

Synthesis of natural arm swing motion in human bipedal walking

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Abstract

It has historically been believed that the role of arm motion during walking is related to balancing. Arm motion during natural walking is distinguished in that each arm swing is with the motion of the opposing leg. Although this arm swing motion is generated naturally during bipedal walking, it is interesting to note that the arm swing motion is not necessary for stable walking. This paper attempts to explain the contribution of out-of-phase arm swing in human bipedal walking. Consequently, a human motion control methodology that generates this arm swing motion during walking is proposed. The relationship between arm swing and reaction moment about the vertical axis of the foot is explained in the context of the dynamics of a multi-body articulated system. From this understanding, it is reasoned that arm swing is the result of an effort to reduce the reaction moment about the vertical axis of the foot while the torso and legs are being controlled. This idea is applied to the generation of walking motion. The arm swing motion can be generated, not by designing and tracking joint trajectories of the arms, but by limiting the allowable reaction moment at the foot and minimizing whole-body motion while controlling the lower limbs and torso to follow the designed trajectory. Simulation results, first with the constraint on the foot vertical axis moment and then without, verify the relationship between arm swing and foot reaction moment. These results also demonstrate the use of the dynamic control method in generating arm swing motion.

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1. Introduction

There has been a great deal of effort in various research fields to understand bipedal locomotion. In biomechanics, the motivations have included an improved understanding of the basic mechanisms of locomotion, including the role of muscular activation, as well as the design of assistive devices for the disabled (Inman et al., 1981; Perry, 1992; Andriacchi and Alexander, 2000). The purpose of much of the research has involved the clinical desire to treat gait pathologies in patients by analyzing the neuromuscular and musculoskeletal systems. In a complementary manner the robotics community has also been interested in bipedal locomotion for the purpose of controlling humanoid robots by generating gait trajectories and executing them using feedback algorithms. This has been the focus of much ongoing research since walking is one of the most basic

functions required of humanoid robots. Because of their different motivations, the approaches between the biomechanics and robotics communities differ with respect to bipedal locomotion.

Musculoskeletal models of the human body have been implemented in much of the gait research (Taga, 1995; Jo and Massaquoi, 2007; Anderson et al., 2006). These models enable the simulation of gait more realistically. A great deal of effort has been directed at applying these techniques to clinical purposes (Andriacchi and Alexander, 2000). In robotics, with the introduction of many new humanoid robots over the past 10 years (Hirai et al., 1998; Daneko et al., 2002; Kim et al., 2005), there has been an emerging interest and challenge to provide humanoids with natural gait behavior (Kajita et al., 2003; Pratt and Pratt, 1998; Kuo, 1995; Nazir et al., 2002). It is believed that further sharing of these tools and techniques will continue to benefit both the biomechanics and robotics communities (Azevedo et al., 2007).

It is noteworthy that the main focus of gait research has been on lower body motion, rather than the secondary

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motion of the arms. Consequently, the models used in gait studies have often been simplified to exclude the arms. However, the relationship between arm swing motion and the vertical moment on the foot has been explored (Elftman, 1939; Li et al., 2001) and briefly discussed (Inman et al., 1981; Perry, 1992; Jackson et al., 1978). The main thrust of these studies is that arm swing motion helps stabilize body motion, specifically angular motion about the vertical axis of the foot. The experiments in Li et al. (2001) clearly demonstrate that the moment about the vertical axis of the foot increases when arm swing is prevented in male subjects (this effect is not significant for female subjects). This finding has been used in the design of walking machines (Collins et al., 2001) to reduce the vertical axis moment on the foot; this design, however, was not able to provide stable yaw control.

There has also been analysis of arm swing in the sagittal plane using pendulum models (Wagenaar and Van Emmerik, 2000; Kubo et al., 2004). These studies correlate the magnitude and phase of arm swing with the speed of walking. Popovic et al. (2004) demonstrate that spin angular momentum is regulated during human walking and this idea is used in control.

Building upon past work, in this paper we attempt to gain further insight into why and how out-of-phase arm swing motion is generated. We focus on the effect of reaction moment about the vertical axis of the foot (Fig. 1). It is believed that further study of arm motion will greatly aid in the understanding of the walking pattern overall.

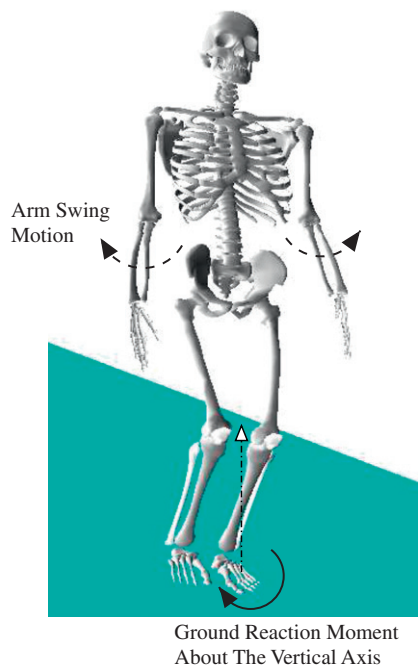


Fig. 1. Illustration of human walking with arm swing on one foot stance. The torsional effect of the out-of-phase arm swing motion about the vertical axis is the same as that of the ground reaction moment about the vertical axis of the stance foot. Out-of-phase arm swing motion, therefore, reduces the reaction moment of the stance foot.

This understanding could prove useful in biomechanics, particularly with regard to the relation between leg muscle activation and upper body motion (Ferris et al., 2006). For humanoid robots, implicit, online generation of natural arm swing during walking could replace pre-planned joint trajectories to resolve problems of potential instability.

The relationship between arm swing and reaction moment about the vertical axis of the foot is explained in Section 2 using an articulated multi-body system model. From this relationship, it is reasoned that human body control implicitly generates the arm swing motion in an effort to reduce the reaction moment about the vertical axis of the foot. By not relying entirely on the feet to generate vertical moments, the walking pattern thus becomes more robust to sudden changes in surface friction.

In order to make this connection, a multi-body dynamic model of the human body, not a simplified model, must be used. Physics-based simulation has been employed in this paper to verify this hypothesis. Human motion control is then recreated using the contact consistent control framework, which is capable of employing the full dynamic model of a human multi-body system, as well as commanding reaction forces and moments directly (Park and Khatib, 2006; Park, 2006).

By using simulation, it was possible to isolate the relationship between arm swing and vertical moment on the foot from other factors such as head, trunk, and pelvic motions as well as variations created by height, weight, sex, and age. A side effect from excluding these factors is that the walking posture may not be as natural as with actual human beings. Compared to human subject data, however, the simulation can provide vital insight. For example, subjects can be instructed not to move their arms in an attempt to study how the absence of arm swing affects the vertical moment on the foot. However, the subject may naturally compensate for lack of arm swing with head, torso, and pelvic motion that produce similar effects on the foot moment. Additionally, the behaviors of male and female subjects can be very different due to cultural influences (Li et al., 2001). Simulation allows the human motion to be consistently controlled, with the flexibility to introduce or remove constraints at will.

2. Analysis of arm swing during human bipedal walking

Out-of-phase arm swing is a common pattern during human bipedal walking. The left arm moves forward when the right leg and torso move forward, and vice versa for the opposing leg and arm (Fig. 1). This arm motion, though natural, is not required for walking motion. For example, we are able to walk even while executing certain manual tasks which constrain the arms from swinging (e.g., holding an object with two hands or carrying a suitcase). However, without any special manual objectives the arm movements follow a consistent pattern. In this section, the reason for this natural arm motion is analyzed qualitatively from a dynamics perspective. Based on the conclusions obtained,

a control approach will be developed in the following section.

2.1. Whole-body coordination

Walking requires the coordination of the whole body. There can be significant coordinated movement in the head, trunk, and pelvis orientation related to the walking speed and pattern. Pelvis rotation, for example, may occur depending on the length of the foot step. That is, the hip may rotate so that the swinging leg can reach farther. In this paper, in order to concentrate on the effect of arm swing, we will only consider a walking pattern that has little change in chest, head, and hip orientation without any specific tasks for the hands or arms.

2.2. The role of arm swing during walking

The out-of-phase motion of arm swing does not create any net change of the center of mass of the system in the horizontal plane, because the locations of the masses of the left and right arms change in exactly opposite directions. Additionally, because the arms rotate in opposing directions about the lateral axis (axis passing through the shoulders) they do not generate any moments about this axis. This can be seen by considering the torque vector exerted on each arm; these vectors are in opposite directions, and of equal magnitude, so they cancel each other out (in this axis). The only net torsional effect from arm motion is a moment about the vertical axis of the torso. This arises due to the inertial effects of arm swing motion about the vertical axis. When viewed from above, the reaction forces from the arms to the torso (one to move one arm forward and the other to move the other arm backward), both contribute to the moment about the vertical axis in the same direction, causing them to add up rather than cancel each other out.

During the left foot stance, a positive torque with respect to the stance foot (counter-clockwise from the top view) is necessary to move the right leg and torso forward. This is illustrated in Fig. 2. This torque will be larger if the hip rotates as well in a positive direction to increase the right foot step length.

The external torque applied to the legs and torso, required to counter the reaction from the motion of leg swing and torso, can come from only two sources: (i) the ground reaction moment about the vertical axis of the stance foot (this reaction moment is transmitted through the stance leg to the torso); (ii) the torsional effect to the torso from arm swing (this was described earlier). While both of these sources of torque can achieve the same effect, the ground reaction moment is limited by ground frictional properties, whereas the torsional effect from arm swing is only limited by arm motion.

In summary, the same motion of the torso and legs can be generated with or without arm swing motion. The trade-off is between the external reaction moment about the

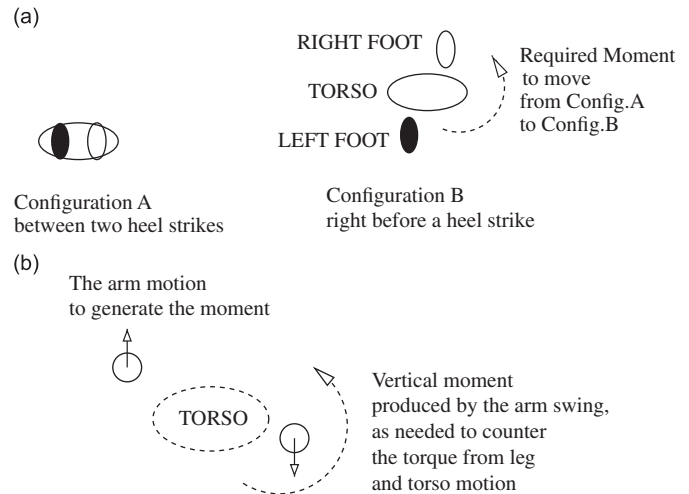


Fig. 2. The relationship between arm swing and reaction moment is illustrated. (a) The required moment about the vertical axis is shown when a person is on the left foot stance while walking. (b) The arm swing motion, producing the moment about the vertical axis, reduces the reaction moment about the same axis of the stance foot.

vertical axis of the stance foot and out-of-phase arm swing motion. Decreasing the foot reaction torque requires increasing the arm swing motion, and vice versa. Arm swing in walking is, therefore, a way of supplementing (or precluding) ground reaction torque at the stance foot.

3. Contact consistent control framework for human motion synthesis

The contact consistent whole-body control framework was used in walking simulation (Park and Khatib, 2006; Park, 2006). This algorithm uses the full dynamics of the multi-body system in the composition of control torques (Fig. 3). The framework allows us to compose motion control in the task space rather than joint angle space. Designing motion in the task space has great advantages over the joint angle space since the task space can be defined using physically meaningful coordinates. In addition, the task space does not necessarily need to have the same degrees of freedom as the whole system. Often, only a small number of tasks can be sufficient for controlling the system. The contact consistent whole-body control framework minimizes motion in the remaining degrees of the freedom after controlling the task coordinates. The framework provides a dynamically consistent method to compose specific control or optimization in the null-space of task control.

In this paper, we want to investigate the relationship between the motion of the arms and the reaction moment about the vertical axis of the foot. Thus, using human skeleton model in this framework, a constraint on the moment about the vertical axis of the supporting foot was imposed, resulting in an implicitly generated out-of-phase arm swing motion. The control torque for this constraint was composed in the null-space of the task space so that

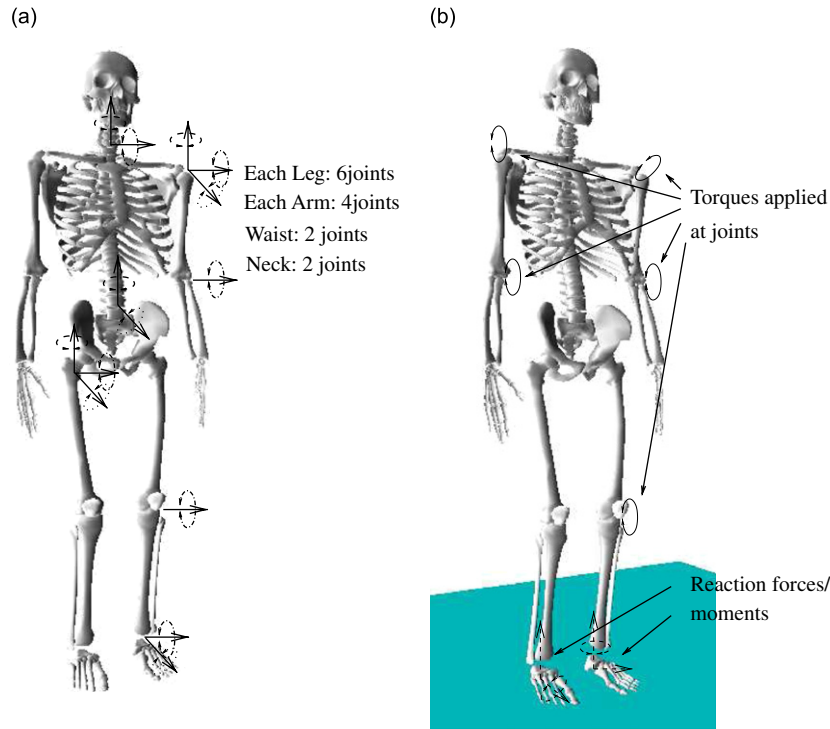


Fig. 3. (a) Kinematic representation of human body. (b) Joint torques and ground reaction forces and moments.

the task coordinates for walking motion were not affected. This method involved applied actuator torques on the arm joints, rather than explicit joint motion trajectories.

3.1. System dynamics in contact

The dynamic equations for the multi-body system in contact (Fig. 3) are described by

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) + J_c^T f_c = \Gamma, \quad (1)$$

where q is the vector of joint angles and Γ is the torque vector. The terms $A(q)$, $b(q, \dot{q})$, and $g(q)$ are the joint space inertia matrix, the vector of Coriolis and Centrifugal forces, and the vector of gravity, respectively. The term, f_c , is the vector of the contact forces and moments and J_c is the corresponding Jacobian.

In describing multi-body dynamics with respect to the ground, we must account for the relative motion between the human and the ground. The above equation accounts for this by including three prismatic and three revolute *virtual joints*, which connect one of the links to the ground.

When the human is in contact with the *ground*, the *constrained dynamic equations of motion* can be derived with the assumption that there is no velocity and acceleration at the contact (Park and Khatib, 2006; Udwadia and Kalaba, 1996).

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) - h_c(q, \dot{q}) = (I - P_c(q))\Gamma, \quad (2)$$

$$f_c = \bar{J}_c^T(q)\Gamma - \mu_c(q, \dot{q}) - p_c(q), \quad (3)$$

where

$$\begin{aligned} \bar{J}_c^T(q) &= (J_c A^{-1} J_c^T)^{-1} J_c A^{-1}, \\ P_c(q) &= J_c^T(q) \bar{J}_c^T(q), \\ h_c(q, \dot{q}) &= J_c^T(q) (\mu_c(q, \dot{q}) + p_c(q)), \\ \mu_c(q, \dot{q}) &= (J_c A^{-1} J_c^T)^{-1} (J_c A^{-1} b(q, \dot{q}) - \dot{J}_c \dot{q}), \\ p_c(q) &= (J_c A^{-1} J_c^T)^{-1} J_c A^{-1} g(q). \end{aligned} \quad (4)$$

3.2. Control using task space

Given the task space coordinates (or the operational space coordinates; Khatib, 1987), x , the corresponding Jacobian is defined as

$$\dot{x} = J\dot{q}. \quad (5)$$

The task coordinates for walking in this paper were the position of the swinging foot, the system center of mass in the lateral (or horizontal) plane, the height of the hip, and the orientation of the hip, chest, and head. The dynamic equations of motion in the relevant task space coordinates, x , is

$$A(q)\ddot{x} + \mu(q, \dot{q}) + p(q) = F, \quad (6)$$

where

$$\begin{aligned} A(q) &= [JA^{-1}(I - P_c)J^T]^{-1}, \\ \bar{J}^T &= \Lambda JA^{-1}(I - P_c), \\ \mu(q, \dot{q}) &= \bar{J}^T b(q, \dot{q}) - \Lambda \dot{J} \dot{q} + \Lambda JA^{-1} J_c^T A_c \dot{J}_c \dot{q}, \\ p(q) &= \bar{J}^T g(q). \end{aligned} \quad (7)$$

The terms, $A(q)$, $\mu(q, \dot{q})$, and $p(q)$ are the task space inertia matrix, Coriolis/centrifugal force and gravity force, respectively.

The control force, F , in the task space can be composed using the task space dynamics for the desired acceleration or force. Given this task force, F , the relation between the task force and joint torque is used to compute the torque for the physical joint excluding the virtual joints.

$$F = \bar{J}^T \Gamma = \bar{J}^T (S^k)^T \Gamma^k, \quad (8)$$

where \bar{J} is the *dynamically consistent inverse* of the task Jacobian. The superscript, k , denotes the physical joints excluding the virtual joints. The term, Γ^k , represents the physical joint torque vector and S^k is the corresponding selection matrix.

The torque for the given task, which minimizes the acceleration energy of the system (Bruyninckx and Khatib, 2000), is

$$\Gamma_{\text{task}}^k = (J^k)^T F, \quad (9)$$

where

$$(J^k)^T = \bar{J}^T (S^k)^T. \quad (10)$$

The notation, $\bar{(\cdot)}$, represents the dynamically consistent inverse of the quantity (\cdot) .

3.3. Reaction forces and moments

Reaction forces and moments are generated as a result of the dynamics of the human body and the joint torques. The augmented torque for the constraint on the vertical moment of the stance foot is composed using the null-space projection matrix, $(N_{\text{task}}^k)^T$, such that the composite torque does not interfere with the task. The composite torque is

$$\Gamma^k = \Gamma_{\text{task}}^k + (N_{\text{task}}^k)^T \Gamma_{\text{aug}}^k, \quad (11)$$

where

$$(N_{\text{task}}^k)^T = I - (J^k)^T \bar{(J^k)^T}. \quad (12)$$

The first term, Γ_{task}^k , is the torque for task execution and the second term, $(N_{\text{task}}^k)^T \Gamma_{\text{aug}}^k$, is to impose the constraint on the vertical reaction moment of the stance foot. The simulation in the following section compares the results with the second term and without it.

The reaction moment about the vertical axis is computed using the selection matrix, S_c ,

$$\begin{aligned} m_{\text{vertical}} &= S_c f_c \\ &= S_c f_{c,\text{task}} + S_c \bar{J}_c^T (S^k)^T (N_{\text{task}}^k)^T \Gamma_{\text{aug}}^k, \end{aligned} \quad (13)$$

where

$$f_{c,\text{task}} = \bar{J}_c^T (q) (S^k)^T \Gamma_{\text{task}}^k - \mu_c(q, \dot{q}) - p_c(q). \quad (14)$$

In the simulation presented in the following section, the desired vertical moment, $m_{\text{vertical}}|_{\text{desired}}$, was set to zero for

the natural arm swing motion.

$$\begin{aligned} S_c \bar{J}_c^T (S^k)^T (N_{\text{task}}^k)^T \Gamma_{\text{aug}}^k &= m_{\text{vertical}}|_{\text{desired}} - S_c f_{c,\text{task}} \\ &= \tilde{m}_{\text{vertical}}. \end{aligned} \quad (15)$$

Then,

$$\Gamma_{\text{aug}}^k = (J_c^k)^T \tilde{m}_{\text{vertical}}, \quad (16)$$

where

$$(J_c^k)^T = \overline{S_c \bar{J}_c^T (S^k)^T (N_{\text{task}}^k)^T}. \quad (17)$$

4. Walking simulation

Experiments were conducted in the simulation environment called SAI (Khatib et al., 2002). The SAI simulation environment has been developed at the Stanford Artificial Intelligence Laboratory. Since the human body is modeled as a multi-body system in this paper, physics-based simulation for multi-body systems can be utilized in the

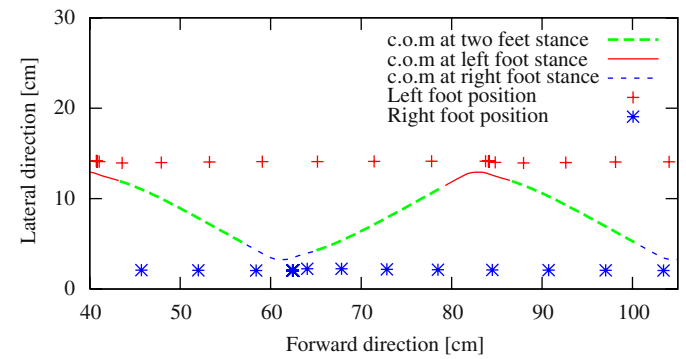


Fig. 4. The trajectories of the center of the mass, left foot, and right foot during *static* walking simulation are plotted. The left and right foot positions were sampled at regular time intervals and plotted with symbols of + and *. The orientation of hip, chest, and head were controlled to maintain upright posture during walking.

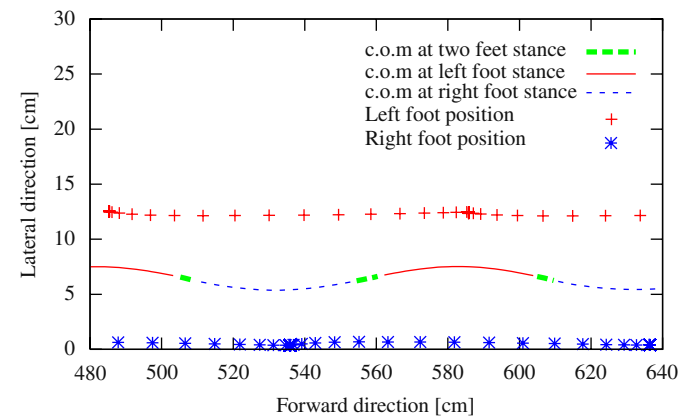


Fig. 5. The trajectories of the center of the mass, left foot, and right foot during *dynamic* walking simulation are plotted. The left and right foot positions were sampled at regular time intervals and plotted with symbols of + and *. The orientation of hip, chest, and head were controlled to maintain upright posture during walking.

verification of the hypothesis on the human control of the arm swing during walking. It computes the forward dynamics of articulated multi-body systems as well as executing collision detection/resolution algorithms for efficient and accurate simulation (Chang and Khatib,

2000; Ruspini and Khatib, 1999). Contact forces and impact forces are computed using the full multi-body dynamics. Friction between objects is modeled as a combination of static and viscous friction. Any multi-body system, built with joints and links, can be composed and

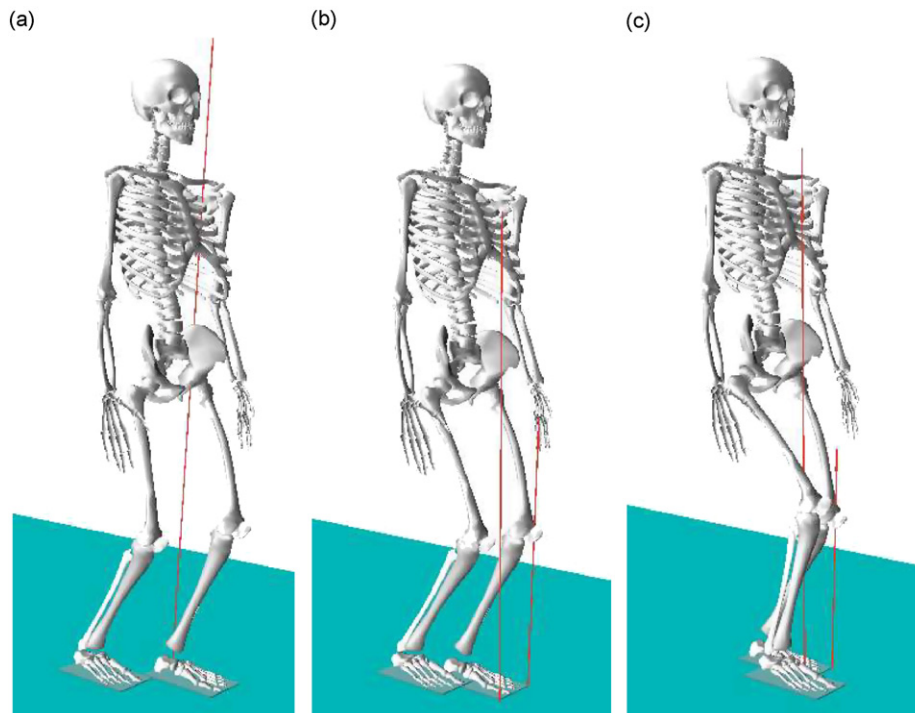


Fig. 6. The simulated result of walking without out-of-phase arm swing motion. The snapshots (a), (b), and (c) are the motion during walking on the left foot stance. There is little arm swing motion.

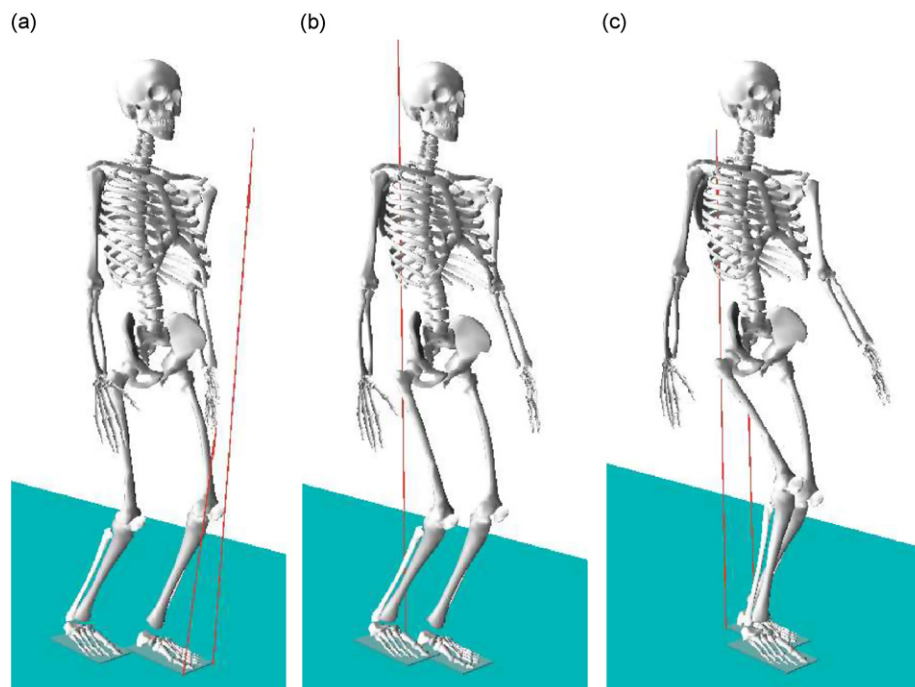


Fig. 7. The simulated result of walking with out-of-phase arm swing motion. The snapshots (a), (b), and (c) are the motion during walking on the left foot stance.

simulated. Commanding torques to each joint, impact forces, and contact forces are all included in the forward dynamic integration.

Walking motion was realized by composing tasks and controlling these tasks to track desired trajectories. The tasks were defined as the position of the swinging foot, the system center of mass in the lateral (or horizontal) plane, the height of the hip, and the orientation of the hip, chest, and head. Note that the motion of the arms was not included as part of the task.

Two types of walking were simulated: static (always stable) and dynamic (only stable while moving). For the slower, static walking case, feet placement was planned to move forward with fixed step size. Then, the COM trajectory was designed to move from one foot to the other during double supporting stance. Feet placement was planned in the same way for the faster, dynamic walking,

while the COM trajectory has been designed to move straight under the constraints of the Zero Moment Point (ZMP) condition (Vukobratovic and Borovac, 2004). The ZMP condition is a set of mathematical constraints which prevent the system from tipping over, and it is a common technique for dynamic walking in the field of humanoid robotics. The desired trajectories for the orientation of the hip, chest, and head were defined to maintain the starting configuration (an upright posture).

The period for static walking was chosen to be 2 s for half of a gait cycle, and the ratio between the duration of the two foot support phase and the one foot support phase was designed to be 2:3. For dynamic walking, the half gait period was 0.5 s and the support phase ratio was 1:9. The trajectories of the feet and center of the mass of the system are shown in Figs. 4 (static walk) and 5 (dynamic walk).

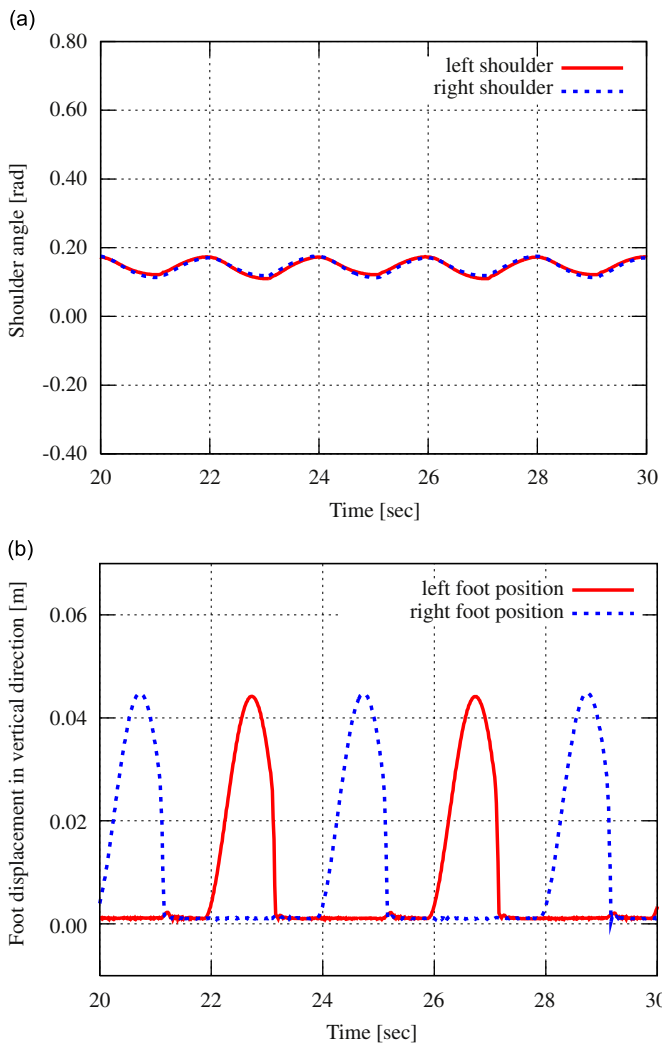


Fig. 8. The shoulder motion (a) and trajectory of feet (b) are plotted during *static* walking simulation allowing the foot moment to be used for desired motion. The arm swing motion is small and both arms are in phase. This shoulder motion is the result of control for the orientation error of the body in the sagittal plane due to the impact forces during the transitions.

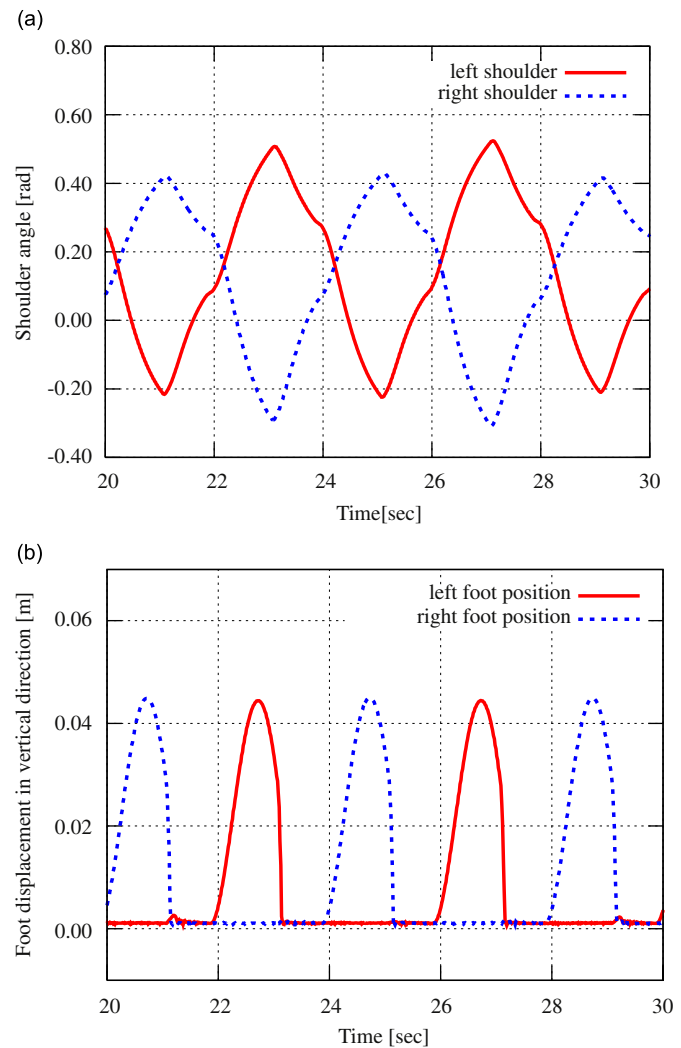


Fig. 9. The shoulder motion (a) and trajectory of feet (b) are plotted during *static* walking simulation constraining the foot moment to be zero. That is, the arms are moving out of phase to generate the necessary moment about the vertical axis to reduce the foot moment to be zero.

5. Generation of arm swing motion

As explained in Section 2, out-of-phase arm motion would be generated if a constraint were imposed on the reaction moment about the vertical axis of the contact foot. Using the control framework in Section 3, the joint torques of the human can be commanded to generate walking motion while the reaction moment about the vertical axis of the stance foot is zero during single stance. Two experiments were conducted: one without any limit on the reaction moment about the vertical axis of the foot, and the other with a limit specifying the reaction moment to be zero. The snapshots of these two conditions are shown in Figs. 6 and 7. In both cases the same trajectory tracking performance for the specified tasks was demonstrated. That is, the walking patterns of the feet, body orientation, and center of mass position were the same. Out-of-phase

arm swing motion was generated in the second case due to the constraint on stance foot reaction moment.

For the static walking case, shoulder flexion angle is plotted for the two experiments in Figs. 8 and 9. In Fig. 8, small in-phase swinging of the two arms is observed. Due to impact forces associated with the foot striking the ground body orientation in the sagittal plane is disturbed. In-phase arm swing arises in order to correct the body orientation. This motion is in phase because each arm's motion contributes identically to the moment in the sagittal plane.

Continuing with static walking case, Fig. 9 shows the results associated with limiting the reaction moment to zero. The plot of shoulder angle clearly shows the natural out-of-phase motion that can be observed from normal human walking. This arm motion is the result of constraining the reaction moment of the foot in addition to the motion that was created due to impact forces at the foot (Fig. 8).

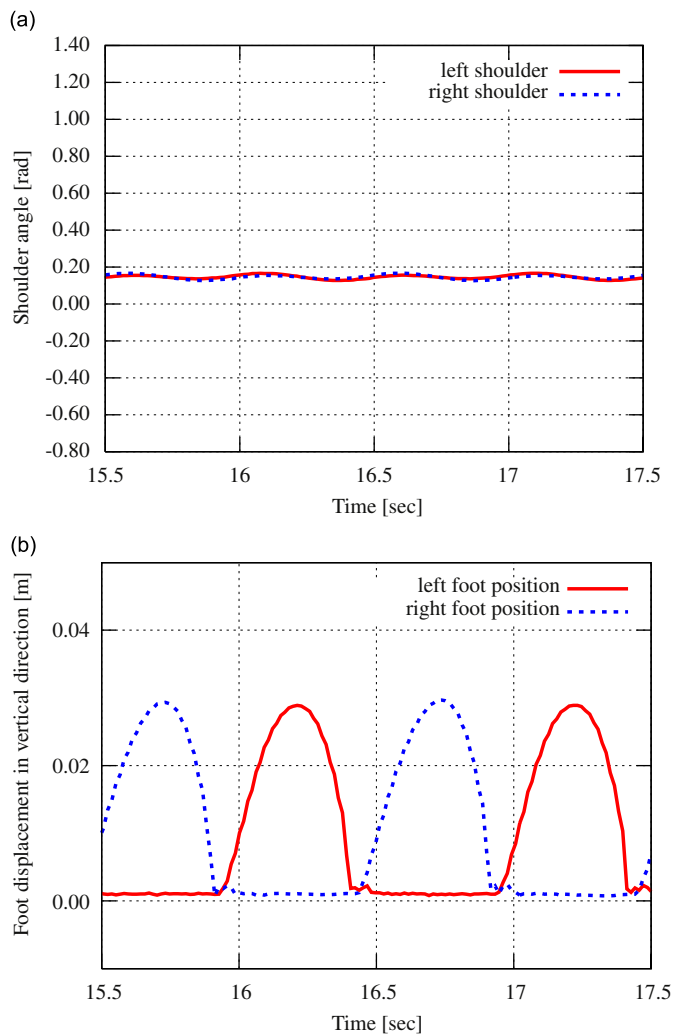


Fig. 10. The shoulder motion (a) and trajectory of feet (b) are plotted during *dynamic* walking simulation allowing the foot moment to be used for desired motion. The arm swing motion is small and both arms are in phase. This shoulder motion is the result of control for the orientation error of the body in the sagittal plane due to the impact forces during the transitions.

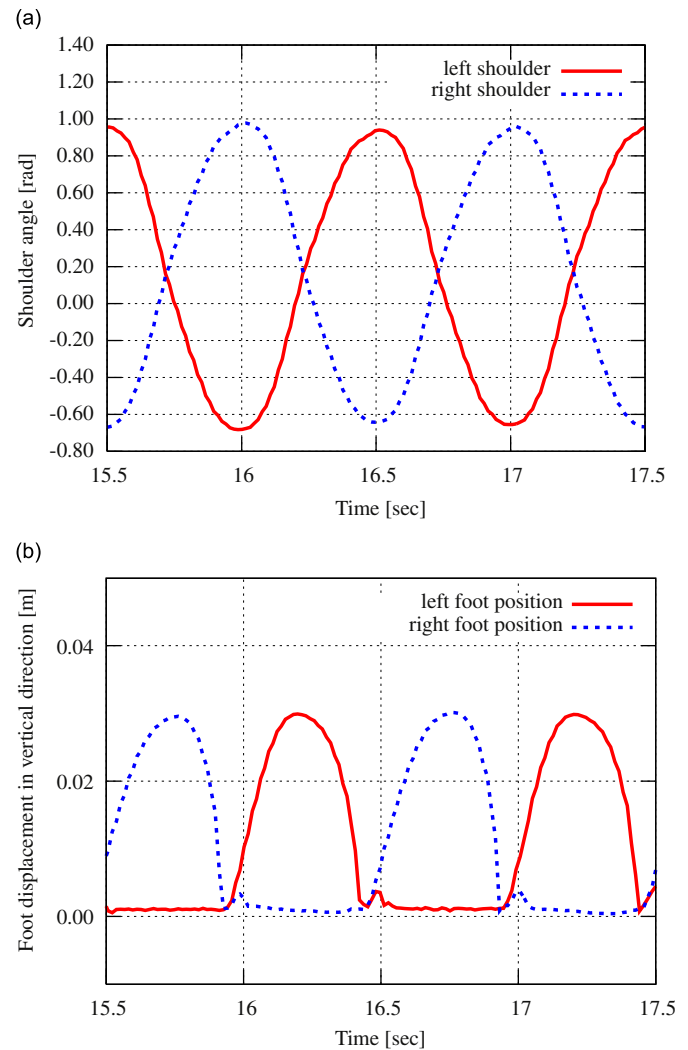


Fig. 11. The shoulder motion (a) and trajectory of feet (b) are plotted during *dynamic* walking simulation constraining the foot moment to be zero. That is, the arms are moving out of phase to generate the necessary moment about the vertical axis to reduce the foot moment to be zero.

The same experiments were conducted for the dynamic walking pattern discussed earlier (Section 4). The results, shown in Figs. 10 and 11, are clearly consistent with the static walking case.

6. Conclusion

In this paper the relation between out-of-phase arm swing motion and reaction moment about the vertical axis of the foot in bipedal walking is explained and verified through physics-based simulation. This out-of-phase arm swing can be generated by imposing a constraint on the reaction moment about the vertical axis of the foot. Humans can walk without arm swing but this requires greater effort of the legs due to greater reaction moment of the foot. It is believed that human walking with arm swing optimizes the motion of the lower limb by minimizing torque loading on the joints and skeletal structure.

The contact consistent whole-body control framework was employed to realize the human walking motion with or without out-of-phase arm swing motion, both for static and dynamic walking patterns. The simulated results in this paper are consistent with the experimental results of adult male subjects in Li et al. (2001). This result provides a verification of the relationship between arm swing and reaction moment and the associated control approach for synthesizing out-of-phase arm swing motion. It supports the belief that out-of-phase arm swing is an active process (albeit subconscious).

Using the simulation environment presented in this paper, many different gait patterns can be generated and simulated for further analysis. Future research will look into the variations of arm swing between these different gait patterns, as well as patterns from actual human walking data. In addition, different shape profiles of the arm swing can be investigated through systematic choice of how much, and when, the reaction moment is limited.

Dynamic simulation with appropriate control design techniques offers great potential in the investigation of the biomechanics of human motion. The control framework demonstrated here represents a useful application in this direction. Ongoing work is being conducted on merging neuromusculoskeletal models into a dynamic simulation environment (De Sapio et al., 2005). This effort will hopefully result in even further detailed investigation of human motion and control.

Conflict of interest statement

There are no conflicts of interest.

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