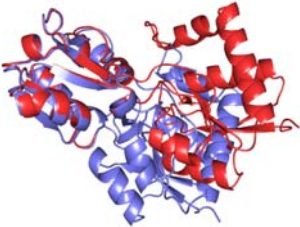


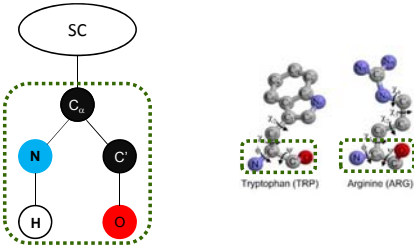
Study of Protein Motion



1

Protein

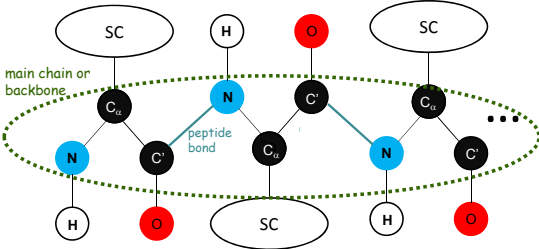
Long sequence of amino-acids (dozens to thousands)



2

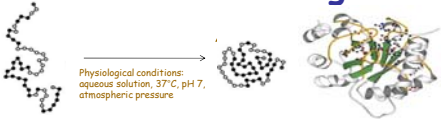
Protein

Long sequence of amino-acids (dozens to thousands)




3

Protein Folding



Physiological conditions:
aqueous solution, 37°C, pH 7,
atmospheric pressure

The folded structure is uniquely determined by the protein sequence but is not fully rigid



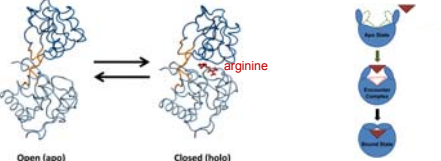
4

Flexibility is necessary ...

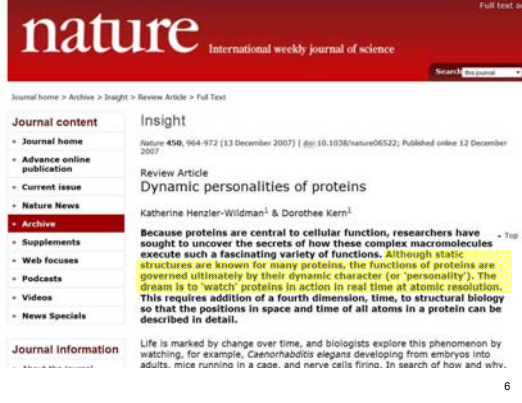
... for a protein to achieve its biochemical functions by binding against other molecules (ligands, proteins)

Binding models:

- Induced fit model
- Conformational selection model



5



6

Experimental Methods

Protein Data Bank (PDB):
Repository of folded structures of **74,732** proteins (August 2011)

- **X-ray crystallography** (65,195 entries)
→ high resolution, but one or few conformations
- **NMR spectroscopy** (9,014 entries)
→ multiple conformations, but for small proteins
- **Cryo-electron microscopy** (373 entries)
→ multiple conformations, but low resolution

7

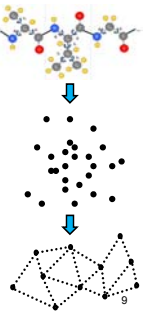
Energy-Based Computer Simulation

- Molecular Dynamics simulation
- Monte Carlo simulation
- Others:
 - coarse-grained force fields,
 - multi-scale modeling,
 - replica exchange,
 - normal mode analysis,
 - elastic network models, ...
- **Advantages:** produce time-dependent information at atomic resolution
- **Drawbacks:** huge running times, enormous amount of data, delicate setup

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Application of Robotics and Motion Planning Techniques

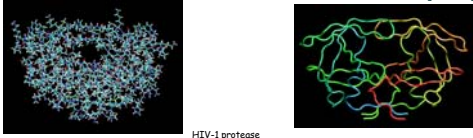
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Transform a cloud representation into a probabilistic roadmap representing protein kinetics.



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Timescales of Protein Motion

Bond/atomic vibration	Water dynamics	Helix forms	Fast correlated conf change	Slow conf change
10^{-15} femtosec MD step	10^{-12} picosec	10^{-9} nanosec one-day MD run	10^{-6} microsec long MD run	10^{-3} millisec where we need to be
				10^0 seconds where we'd love to be



[Pande]
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How can one access directly to relevant timescales?

Bond/atomic vibration	Water dynamics	Helix forms	Fast correlated conf change	Slow conf change
10^{-15} femtosec MD step	10^{-12} picosec	10^{-9} nanosec one-day MD run	10^{-6} microsec long MD run	10^{-3} millisec where we need to be
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[Pande]

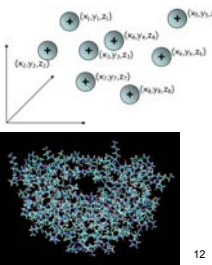
- Simulating a protein over a nanosecond timescale is like simulating human locomotion over a tiny fraction of a footstep, or like trying to understand how to reach the Moon by jumping 1.5 feet in the air.

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Kinematic Model #1: Collection of independent atoms

[Quick reminder: Kinematics studies the motion of objects without consideration of the forces that cause the motion.]

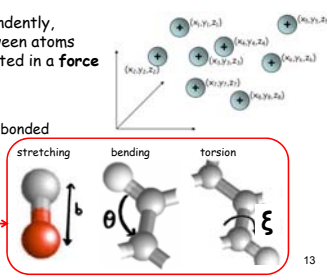
- Atoms can move independently, i.e., all constraints between atoms are eventually represented in a **force field function**
- A conformation is defined by $3 \times n$ parameters (the coordinates of the atom centers)
- All motion frequencies can be simulated



12

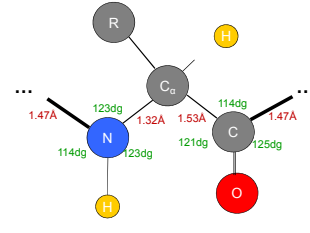
Kinematic Model #1: Collection of independent atoms

- Atoms can move independently, i.e., all constraints between atoms are eventually represented in a force field function

$$F = F_{\text{bonded}} + F_{\text{non-bonded}}$$


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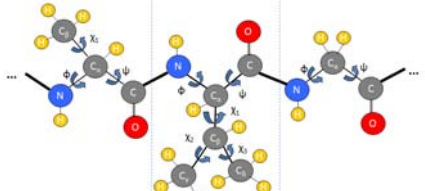
Over picosecond timescales bond lengths and angles average to constants



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Kinematic Model #2: Linkage of connected atoms

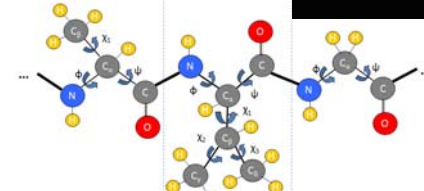
- Bonded atoms are connected by links of fixed lengths
- The only degrees of freedom are the dihedral angles around the simple bonds



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Kinematic Model #2: Linkage of connected atoms

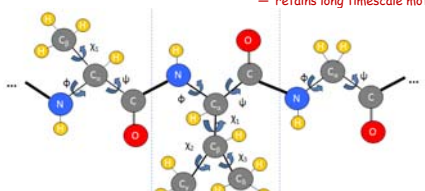
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Kinematic Model #2: Linkage of connected atoms

- Bonded atoms are connected by links of fixed lengths
- The only degrees of freedom are the dihedral angles around the simple bonds
- Free vibrations of the atoms can no longer be generated
- The linkage model
 - encodes terms of the force field,
 - filters out free atomic vibrations,
 - retains long timescale motions

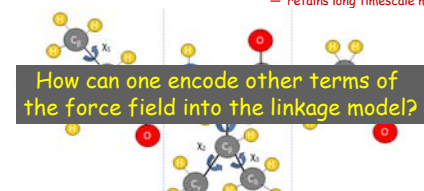


17

Kinematic Model #2: Linkage of connected atoms

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- The only degrees of freedom are the dihedral angles around the simple bonds
- Free vibrations of the atoms can no longer be generated
- The linkage model
 - encodes terms of the force field,
 - filters out free atomic vibrations,
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How can one encode other terms of the force field into the linkage model?



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Van der Waals Forces

- $F_{\text{non-bonded}} = F_{\text{van der Waals}} + F_{\text{Coulomb}}$

Van der Waals forces between two atoms result from induced polarization effect (formation of electric dipoles). They are weak, except at close range.

12-6 Lennard-Jones potential:

$$V(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right]$$

- In a folded conformation, atoms are densely packed against one another. Small perturbation can result into large repulsive vdW terms.
- Atoms are modeled as **hard spheres** with radii $\approx \alpha \times \text{vdW radii}$, where $\alpha = 0.7$ to 0.8
- + **no two hard spheres are allowed to overlap** (volume exclusion constraint)

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Hydrogen Bonds

- $F_{\text{non-bonded}} = F_{\text{van der Waals}} + F_{\text{Coulomb}}$

H-bonds stabilize secondary structure elements and tertiary structure

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Hydrogen Bonds

- $F_{\text{non-bonded}} = F_{\text{van der Waals}} + F_{\text{Coulomb}}$

H-bonds rigidify portions of the protein and create closed cycles in linkage model

21

Hydrogen Bonds

- $F_{\text{non-bonded}} = F_{\text{van der Waals}} + F_{\text{Coulomb}}$

H-bonds rigidify portions of the protein and create closed cycles in linkage model

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Advantages/Drawbacks of Linkage Model

- Fewer DOFs, hence smaller dimensionality of the conformational space
- Many force terms are directly encoded in representation, hence the model can't create motion that would violate these terms
- Most high-frequency motions are de facto filtered out

But:

- Generating kinematically valid conformations can be more difficult

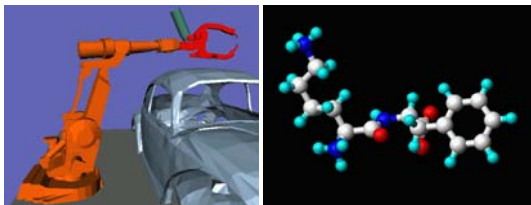
23

Inverse Kinematics Problem

How does a change in the position of an atom affect the rest of the protein?

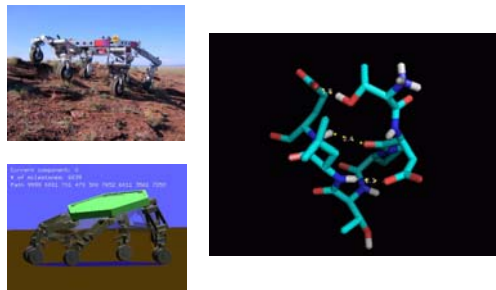
24

Analogy with Robotics



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Analogy with Robotics



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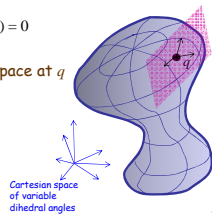
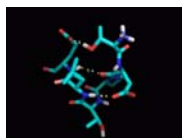
Inverse Kinematics Methods

Null-space motion

How to deform a subset of a protein with more than 6 ϕ and ψ angles without breaking cycles?

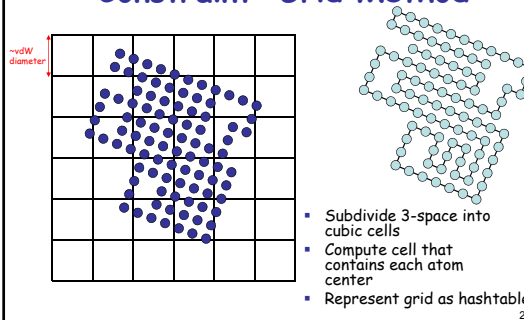
SVD method:

- Cycle closure constraints $\rightarrow F(q) = 0$
- Differentiation $\rightarrow J_F \times dq = 0$
- SVD of $J_F \rightarrow$ Basis of tangent space at q



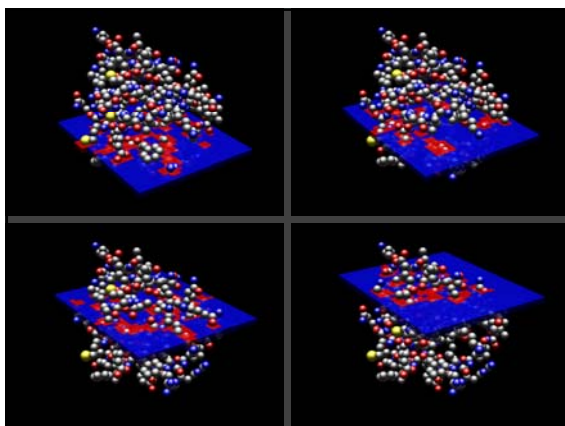
27

Checking Volume-Exclusion Constraint: Grid Method



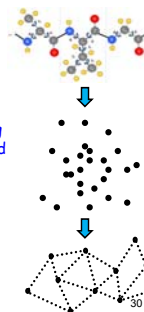
- Subdivide 3-space into cubic cells
- Compute cell that contains each atom center
- Represent grid as hashtable

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Beyond Simulation

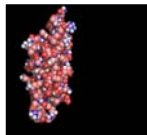
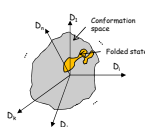
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Kino-Geometric Conformational Sampling

Computational challenges:

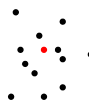
- Requires satisfying often **antagonistic** constraints: kinematic and volume exclusion constraints
- Folded conformations form a relatively **tiny region** of the conformational space. How to hit this region?



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Kino-Geometric Conformational Sampling

- ROCK (Rigidity Optimized Conformational Kinetics) [Zavodsky et al., 2004]
- FRODA (Framework Rigidity Optimized Dynamic Algorithm) [Wells et al., 2005, Farrell et al., 2010]
- KGS (Kino-Geometric Sampling) [Yao et al. 2011]
- PEM (Protein Ensemble Method) [Shehu et al., 2006]

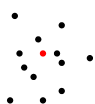


- Initialize conformation distribution Δ to $\{q_{given}\}$
- Iterate
 - Pick q from Δ
 - Deform q into new conformation q_{new}

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KGS: q is picked at random with probability inverse to sampling density
- Select stable H-bonds in q
ROCK and FRODA: select H-bonds with energy less than a threshold
KGS: uses a regression tree trained from Molecular Dynamics data
- Perform rigidity analysis in q
ROCK, FRODA, and KGS: transform kinematic constraints into distance constraints between atoms, run Pebble Game algorithm to identify all rigid groups of atoms
- Deform q into q_{new}
ROCK: Perturb dihedral angles and close cycles by minimizing to zero a measure of closure violation
FRODA: Perturb all atom positions and reform rigid groups of atoms
KGS: Perturb dihedral angles in null space
- Check q_{new} for volume exclusion

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Statistics for Two Proteins

- 2EZM**
atoms: 992
rigid groups: 503
cycles: 47
- 2LAO 84**
atoms: 3649
rigid groups: 1023
cycles: 84



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Kino-Geometric Conformational Sampling

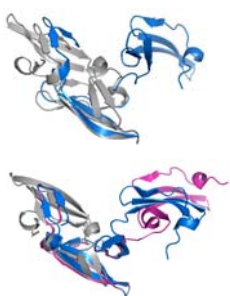
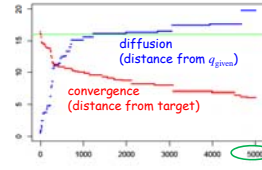
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KGS Sampling: 2EZM

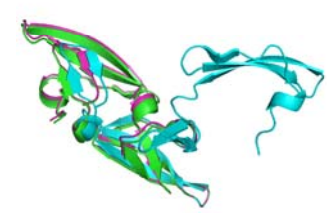
- 992 atoms
- Given conformation in grey
- Target conformation in blue at RMSD 16Å from given conformation (hinge and twist)

Running time on a dual quad-core 3GHz computer: 93 minutes, 1.12 seconds per sample 37

KGS Sampling: 2EZM

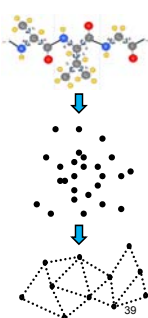
- "Path" from given conformation to sampled conformation closest to target:



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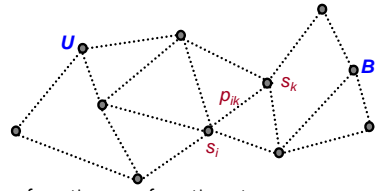
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Graph-Based Model: States and Transitions

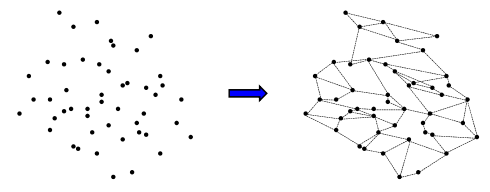


- State: conformation or conformation set
- Transition: probability of going from one state to another (Markov model)
- Compact representation of a huge collection of possible motion paths
- $P_B(s)$ = probability that protein reaches B from s before reaching U

$$P_B(s_i) = \sum_k P_{ik} \times P_B(s_k)$$

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States are conformations

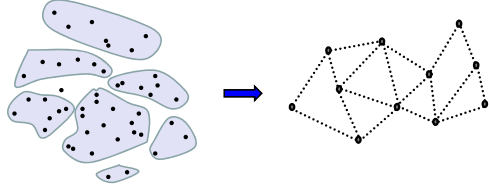


- Sampling distribution can be generated using kino-geometric methods, or by sub-sampling many short MD simulation trajectories, or by other methods

[Apaydin et al., 2003] [Singhal et al., 2004]

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States are conformation sets



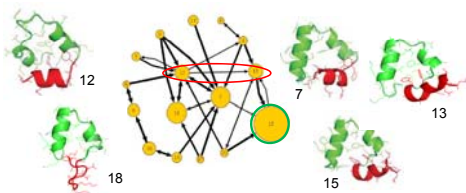
- Sampled conformations are clustered to produce a more compact model and better satisfy the Markov assumption
- Each cluster is computed to approximately match a basin of attraction of the energy landscape

[Chodera et al., 2007] [Chiang et al., 2010]

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Applications

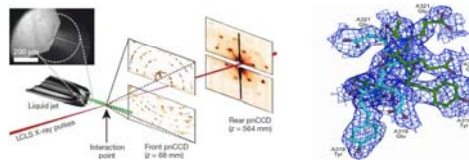
- Analysis of the dynamics of small proteins: alinine dipeptide, villin, tryptophan zipper beta hairpin



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New Project (with SLAC)

- Interpretation of electron-density maps generated with **femtosecond X-ray protein nanocrystallography** [Chapmann et al., Nature, Feb 2011]



- Combination of state-of-the-art kino-geometric sampling with a (hopefully) revolutionary experimental method

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