A GENERAL ALGORITHM FOR BEAM SELECTION IN RADIOSURGERY

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Abstract: Radiosurgery is a technique in which narrow, focused beams of radiation are used to destroy tumors. The treatment-planning problem in radiosurgery is to find a set of feasible beam configurations that will destroy the tumor without harming the healthy tissue surrounding it. Depending on the radiosurgical system being used, solving this problem can involve searching a very large space of possible configurations. Furthermore, because tumors vary in size and shape as well as in their location relative to critical structures, it is important to have an algorithm that is general enough to be applicable to a wide range of tumor types. In this paper, we present a general algorithm for beam selection. This algorithm first narrows down the search space by defining a set of promising beam configurations, and then iteratively refines this initial set to produce a satisfactory solution. The attractiveness of the algorithm stems from its empirically proven ability to produce good treatment plans for a wide range of tumor types, including tumors in the brain, spine, and prostate.

1. INTRODUCTION

In radiosurgery narrow, focused beams of radiation are used as surgical tools to ablate tumors. The goal of radiosurgery is to deliver a lethal dose of radiation to the tumor, so as to effectively remove it, while minimizing the dose to the surrounding healthy tissues in the body. The treatment-planning problem is to find a set of feasible beam configurations that will accomplish this goal. In general, radiosurgery treatment planning occurs in three phases: (1) the *Data Preparation Phase,* which involves the specification of the desired dose distribution that will destroy the tumor while minimizing damage to healthy structures, (2) the *Inverse Dosimetry Phase,* which involves the selection of beam configurations that will achieve the specified dose distribution, and (3) the *Direct Dosimetry Phase,* which involves precise evaluation of the final dose distribution produced by these beam configurations and iteration until this distribution is satisfactory. Although the first and third phases should be performed by a trained radiation oncologist, the second phase lends itself well to automation, and is, thus, the focus of our work.

The radiosurgical system that we are using is illustrated in Fig. 1 and has been described previously (Adler et al., 1994). The key feature to notice is that the linear accelerator that produces the radiation beam is mounted at the end of a six-degree-of-freedom robot arm. Thus, where conventional radiosurgical systems are limited by restrictive kinematics, our system provides much flexibility in defining beam configurations. Each beam configuration is defined by four parameters: the *position* of the radiation source, the *orientation* of the beam, the dose *weight* (the amount of radiation sent along the beam), and the collimator *diameter*. Typically several hundred beam configurations are used per treatment.

In this paper, we present our algorithm for selecting treatment beams. A complete algorithm for beam selection would always succeed in finding a set of feasible beam configurations that satisfy the dose constraints whenever such a set exists. However, such an algorithm seems unrealistic because the search space is extremely large. Our algorithm, thus, provides a good and reasonably fast approximation to a complete algorithm; it finds a solution with high probability when one exists, and is general enough to be applicable to a wide variety of cases without requiring custom tailoring. Our overall approach is to start by making our best guess as to what the solution will look like and then to refine this original estimate iteratively to produce a satisfactory solution.

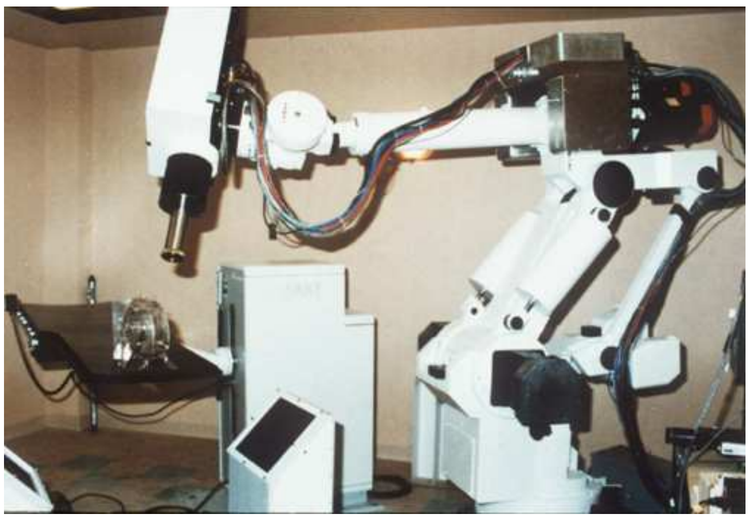

Fig. 1: Image-Guided Robotic Radiosurgical System

2. OVERVIEW OF THE ALGORITHM

The input to our beam-selection algorithm is a description of the geometry -- the size, shape, and location of the tumor and of the critical structures, a set of constraints describing the required dose distribution, and a maximum number of beams to be used (which determines to the time required to deliver the treatment). The output of the algorithm is either a set of beam configurations that will produce the prescribed dose distribution or an indication that such a set cannot been found.

Going from a desired dose distribution directly to a set of beam configurations by an analytical method is extremely difficult, if not impossible (Barth, 1990, Brahme, 1988, Cormack and Cormack, 1987, Kooy and Barth, 1990). A more manageable problem is to evaluate a given set of beam configurations in terms of their effectiveness in treating the tumor. Our algorithm (Fig. 2) first narrows the search space by finding a set of beam positions and orientations that are likely to work. It defines this initial set of beam position and orientations by randomly sampling from a set of beams that all converge on the tumor. If equal weights and diameters were to be assigned to the initial sample, a region of high dose would be created at the tumor; however, the dose constraints of the radiation oncologist may not be met. The next phase of planning, thus, evaluates this initial sample to determine if it can be made to meet the constraints, by manipulating dose weights and collimator diameters. If it can, then a solution has been found. If it cannot, then the constraints are widened (i.e., the problem is made easier) until a solution can be found. The beams in the solution to the less constrained problem are used to guide subsequent iterations until a solution is found or the maximum number of iterations is reached.

1. While *BeamList* contains less than *TotalBeams* beams, add a new beam to *BeamList.*
2. Run linear program to find *doseweights* that satisfy the dose constraints.
3. If the constraints can be satisfied, then
   (a) If the current constraints meet the specified constraints, return solution.
   (b) Else, tighten the constraints, and run the linear program.
4. If the constraints cannot be satisfied, repeat the following until a solution is found:
   (a) Relax the constraints.
   (b) Run the linear program.
5. For each beam in BeamList
   (a) If *doseweight* < *SmallWeight*, delete the beam from *BeamList*.
6. Go back to 1.

Fig. 2: Algorithm for Beam Selection

At the core of this algorithm is the ability to produce an optimal set of beam weights and diameters given a set of beam positions and orientations. This is a very powerful tool: it allows us to find those beams that contribute most directly to solving the problem. Furthermore, because the weights can be very small, or even zero, we can theoretically insert a large number of beams in the input, and allow the algorithm to choose, in an optimal fashion, those beams that should make up the plan. However, this approach involves running a complex linear program with many geometric constraints. Our algorithm must, thus, strive to do two things. First, it must keep the number of constraints and variables that are input to the linear program manageable. Though increasing the number of beams randomly sampled at the start of beam selection would increase the probability that a solution is found up front, it would also, in practice, make the linear program impossible to run. We therefore maintain a bound on how many configurations can be considered per iteration, usually between 1000-1500, and rely on the iterative process to narrow in on the good shooting positions and orientations. Second, we must minimize the number of times that we iterate through the linear-programming loop. Initially this is accomplished by choosing beams that all converge on the tumor and that have empirically been shown to lead to good solutions. In subsequent iterations, this method is augmented by also using information from the previous iteration, with the goal of converging on a solution rapidly.

3. DETAILS OF THE ALGORITHM

In the first phase of planning, the radiation oncologist views successive images from a computed tomography (CT) or magnetic-resonance (MR) study of the patient's anatomy. The radiation oncologist delineates regions of interest (the tumor and any dose-sensitive or critical regions that may be at risk), and assigns constraints on the amount of radiation in and around these structures of interest. For example, he might specify that the tumor needs to receive between 2000 and 2200 rads, that the critical structure should receive no more than 500 rads, and that the dose around the tumor should fall off at a rate of 100 rads per millimeter. This constraint information coupled with the geometric description of the contours from the imaging data is the input to our automatic planning system. Automatic planning then proceeds along the following steps:

(1) **Reconstruction:** The contour data defined by the radiation oncologist must be converted into a useful representation for planning. The automatic planning system requires two representations of the data. The first is a *triangulated surface representation* of the tumor, to be used in defining the initial set of beams, and the second is a *volumetric representation* of the relevant areas, to be used in formulating constraints for the linear program, and in performing dose calculations. To form the triangulation, the system first connects the contours to one another using Fuchs's algorithm (Fuchs et al., 1977). It then triangulates the top and bottom polygons using Delaunay triangulation (Sloan and Houlsby, 1984). The system defines the volume representation within a three-dimensional grid that encompasses all the regions of interest. Each voxel in this grid represents one cubic millimeter. The system assigns each voxel a certain value depending on which region of interest, if any, contains it.

(2) **Initial Beam Selection:** The goal of initial beam selection is to narrow the search space to a set of beam positions and orientations that have a high likelihood of being able to satisfy the specified constraints. The system selects beam positions and orientations such that the beams will all intersect the tumor and converge to form a region of high dose that matches the three-dimensional shape of the tumor. It defines each beam by two parameters: a *target point*, at which the beam is aimed, and a *source point*, where the beam originates.

The system defines target points such that they are normally distributed on the surface of the tumor, and later places additional target points in regions of high curvature. To distribute points normally on the tumor surface, the system uses the triangulated tumor representation. It assigns a certain number of beams to each triangle in the triangulation proportional to that triangle's area relative to the total surface area. It then distributes that number of target points in the triangle using Turk's algorithm (Turk, 1990). Because it is often most difficult to satisfy the constraints of the radiation oncologist in regions of the tumor where there is high curvature, more beams are considered in these regions in hopes of increasing the probability of finding effective beams for treating these areas. To detect regions of high curvature, the system applies the method described in (Turk, 1992). The location of the target points can be adjusted somewhat depending on the tradeoffs implicit in the desired dose distribution: a more homogenous distribution of tumor dose is obtained if the target points are placed on a slightly expanded representation of the tumor; whereas, a faster falloff of the dose around the tumor is obtained if a slightly contracted representation of the tumor is used.

The system associates each target point with a source point. It selects source points randomly from among the points on a feasible template path that the robot arm can traverse safely. We have chosen this randomized method over more deterministic methods because we have found that, although deterministic methods often work well for certain shapes, they fail on others. Randomization is more general. It works by selecting more beams than will be used, and by relying on refinement and iteration to prune out the bad directions and retain the good directions.

(3) **Constraint Formulation:** The radiation oncologist is able to constrain the dose in three ways. First, he can specify the range of dose that each of the regions of interest can receive. Second, he can constrain the rate of dose falloff around the tumor. Third, he can constrain the rate of dose falloff within the critical regions, based on their distance from the tumor. The automatic

planner then formulates linear constraints by examining the arrangement of the beams within the regions of interest, as described previously in (Tombropoulos et al., 1995). For each voxel within a region of interest, the system determines the beams that intersect it. The system then generates linear constraints based on the constraints for the containing region of interest: if the voxel is in the tumor, then the constraints would ensure that the sum of the dose produced by the beams that intersect that voxel is within the prescribed range for the tumor. This method can be augmented to also include constraints for optimizing the selection of collimator diameters (Tombropoulos et al., 1995).

(4) **Refinement and Iteration:** The next step in planning is to evaluate the effectiveness of the selected beam positions and orientations. This is done by solving the constraints formulated in the previous step using linear programming. We use the MINOS linear-programming system (Murtagh and Saunders, 1993). If the problem is feasible, MINOS returns a set of dose weights; otherwise it indicates that the problem is infeasible. If satisfactory dose weights were found, the planner stops iterating. If weights were not found, it must select a new set of beam positions and orientations to test. The new set could be generated with the randomized method for initial beam selection; after a sufficient number of iterations, a set containing enough good beams probably would be found. However, because we want to converge on a solution in as few iterations as possible, we exploit the fact that much information about what characterizes a good beam can be gained from solving similar, but easier, problems. The system, thus, runs the linear program again with a more relaxed set of constraints until it finds a solution; it retains beams that were assigned substantial weights in the solution, tightens the constraints again, and iterates back to the beam-selection phase (see Fig. 2).

(5) **Plan Evaluation:** Once a set of beam configurations has been found, the dose distribution is calculated and presented to the radiation oncologist to review. If he is satisfied with it, then treatment can be delivered. If there are parts that are not satisfactory, then new constraints can be added and/or existing constraints can modified. Because the problem will be very similar to the original problem, however, the planning system can start with the current set of beams and, thus, converge at a faster rate.

4. EXPERIMENTAL RESULTS

Another exciting feature of the radiosurgical system for which we designed our planner is that it provides for frameless location of the tumor (Adler et al., 1994). Not requiring a stereotactic frame means that treatments can be fractionated easily, and that extracranial tumors can be treated. This advance holds promise for extending the scope of radiosurgery beyond the treatment of small intracranial tumors. We have begun to experiment with cases involving a much wider range of tumors than those conventionally treated with radiosurgery. Here, we show results for one case involving a brain tumor and one involving a prostate tumor.

4.1 Brain-Tumor Case

Fig. 3 illustrates a case involving a tumor in the brain with one critical structure, the brain stem.

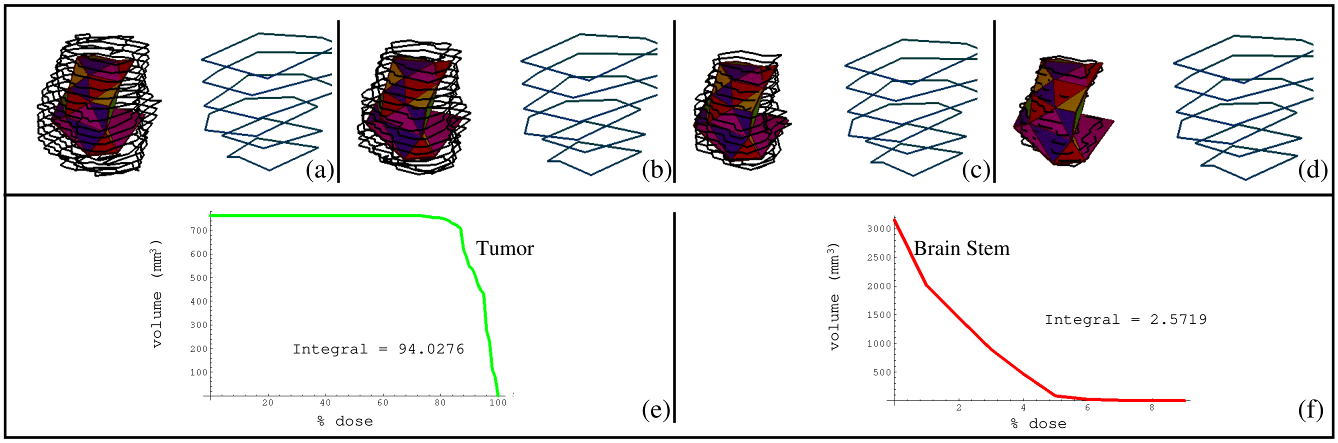

Fig. 3: Dosimetry results for the brain-tumor case. (a)-(d) The triangulated tumor representation encompassed by the 50%, 60%, 70%, and 80% isodose curves, respectively. The contours to the right represent the brain stem. (e) The tumor dose-volume histogram (f) The brain-stem dose-volume histogram.

Because the critical structure is not very close to the tumor, the radiation oncologist gave high priority to attaining a homogeneous dose within the tumor, without worrying as much about a rapid dose falloff around the tumor. However, as illustrated in Fig. 3a-d the distance between the various isodose curves is still small. The 80% isodose curve closely conforms to the tumor volume, and intersects very little of the surrounding healthy tissue. The tumor dose-volume histogram (Fig. 3e) illustrates that the planner succeeded in producing a homogeneous dose distribution within the tumor (with less than 6% inhomogeneity in the tumor) while keeping those dose to the critical structure low (Fig. 3f).

4.2 Prostate-Tumor Case

Fig. 4 illustrates a prostate case where the tumor is embedded between two critical structures, the rectum (left) and the bladder (right). In treating this case, the radiation oncologist chose to give a higher priority to minimizing dose in the critical regions than to achieving a homogeneous dose in the tumor. Furthermore, because some degree of fractionation is desirable for this case, the number of treatment beams was limited to 80, so as to limit treatment time. Notice that the isodose curves follow the tumor shape very closely, and that the fall-off of dose is rapid, i.e. the distance between them is small. However, the dose-volume histograms (Fig. 4e-g) illustrate that this rapid falloff was achieved at the expense of tumor-dose homogeneity.

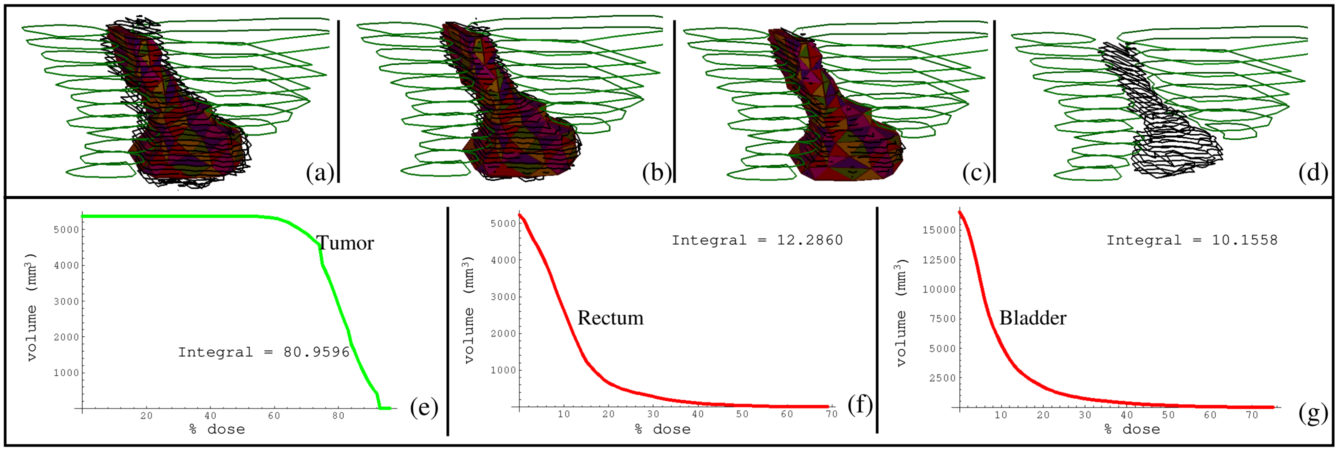

Fig. 4: Dosimetry results for prostate-tumor case. (a)-(d) The 50%, 60%, 70%, and 80% isodose surfaces, respectively. Not that the 70% curve conforms to the tumor shape, while the 80% curve is, for the most part, contained within the tumor. The contours representing the rectum (left) and the bladder (right) are shown surrounding the triangulated representation of the tumor. (e) The tumor dose-volume histogram. (f) The rectum dose-volume histogram. (g) The bladder dose-volume histogram.

5. CONCLUSION

We have described a new algorithm for beam selection in radiosurgical treatment planning. This algorithm allows us to harness the power of a promising new robotic technology. Where current radiosurgical systems have limited radiosurgical treatment possibilities with their restricted kinematics and localization requirements, this new imaged-guided robotic radiosurgical system opens many new possibilities. Our beam-selection algorithm exploits the robotic system's flexibility to produce plans that are general enough to be applicable to wide range of tumor types. We have shown these plans to be effective for a variety of cases, including tumors in the brain and prostate. Our algorithm is efficient enough to be applied in a hospital setting, and to allow for interaction with the radiation oncologists.

REFERENCES

Adler, J. R., Schweikard, A., Tombropoulos, R., and Latombe, J.-C. (1994). Image-Guided Robotic Radiosurgery. In: Proceedings of the First International Symposium on Medical Robotics and Computer Assisted Surgery, 2, pp. 291-297.

Barth, N. H. (1990). An Inverse Problem in Radiation Therapy. In: Int. J. Radiation Oncology Biol. Phys., 18(2), pp. 425-431.

Brahme, A. (1988). Optimization of Stationary and Moving Beam Radiation Therapy Techniques. In: Radiother. Oncol., 12, pp. 129-140.

Cormack, A. M. and Cormack, R. A. (1987). A Problem in Rotation Therapy with X-rays: Dose Distributions with an Axis of Symmetry. In: Int. J. Radiation Oncology Biol. Phys., 12(12), pp. 1921-1925.

Fuchs H., Kedem, Z. M., and Uselton, S. P. (1977). Optimal Surface Reconstruction from Planar Contours. In: Communications of the ACM, 20(10), pp. 693-702.

Kooy, H. M. and Barth, N. H. (1990). The Verification of an Inverse Problem in Radation Therapy. In: Int. J. Radiation Oncology Biol. Phys., 19(2), pp. 433-439.
Murtagh, B. A. and Saunders, M. A. (1993). Minos 5.4 User's Guide. Technical Report SOL 83-20R, Department of Operations Research, Stanford University, California, March.
Sloan, S. W. and Houlsby, G. T. (1984). An Implementation of Watson's Algorithm for Computing 2-D Delaunay Triangulations. In: Adv. Eng. Software, 6(4), pp. 192-197.
Tombropoulos, R., Schweikard, A., Latombe, J.-C., and Adler, J. R. (1995). Treatment Planning for Image-Guided Robotic Radiosurgery. In: Proceedings of the First International Conference on Computer Vision, Virtual Reality and Robotics in Medicine, pp. 131-137.
Turk, G. (1990). Generating Random Points in Triangles. In: Glassner, A. S. (editor). Graphics Gems, Academic Press, Inc., San Diego, pp. 24-28.
Turk, G. (1992). Re-Tiling Polygonal Surfaces. In: Computer Graphics, 26(3), pp. 55-64.

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