Thin Junction Tree Filters for Simultaneous Localization and Mapping

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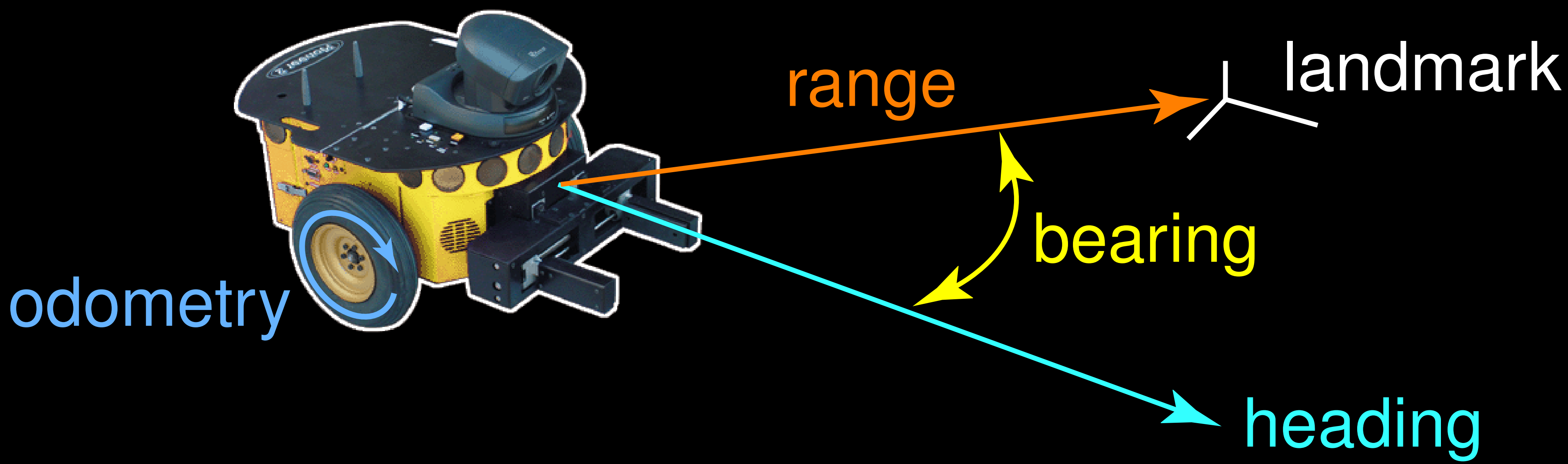
Simultaneous Localization and Mapping

A mobile robot navigating in an unknown environment must incrementally

1. build a map of its surroundings and
2. localize itself within that map.



The traditional approach: Kalman filters

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- The belief state is a Gaussian $\mathcal{N}(\mu_t, \Sigma_t)$
- Time & space complexity: $\Theta(N_t^2)$

Thin junction tree filters

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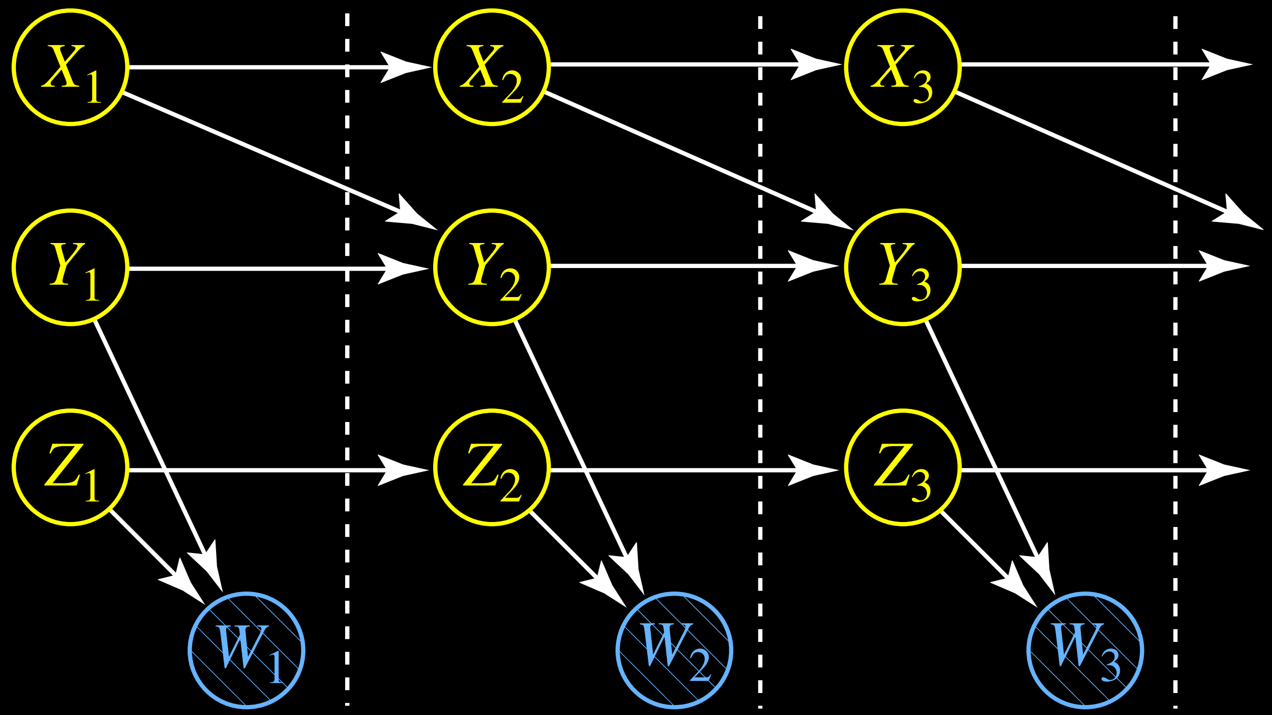
  space complexity: $O(N_t)$

  time complexity: $O(N_t)$ or $O(1)$

Filtering in dynamic Bayesian networks

A dynamic Bayesian network (DBN) is a compact representation of a complex stochastic process.



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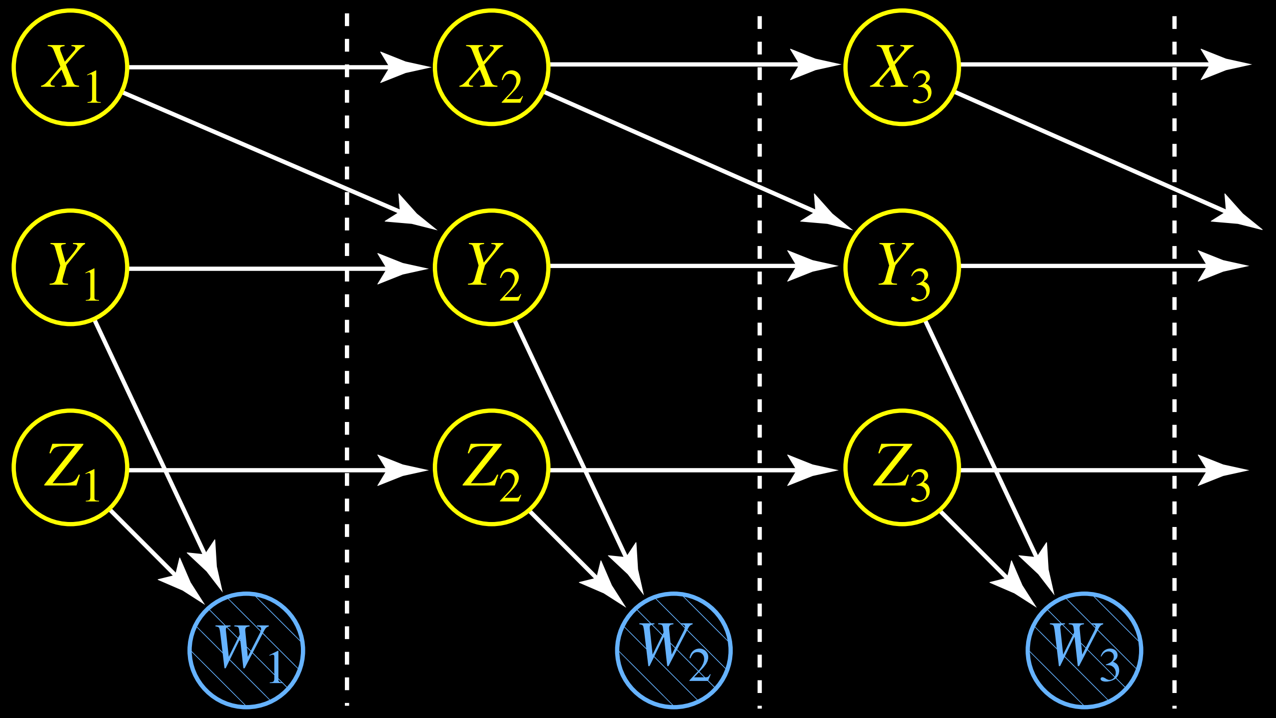

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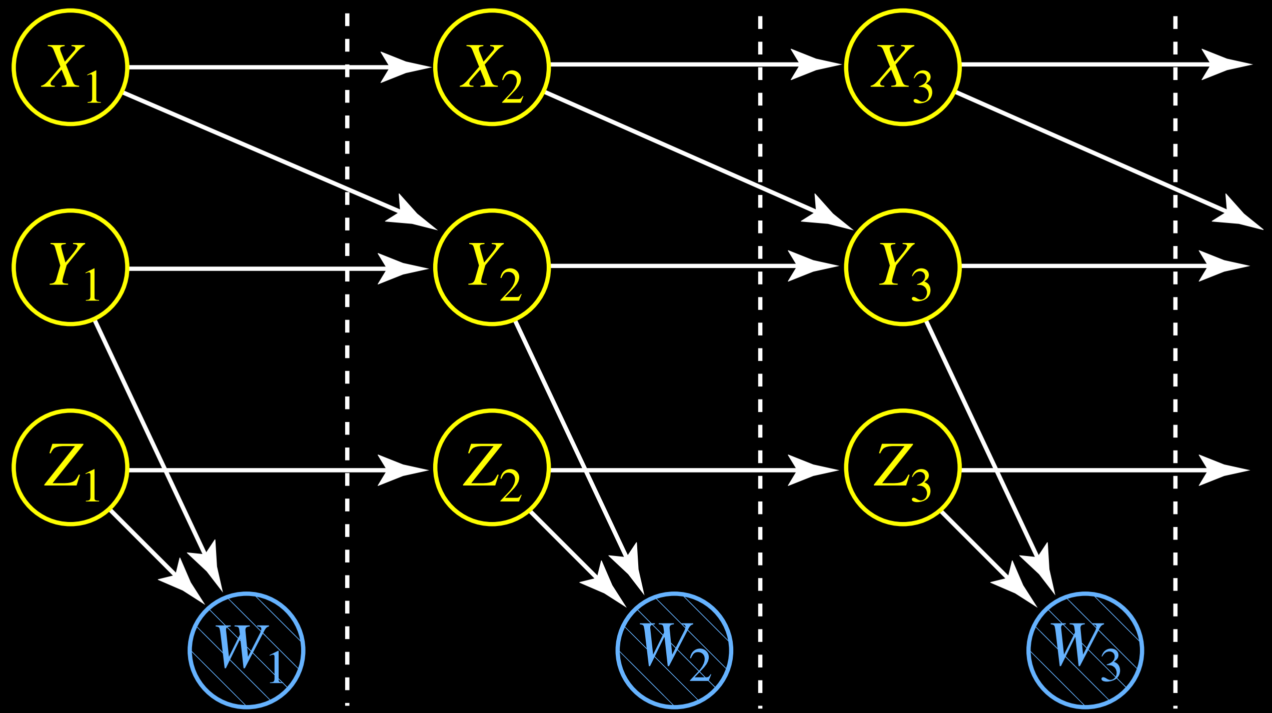

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Filtering: iteratively update $b_t \xrightarrow{\overline{w}_t} b_{t+1}$

Complexity of filtering in DBNs

The DBN is compact, but the belief state is not:

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Exact filtering in DBNs is intractable.

The Boyen & Koller (1998) Algorithm

Choose a fixed, tractable form for the belief state and project to the closest density of that form:

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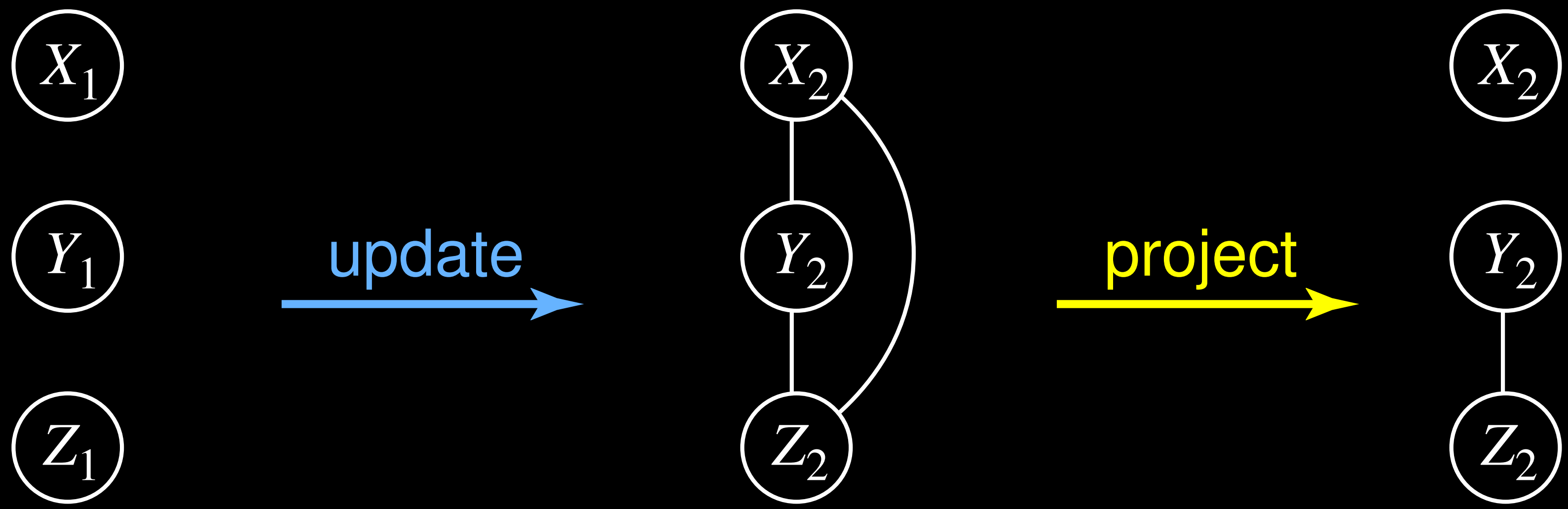

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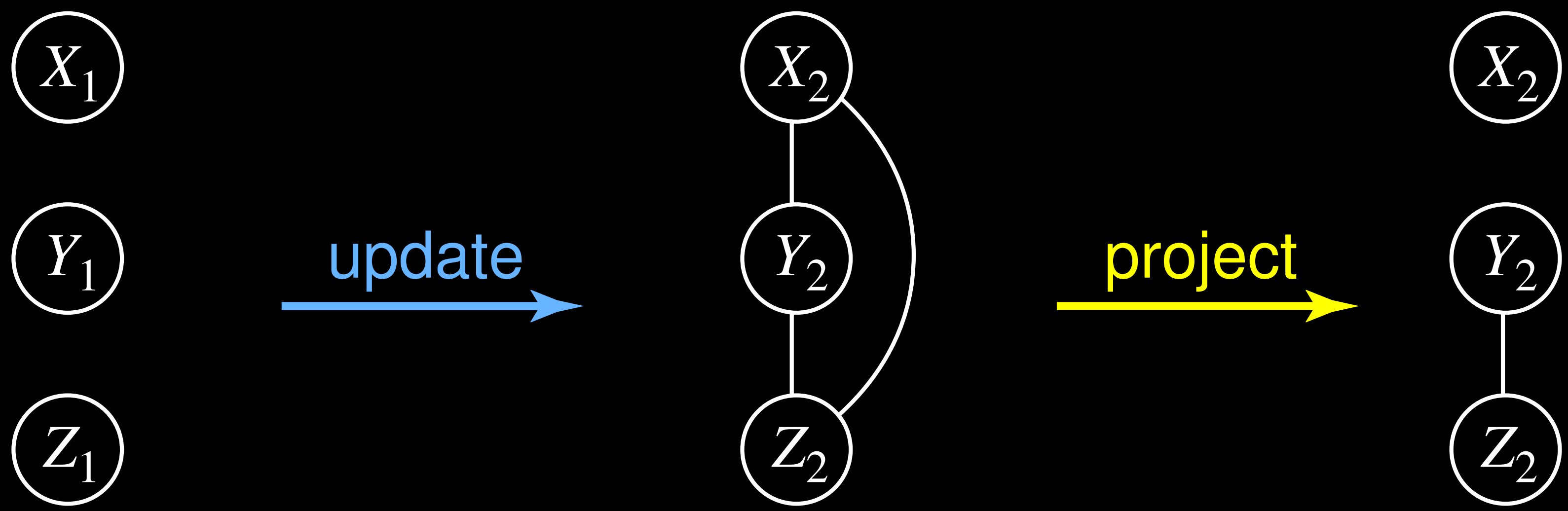

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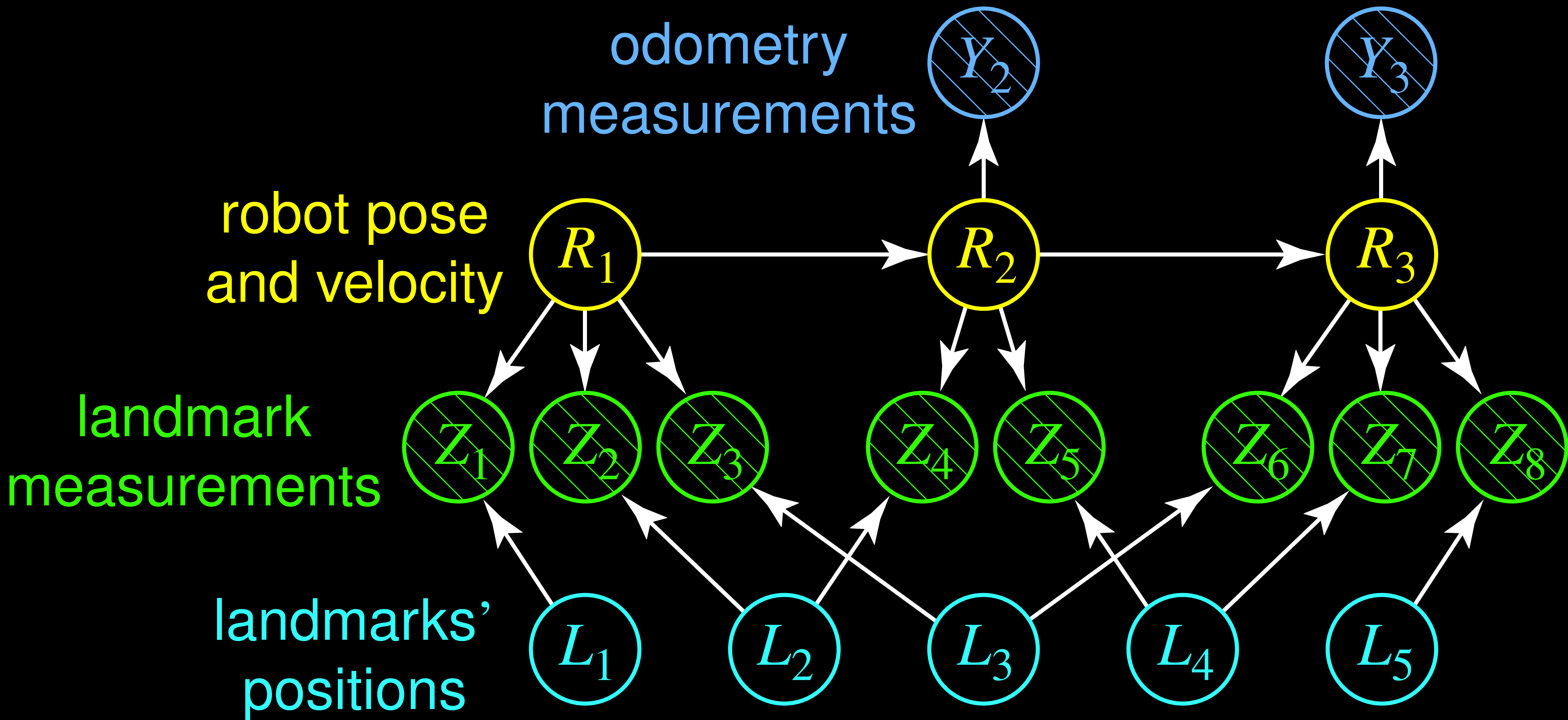
Problem: what is the best tractable form?

An example SLAM DBN



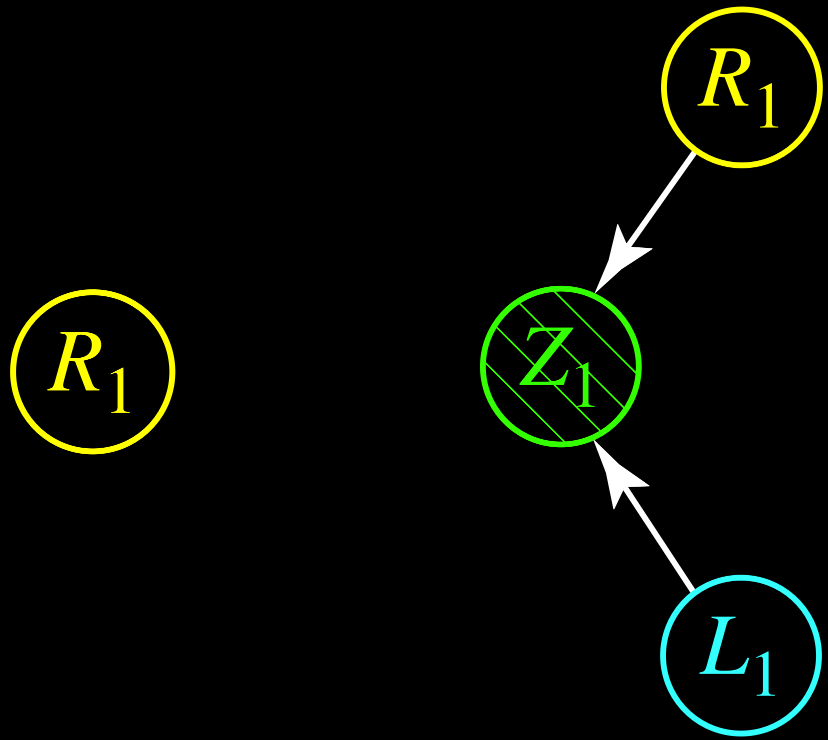
Filtering the SLAM DBN: estimation

$$p(r_1)$$

R_1

Filtering the SLAM DBN: estimation

$$p(r_1, l_1 \mid \overline{z}_1) \propto p(r_1) \cdot p(l_1) \cdot p(\overline{z}_1 \mid r_1, l_1)$$



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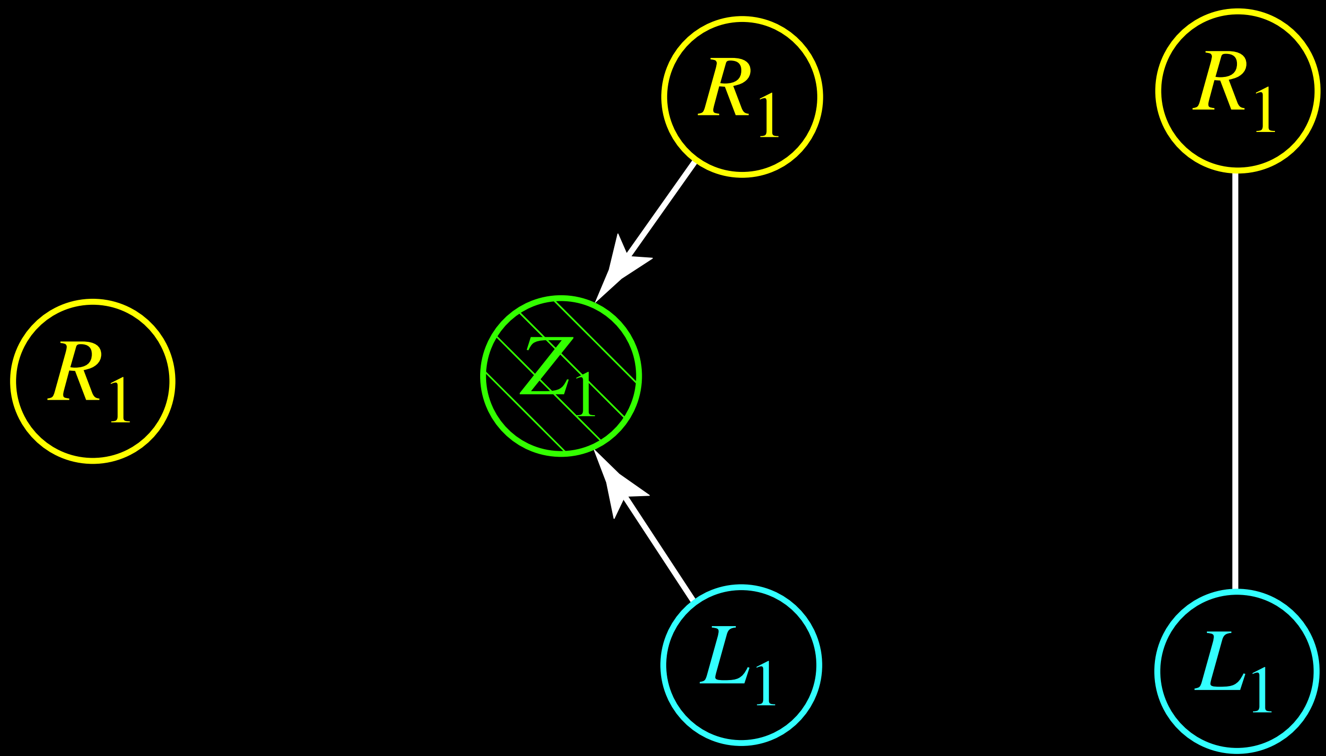

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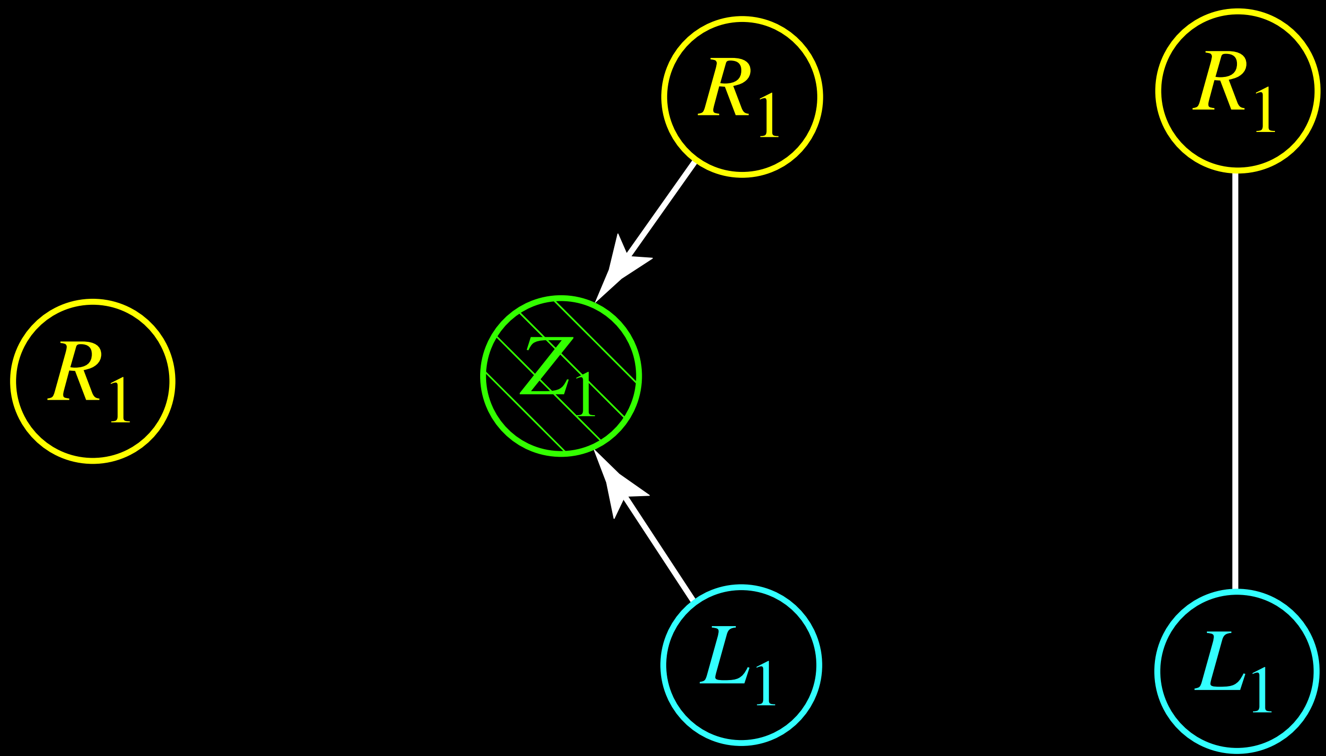
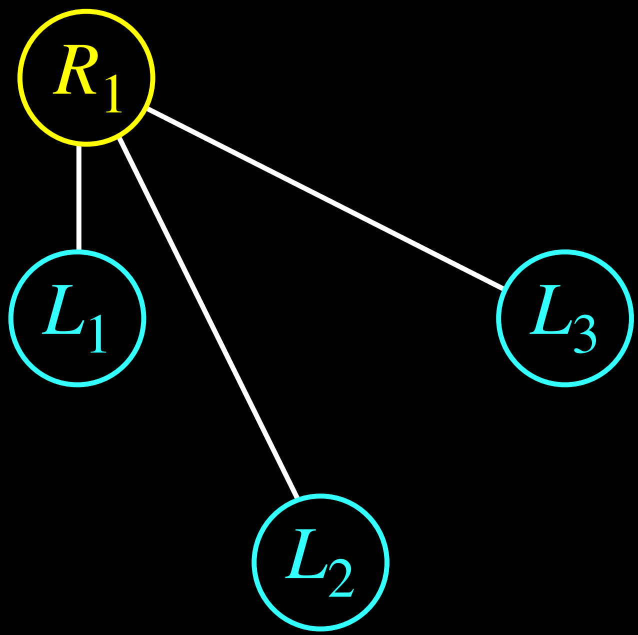


Observing landmark i connects R_t and L_i.

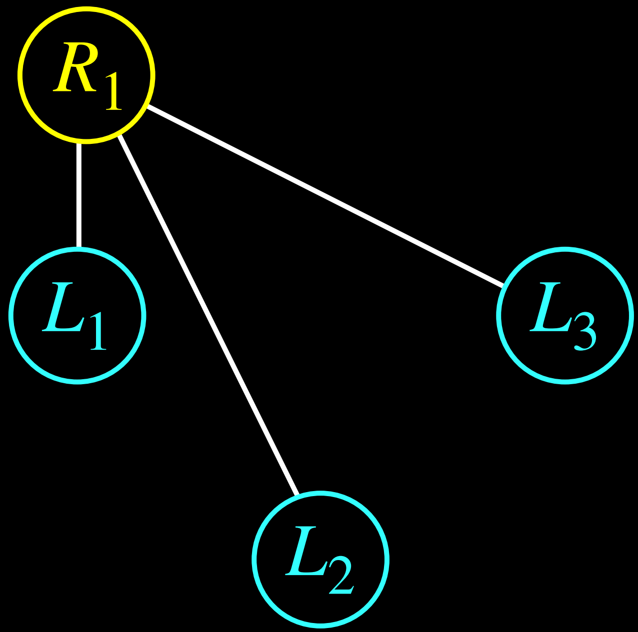
Filtering the SLAM DBN: prediction

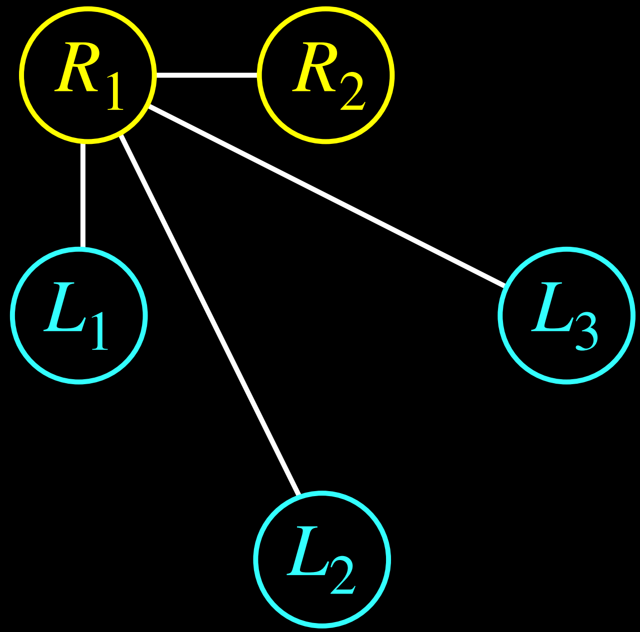
$$p(r_1, l_{1:3} \mid \overline{z}_{1:3})$$



Filtering the SLAM DBN: prediction

$$p(r_{1:2}, l_{1:3} \mid \overline{z}_{1:3}) = p(r_1, l_{1:3} \mid \overline{z}_{1:3}) \cdot p(r_2 \mid r_1)$$





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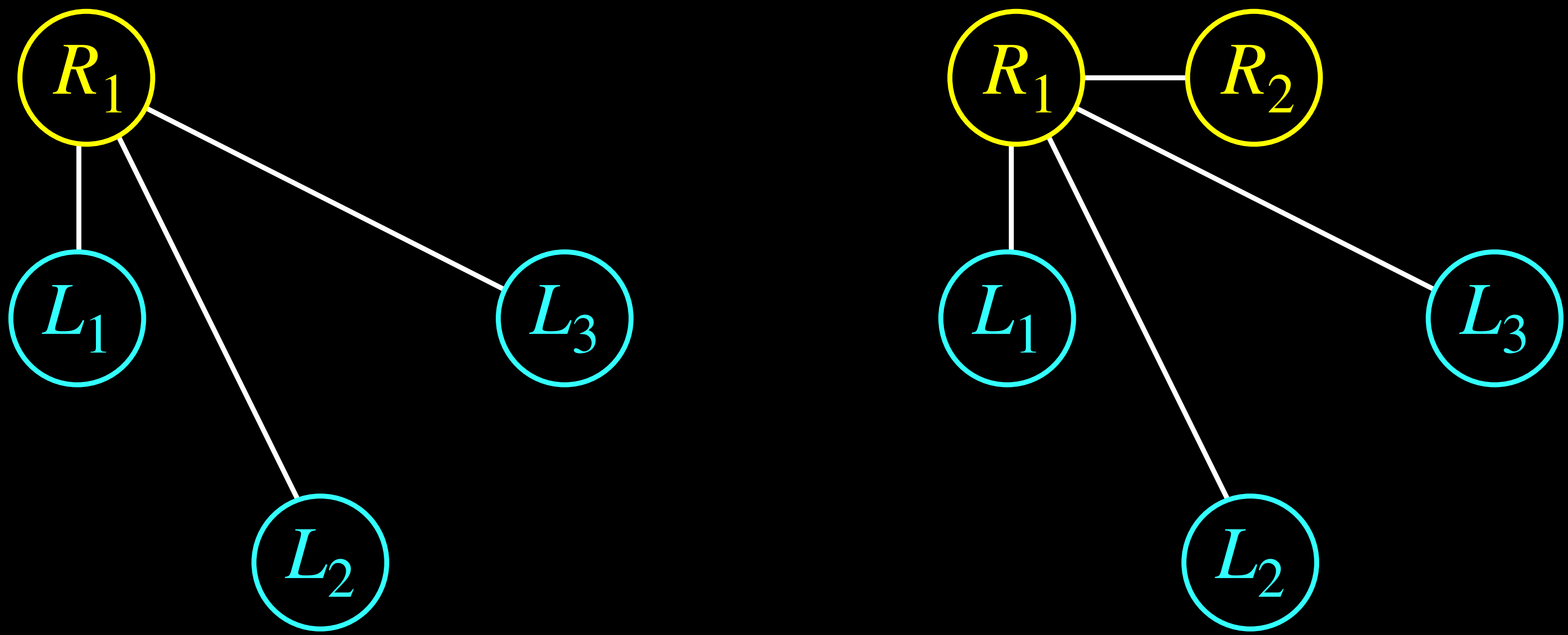
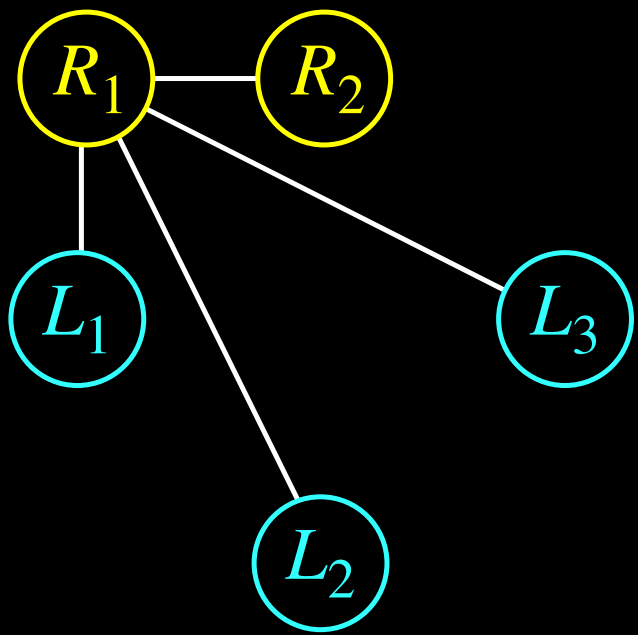


The prediction phase connects R_t and R_{t+1}.

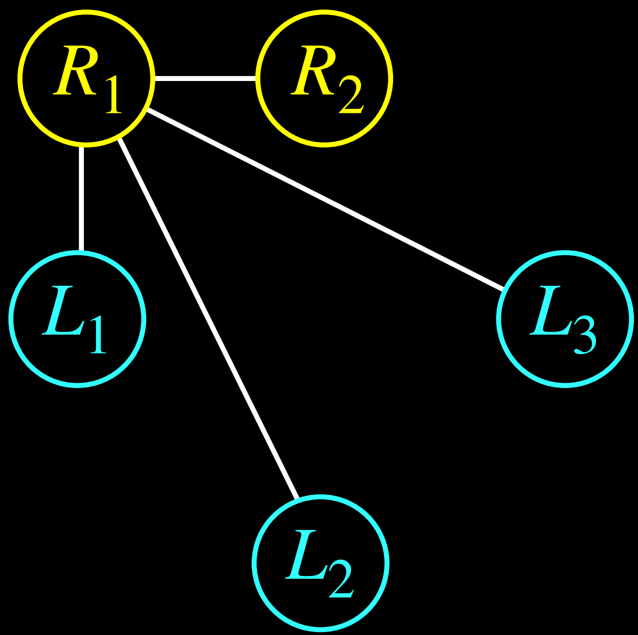
Filtering the SLAM DBN: roll-up

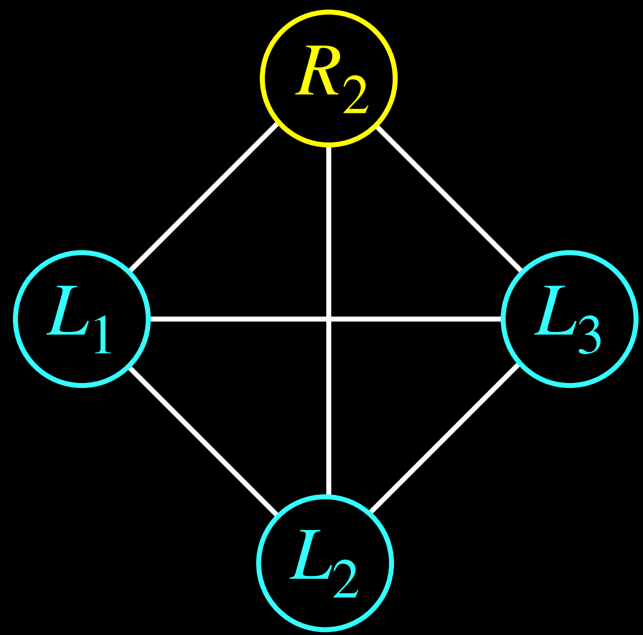
$$p(r_{1:2}, l_{1:3} \mid \overline{z}_{1:3})$$



Filtering the SLAM DBN: roll-up

$$p(r_2, l_{1:3} \,|\, \overline{z}_{1:3}) = \sum_{r_1} p(r_{1:2}, l_{1:3} \,|\, \overline{z}_{1:3})$$





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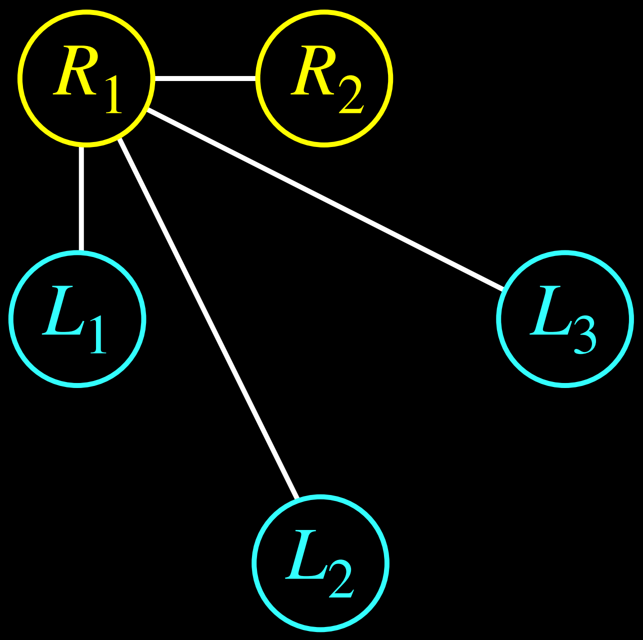
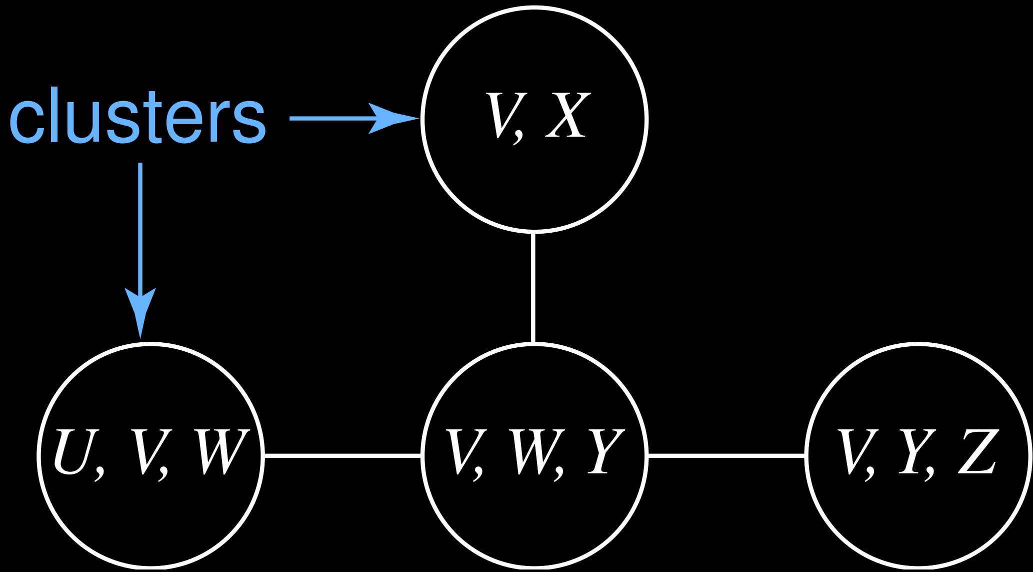


After roll-up, the SLAM belief state has no conditional independencies.

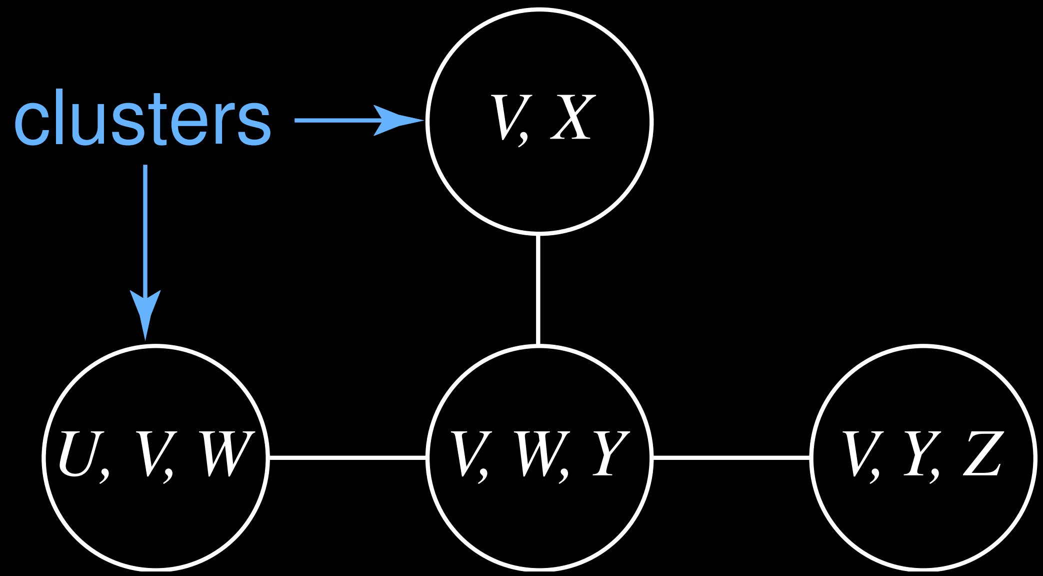
Junction trees

An undirected tree whose nodes are sets of variables. . .



Junction trees

An undirected tree whose nodes are sets of variables. . . with the running intersection property.



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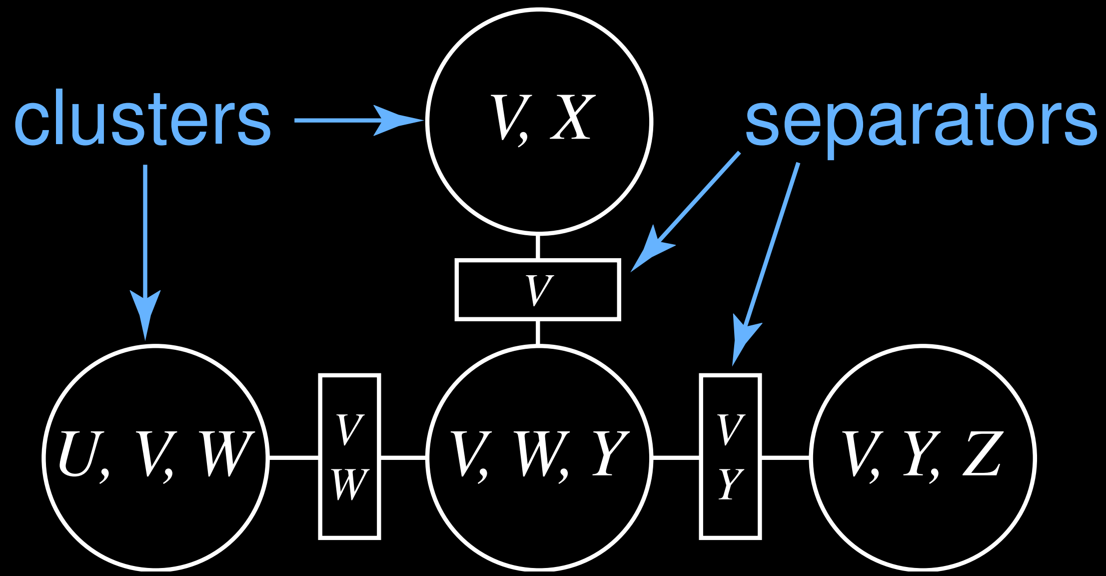

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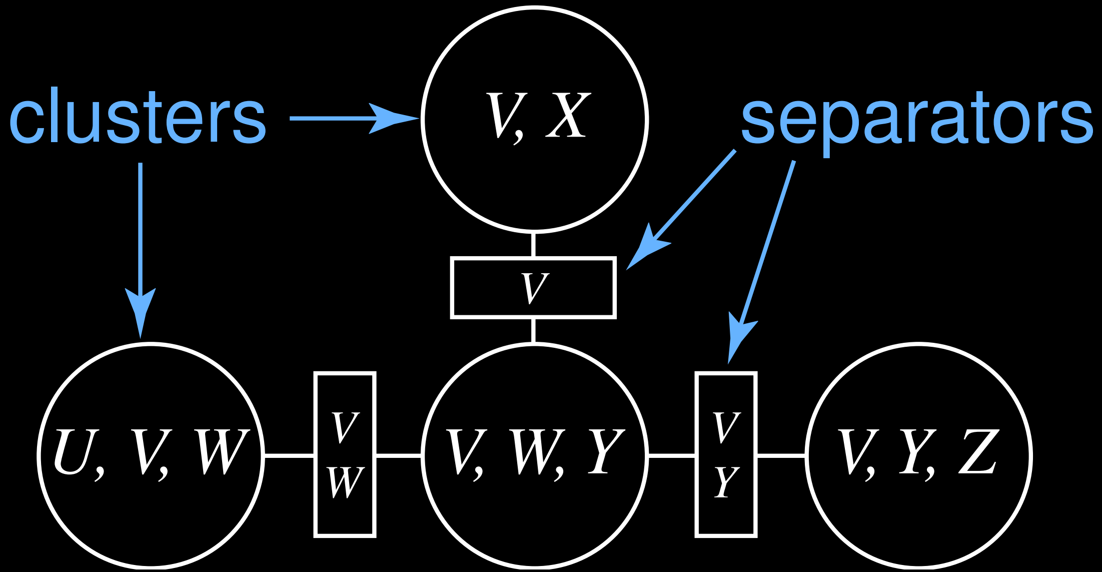
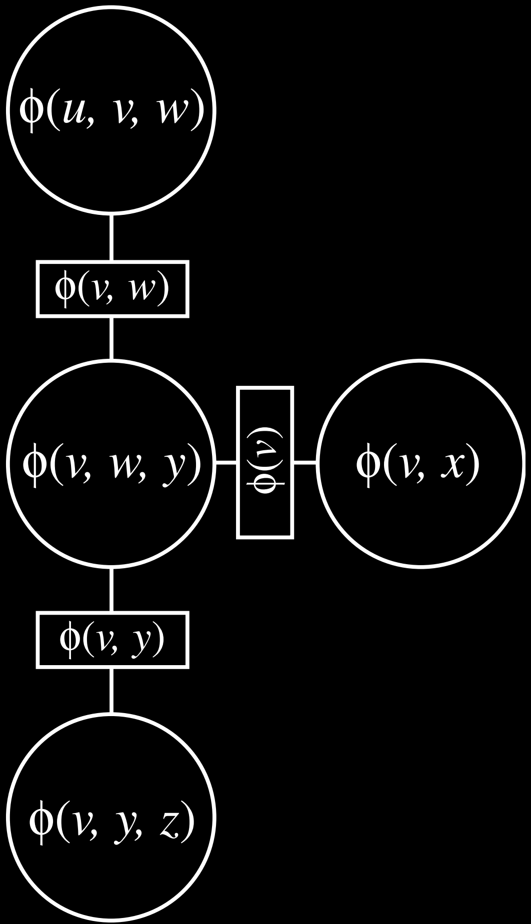


A density p decomposes on T if we can write

$$p(x) = \frac{\prod_C \phi_C(x_C)}{\prod_S \phi_S(x_S)}$$

Junction tree inference

initialize



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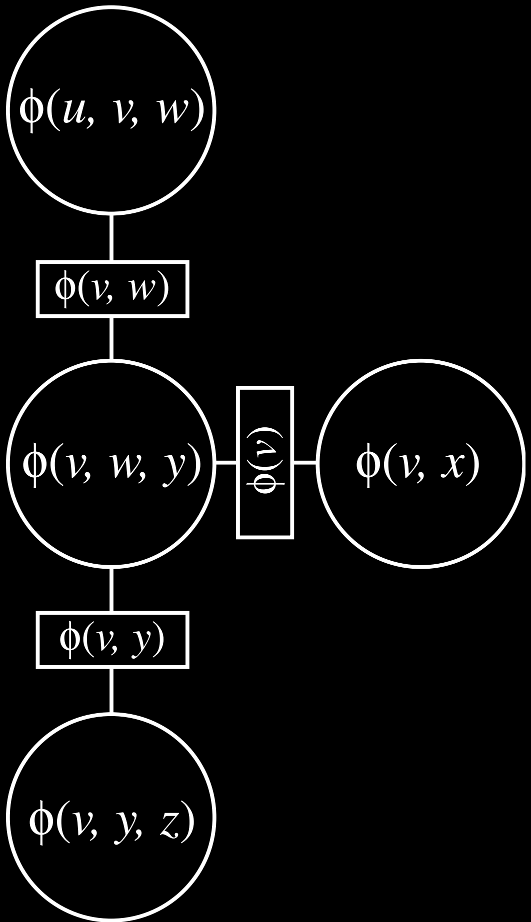

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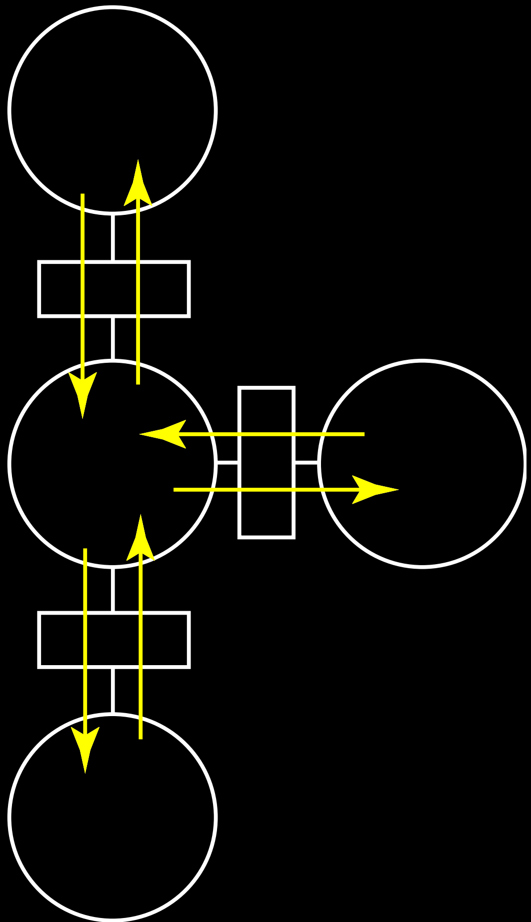
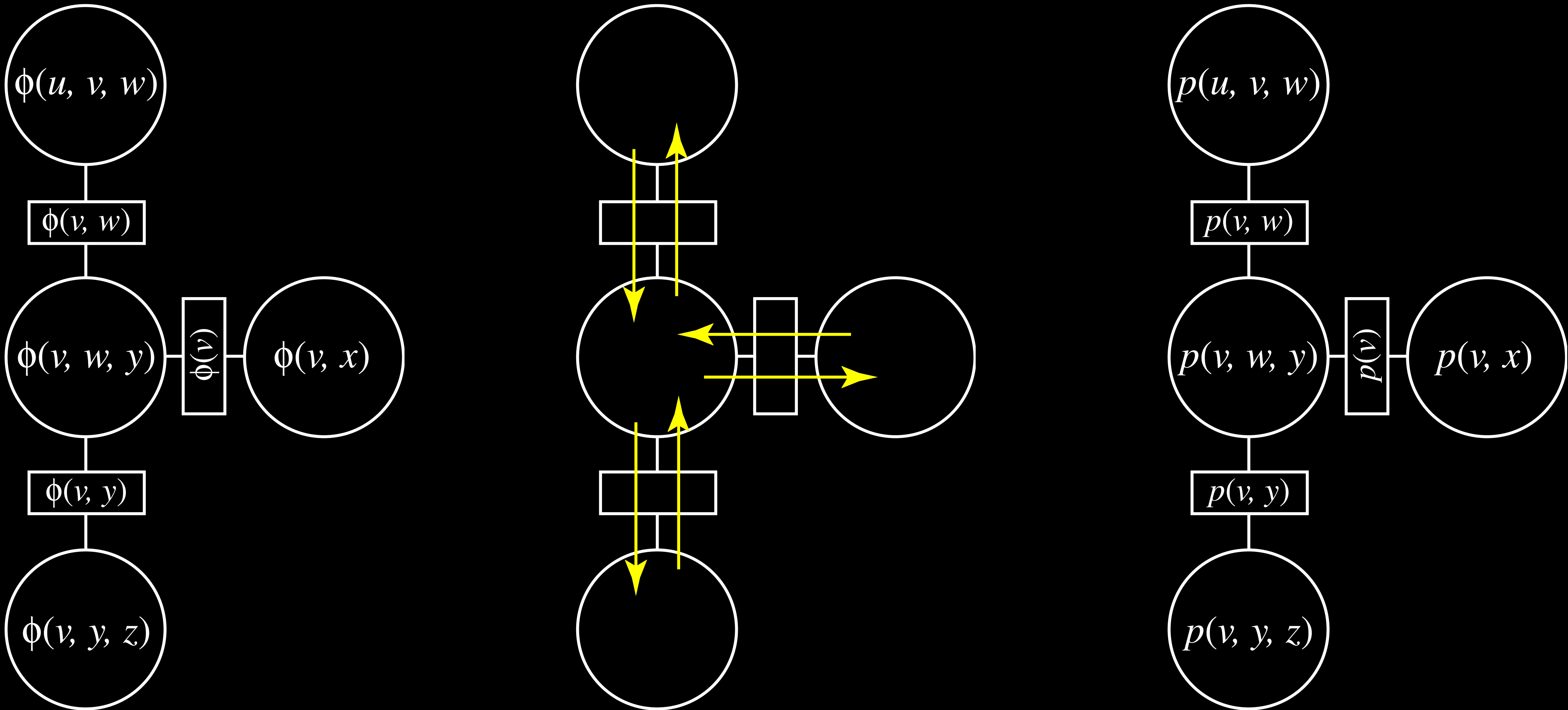
pass messages



Junction tree inference

initialize | pass messages | calibrated



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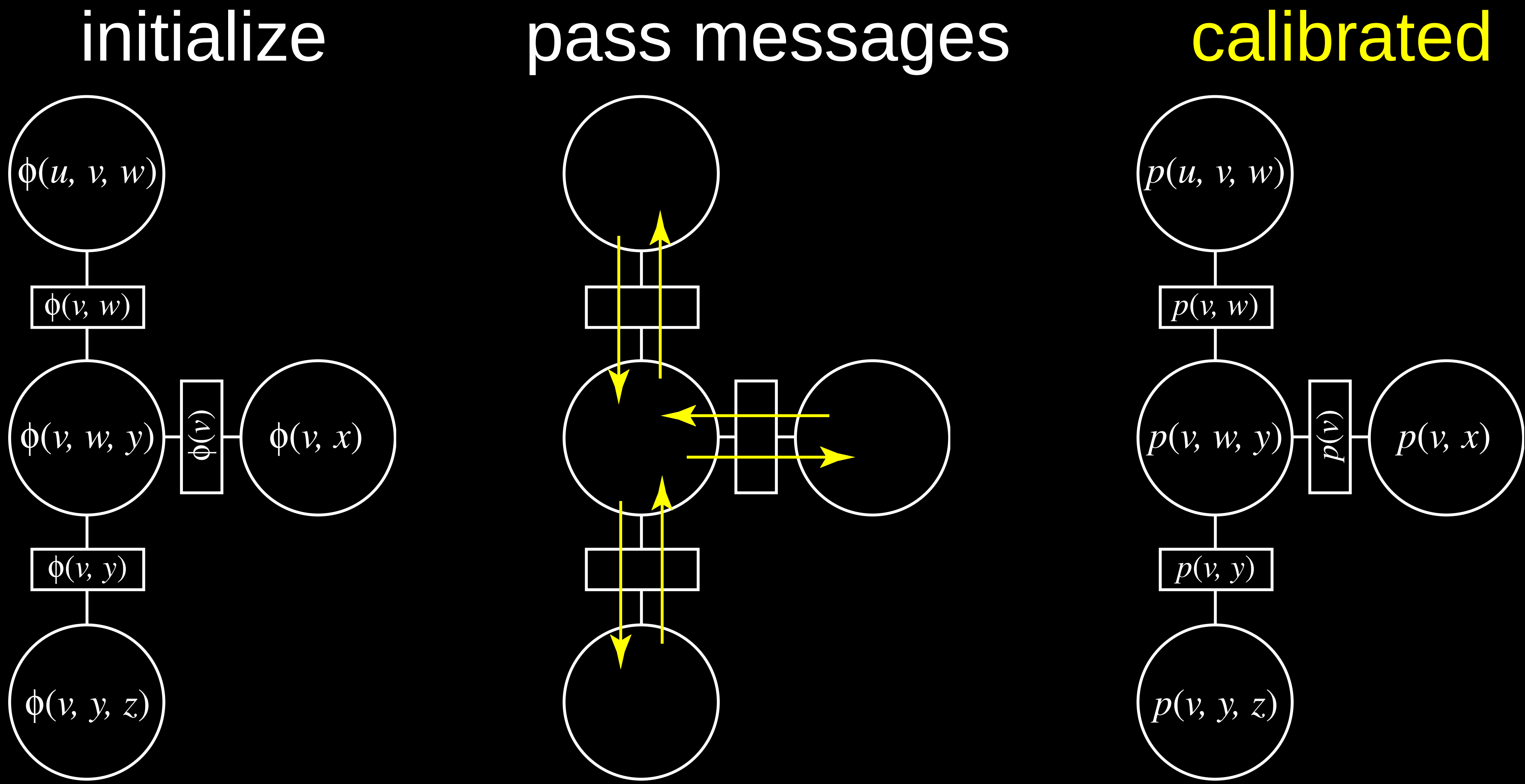
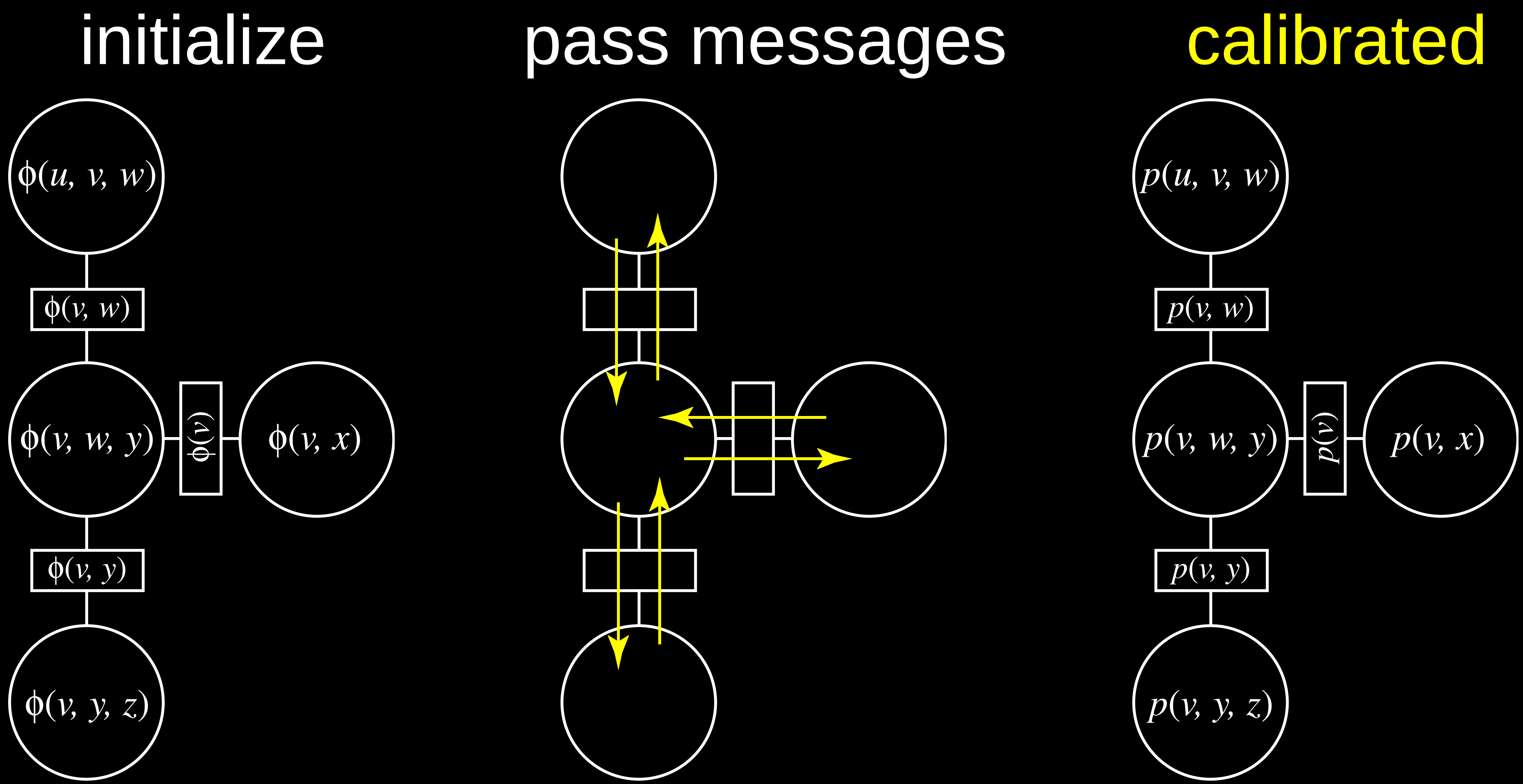


$$p = \frac{\prod_C \phi_C}{\prod_S \phi_S}$$

$$p = \frac{\prod_C p_C}{\prod_S p_S}$$

Junction tree inference



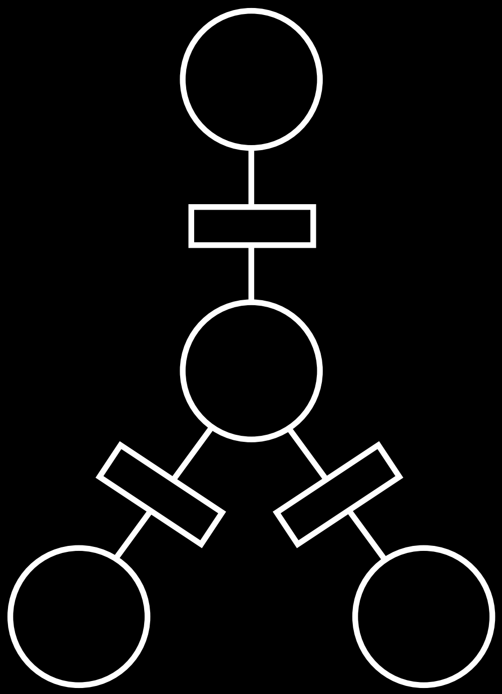
$$p = \frac{\prod_C \phi_C}{\prod_S \phi_S}$$

complexity scales with width

$$p = \frac{\prod_C p_C}{\prod_S p_S}$$

Junction tree filters

The belief state is a calibrated junction tree.



$$p^{(t)} = \frac{\prod_C p_C^{(t)}}{\prod_S p_S^{(t)}}$$

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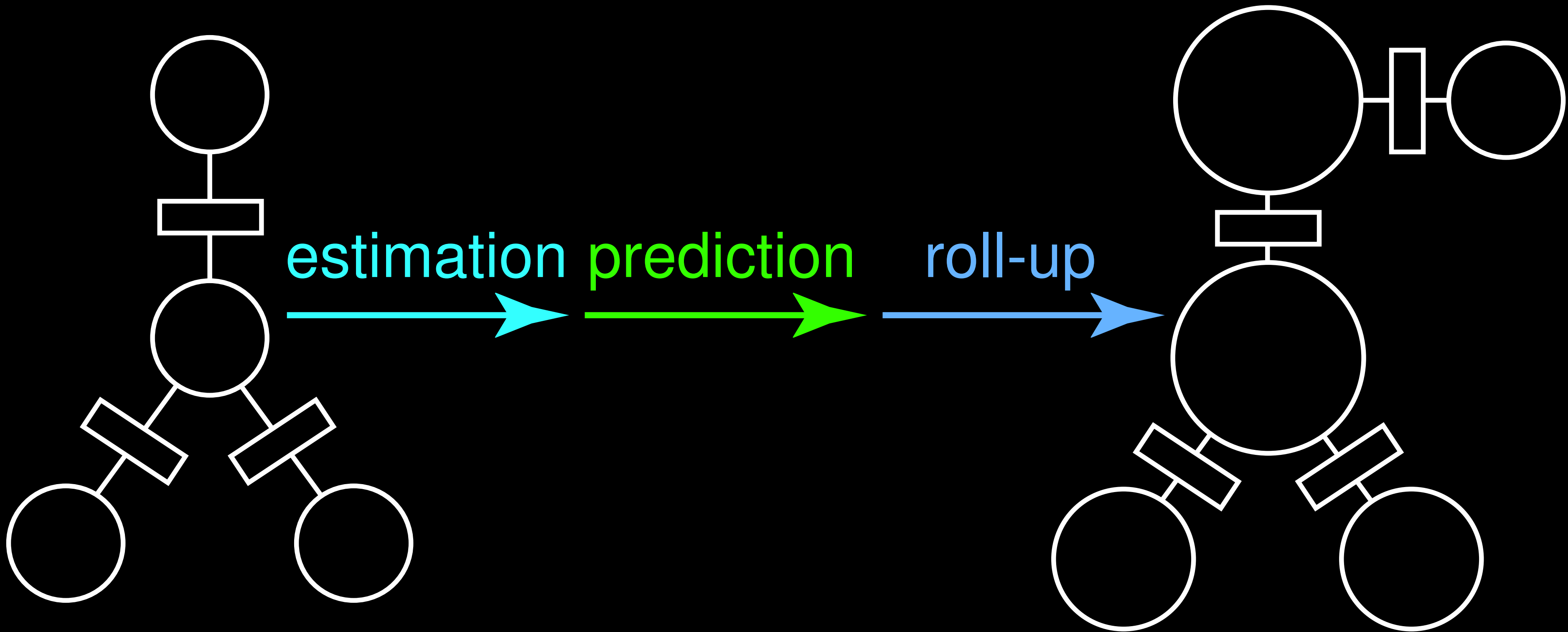
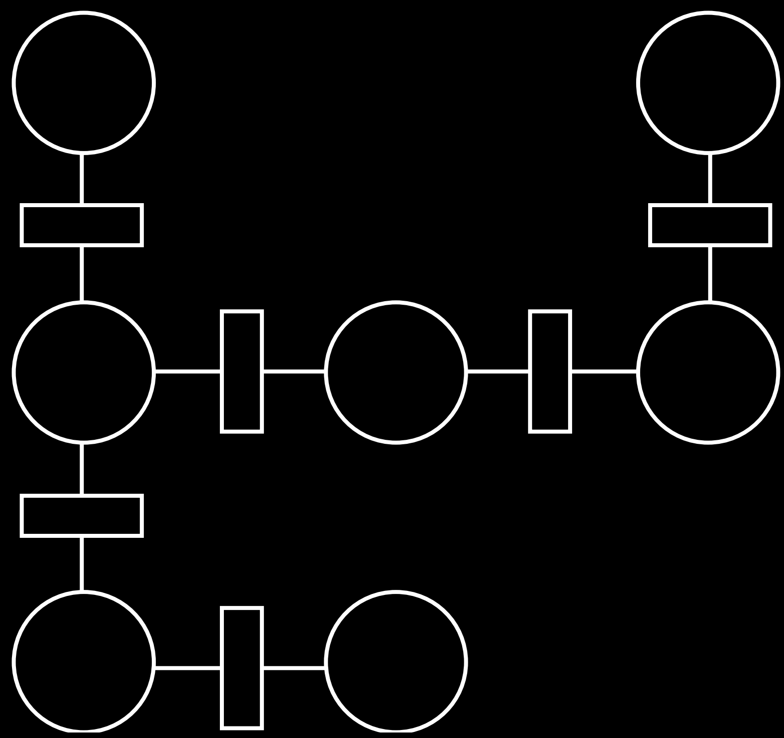


$$p^{(t)} = \frac{\prod_C p_C^{(t)}}{\prod_S p_S^{(t)}} \qquad p^{(t+1)} = \frac{\prod_C p_C^{(t+1)}}{\prod_S p_S^{(t+1)}}$$

Estimation and prediction

To multiply $\psi(x_1, \ldots, x_k)$ into p and recalibrate:



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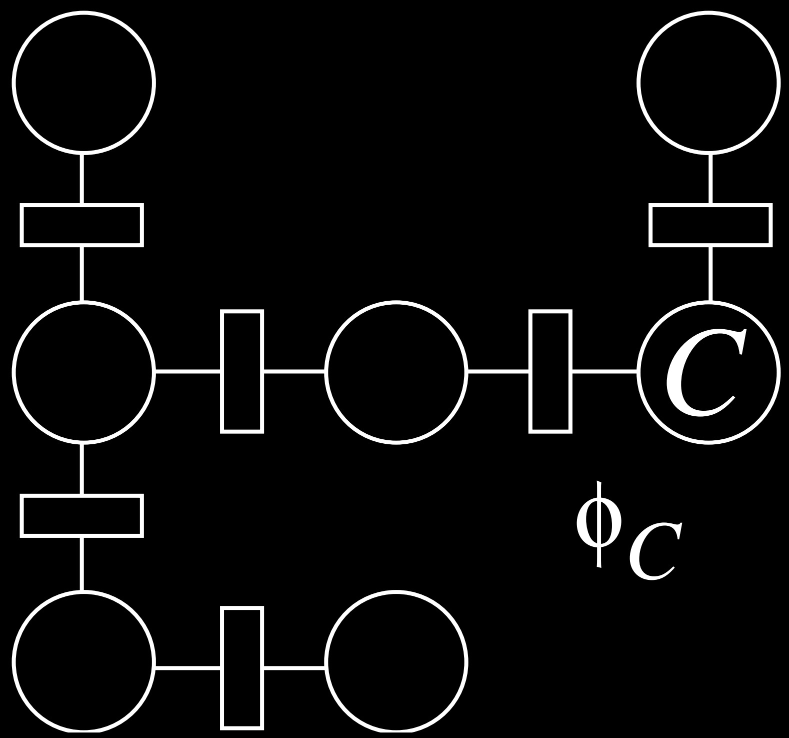

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To multiply $\psi(x_1, \ldots, x_k)$ into p and recalibrate:

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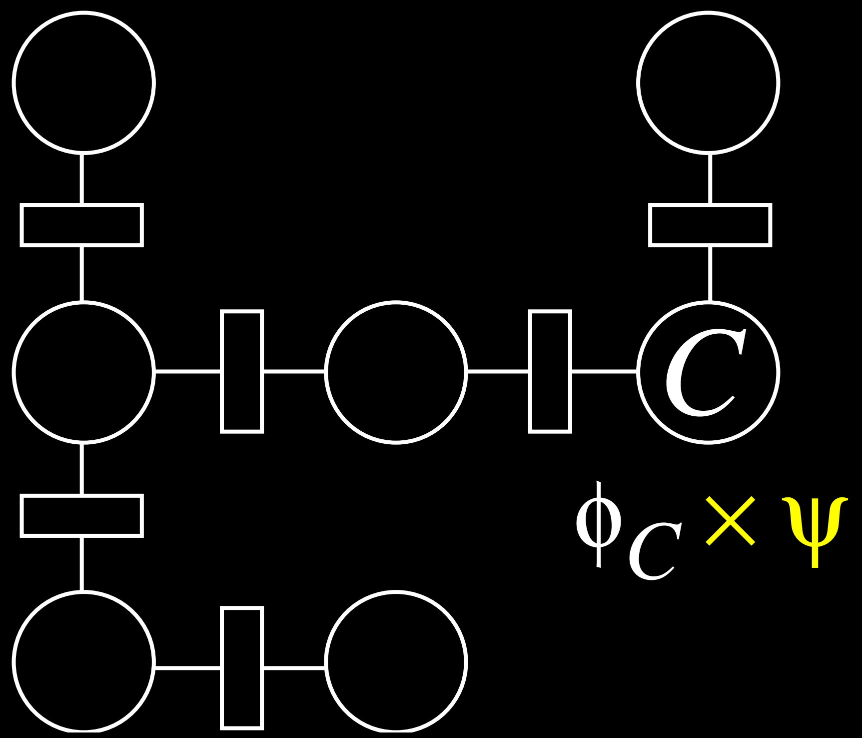
2. Multiply ψ into ϕ_C.



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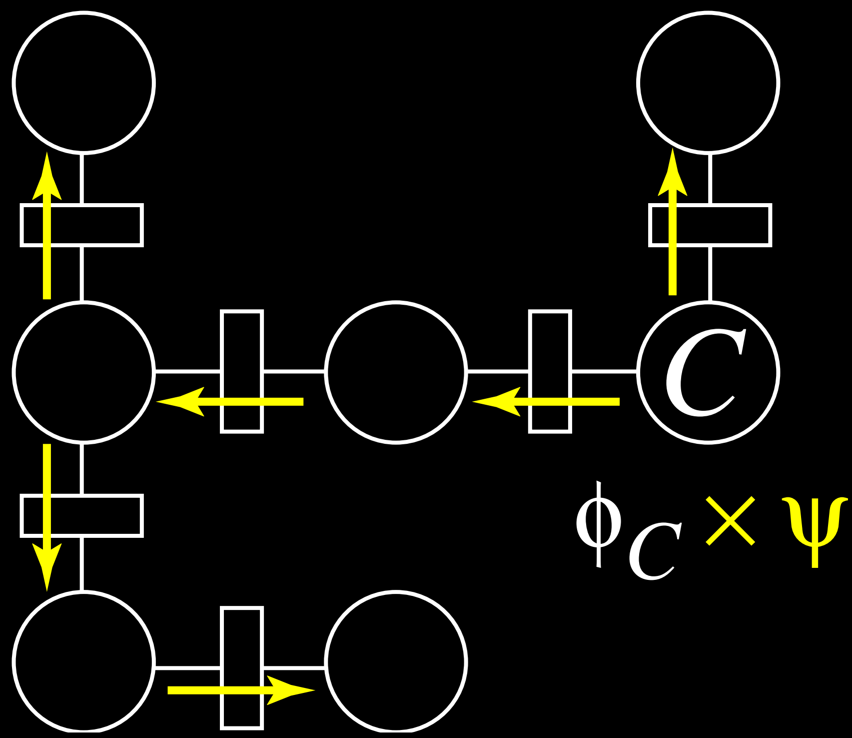
3. Distribute evidence from C (if needed).



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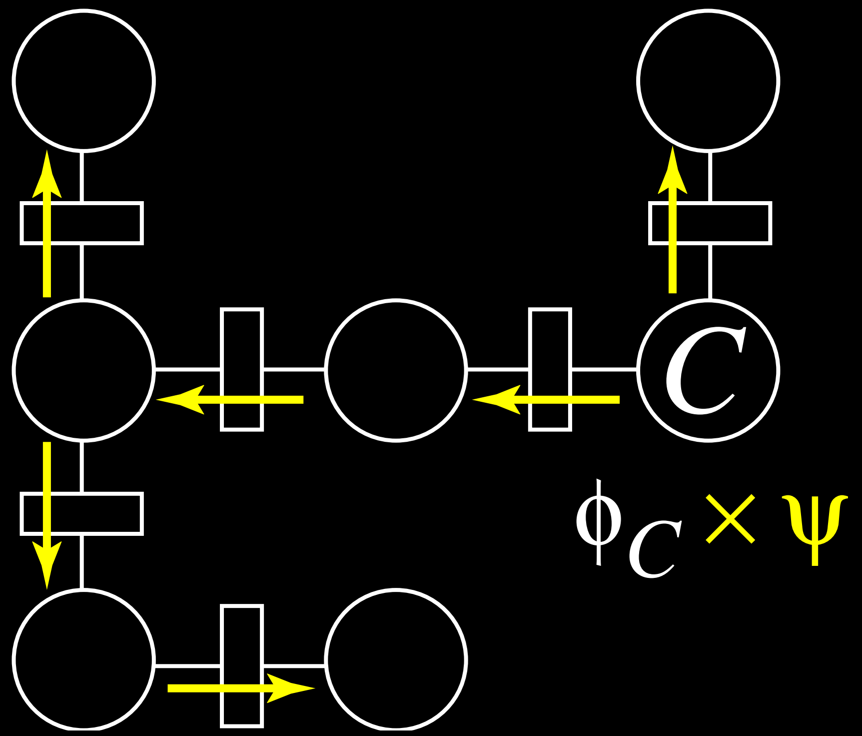
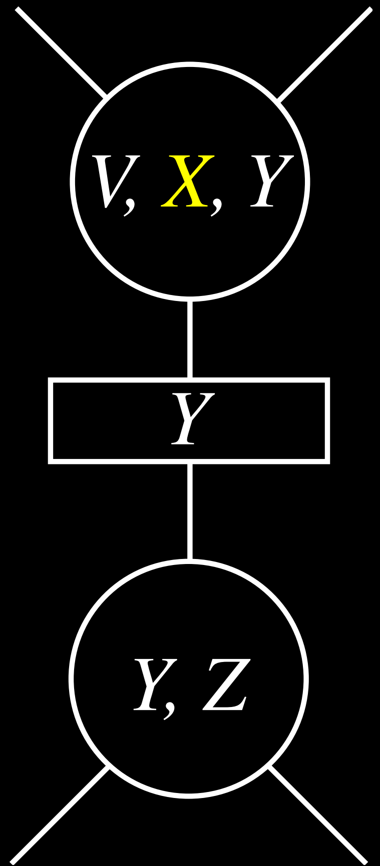


If there is no cover, we must make one.

Pushing variables to create covers



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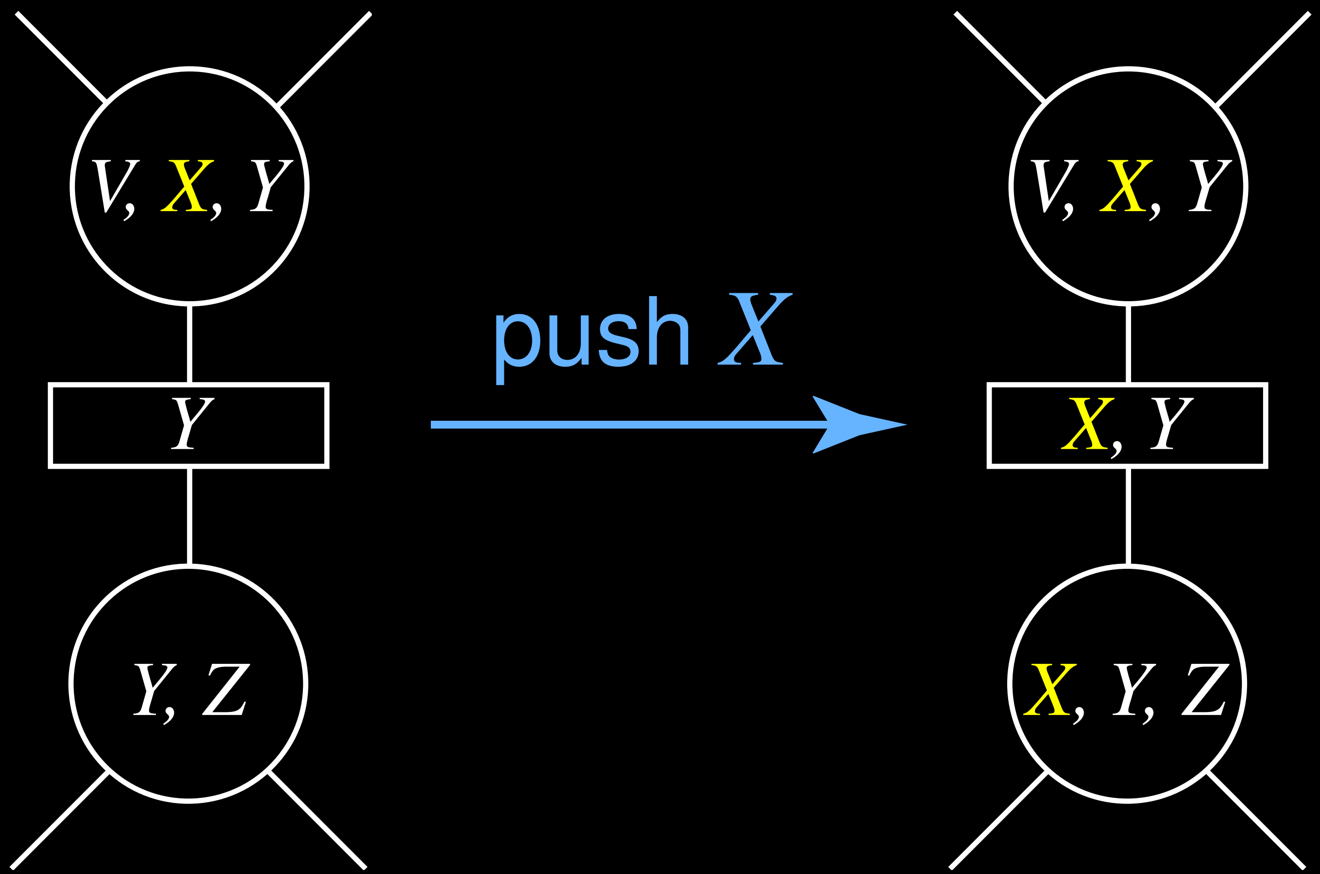

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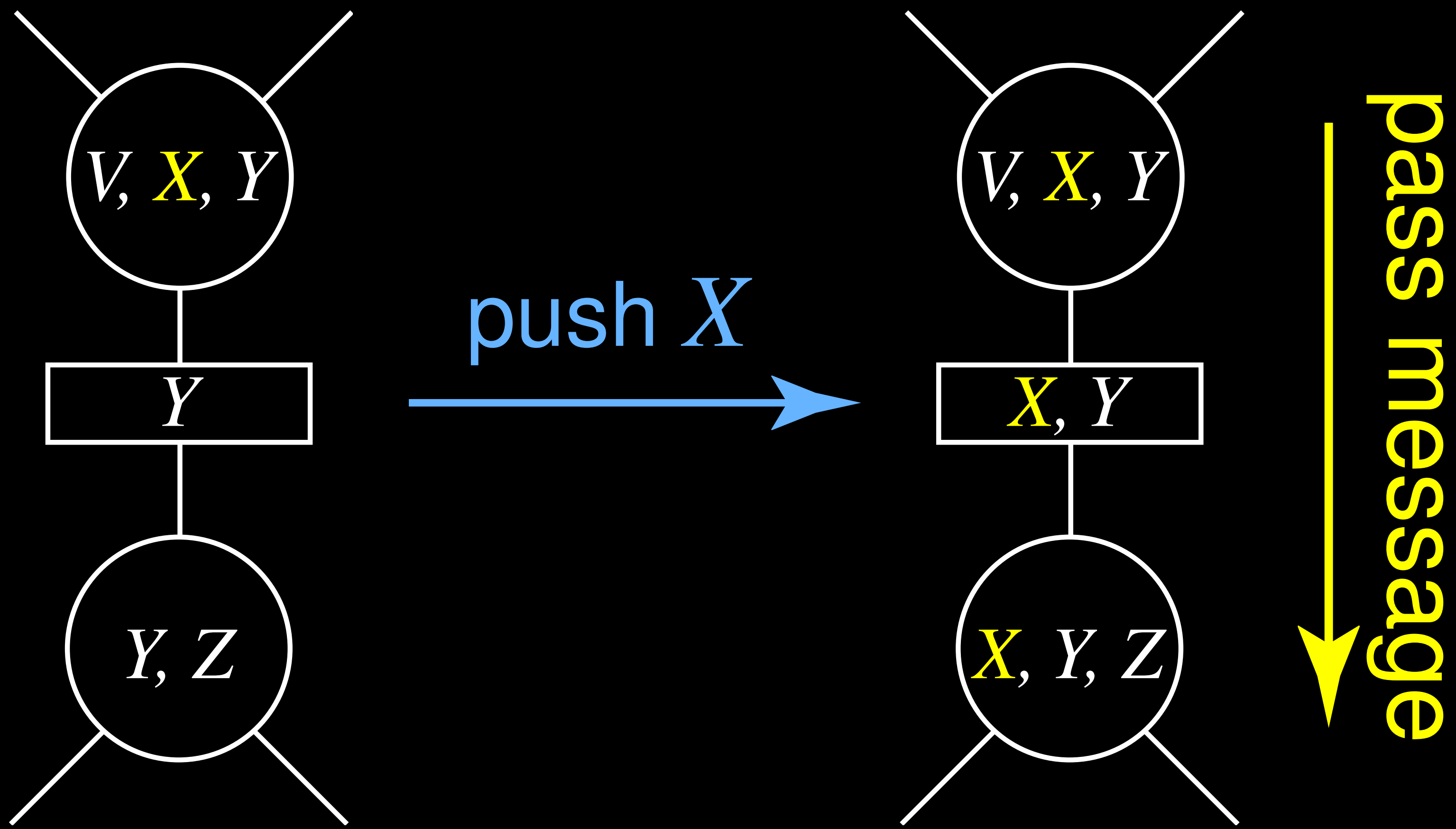
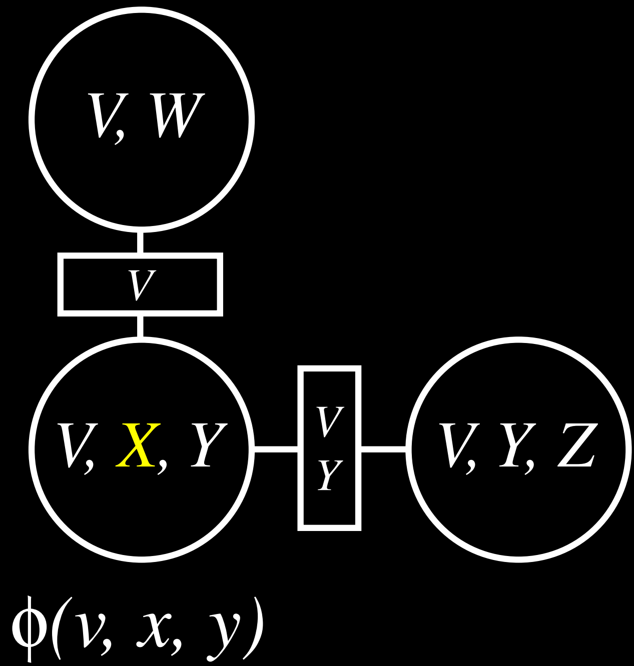


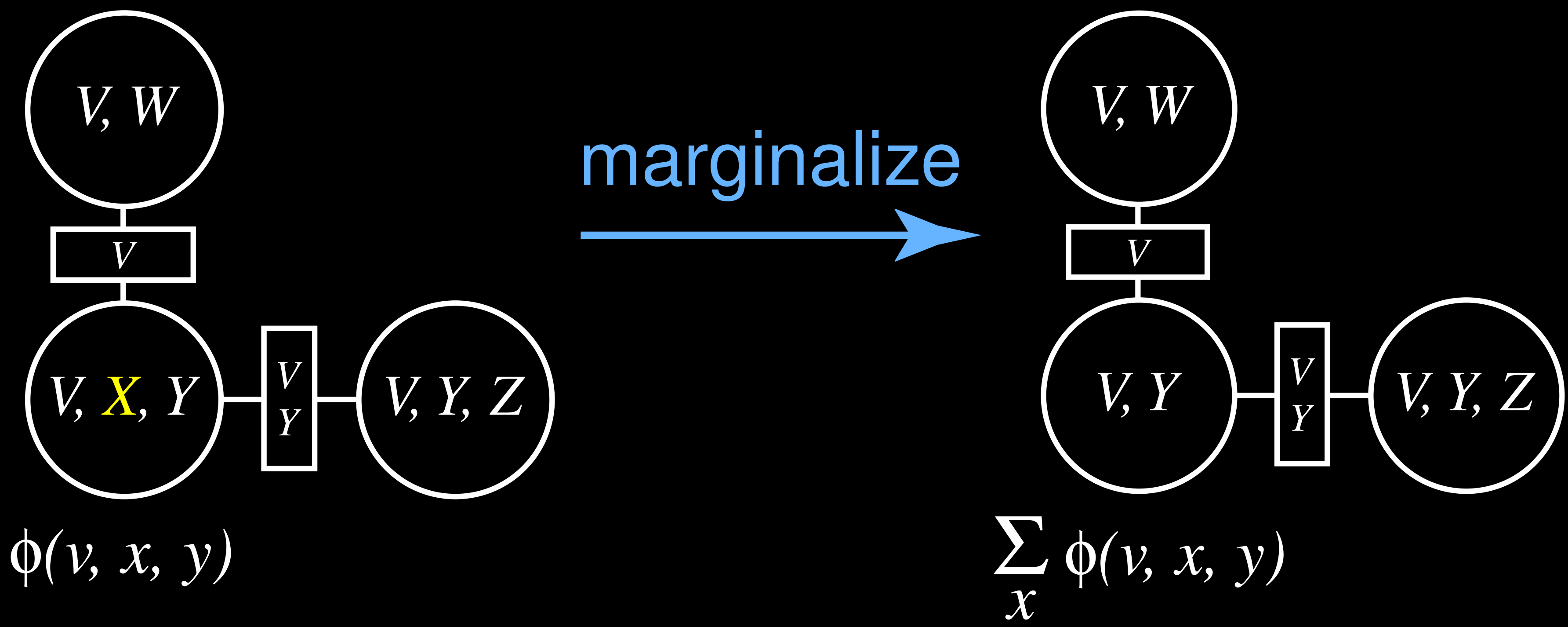
Roll-up

To marginalize out a variable X that is in only one cluster C . . .



Roll-up

To marginalize out a variable X that is in only one cluster C . . . marginalize X out of ϕ_C.



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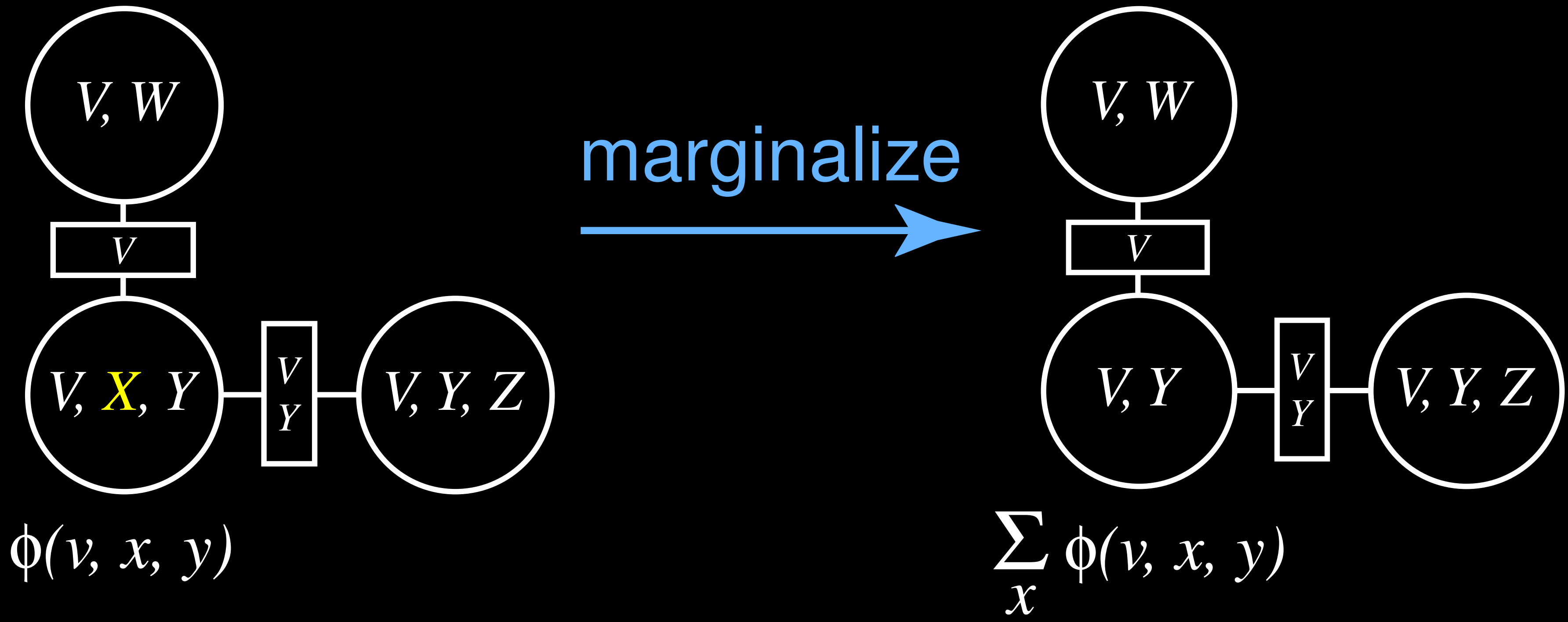
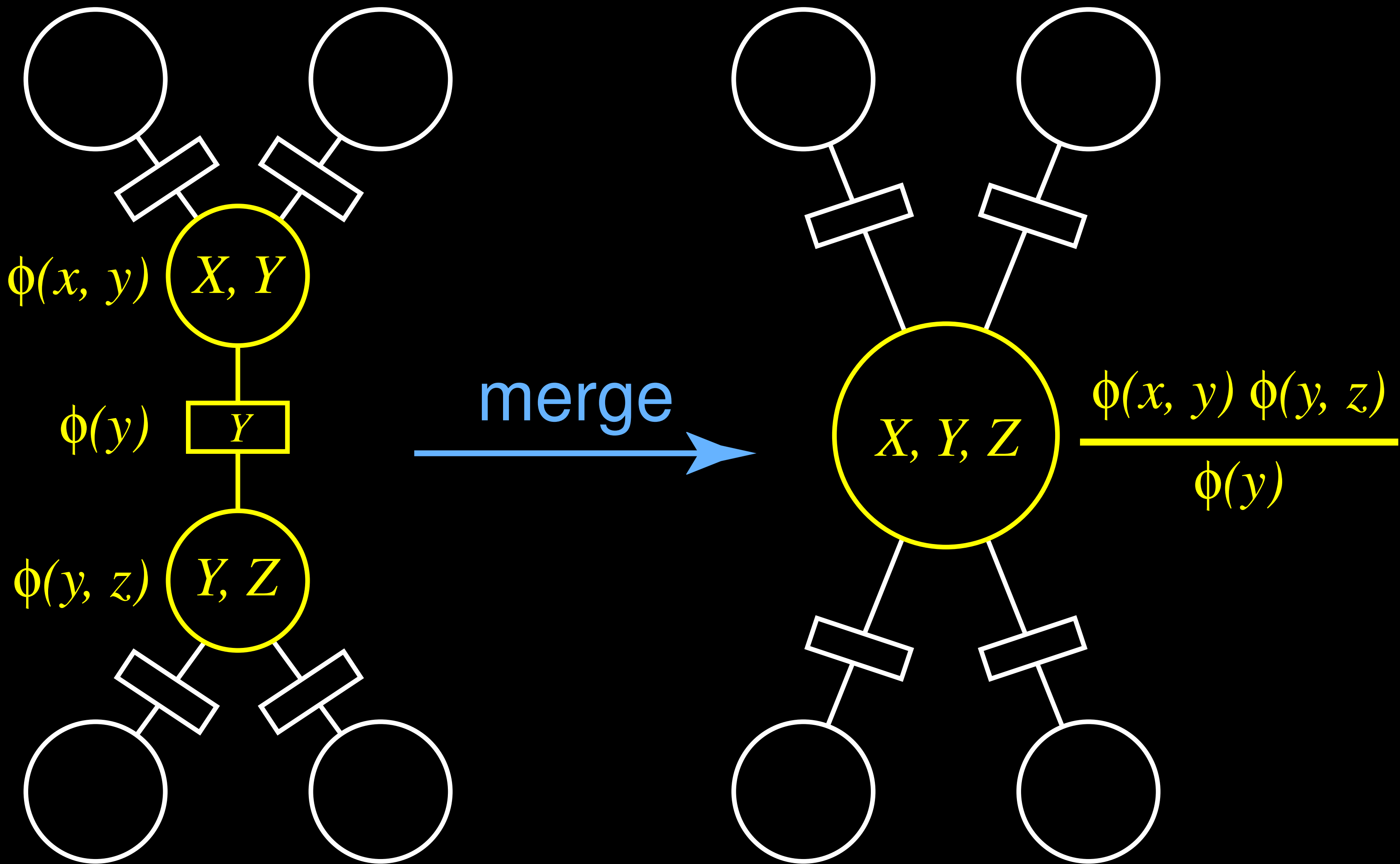


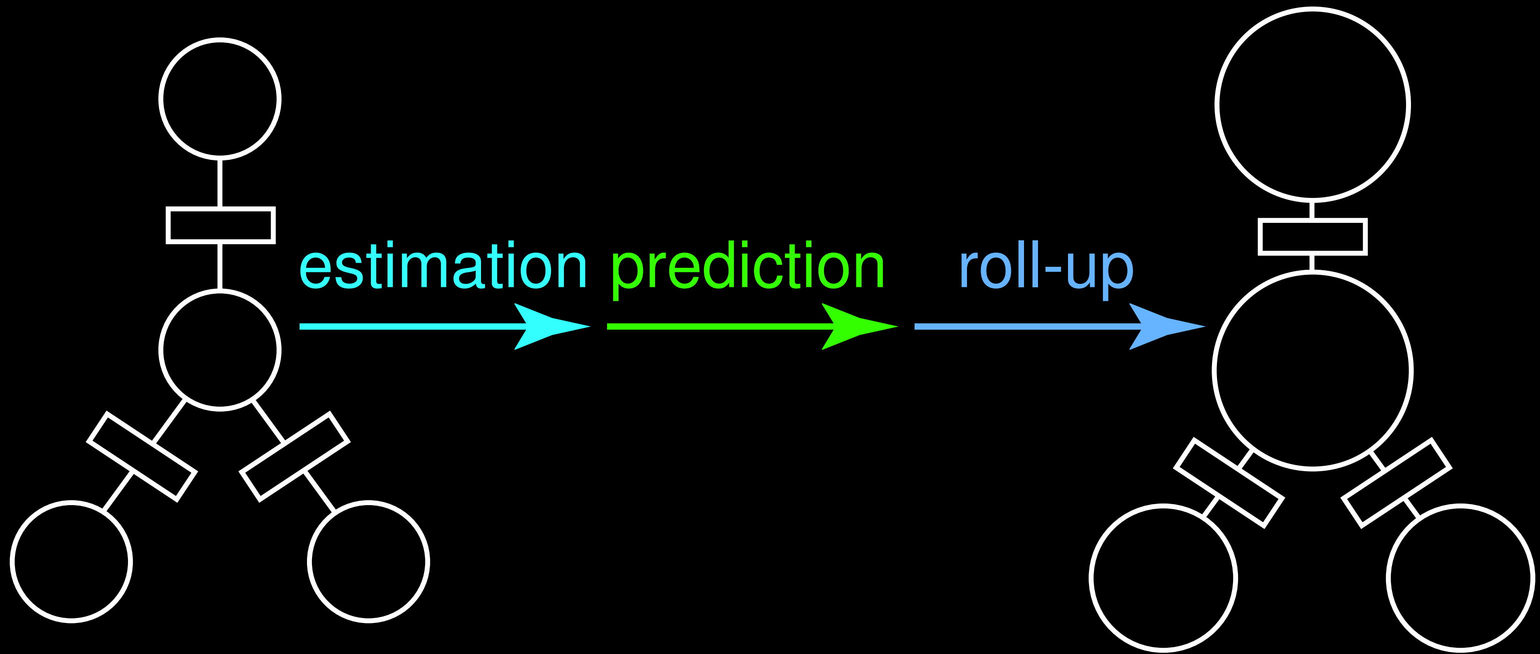
If X is in more than one cluster, we must first merge the clusters containing X...

Merging adjacent clusters



Thin junction tree filters (TJTF)

Pushing and merging increase the width of the junction tree, and therefore the complexity.



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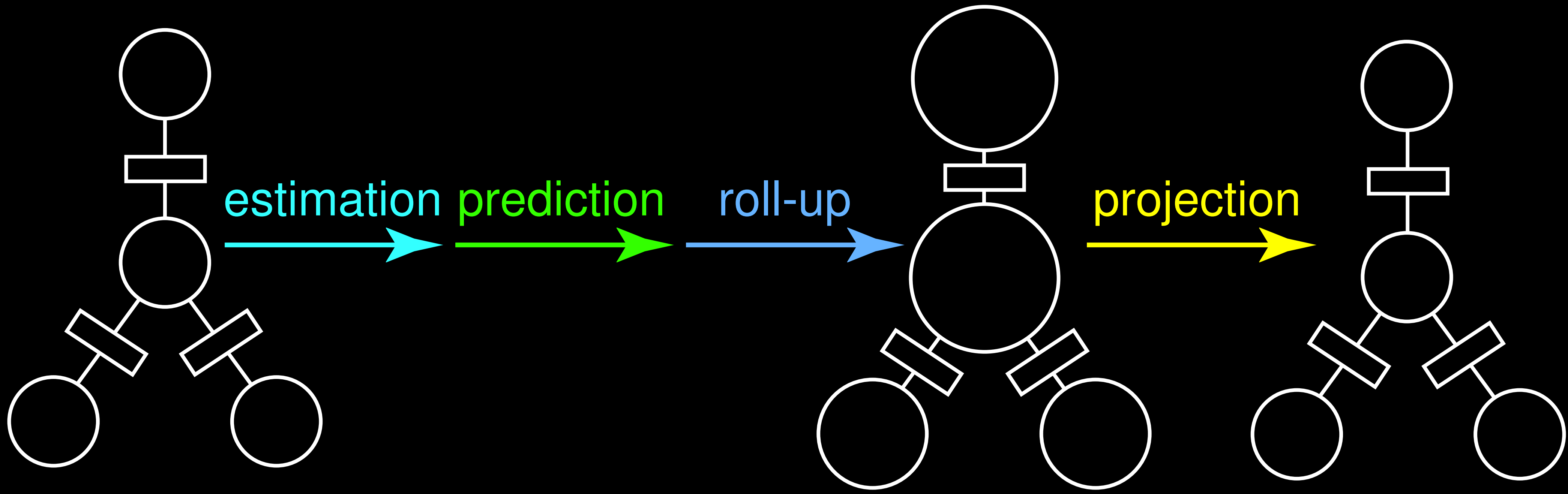

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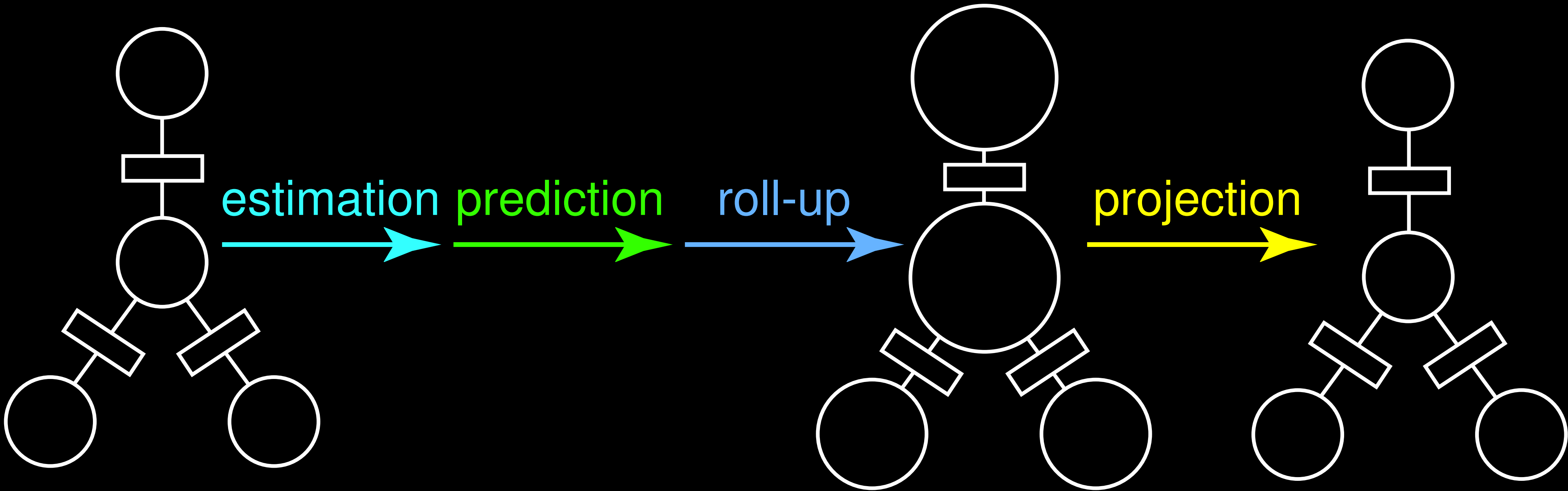
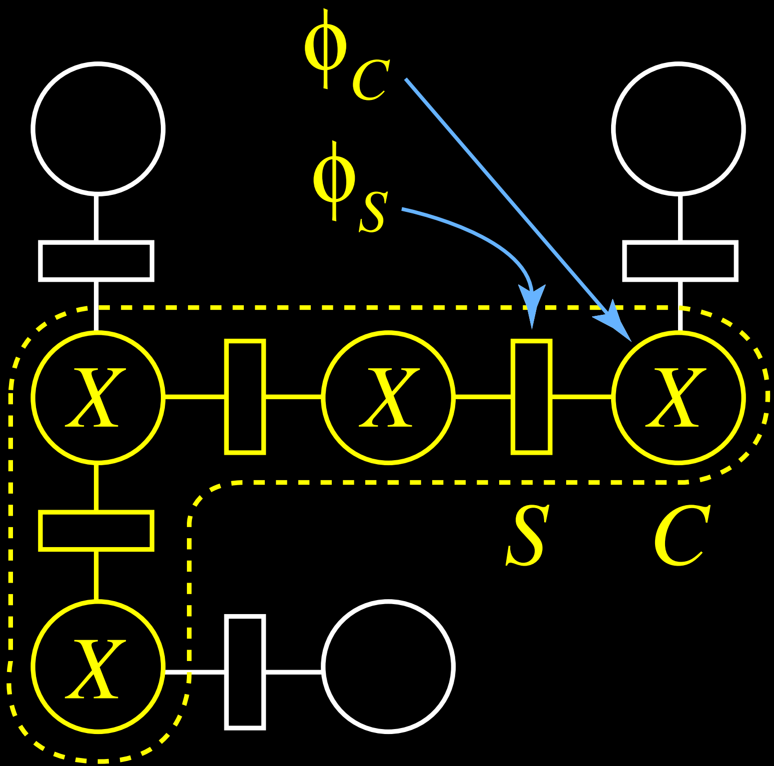


TJTF chooses the projection adaptively to minimize the approximation error.

Variable contraction



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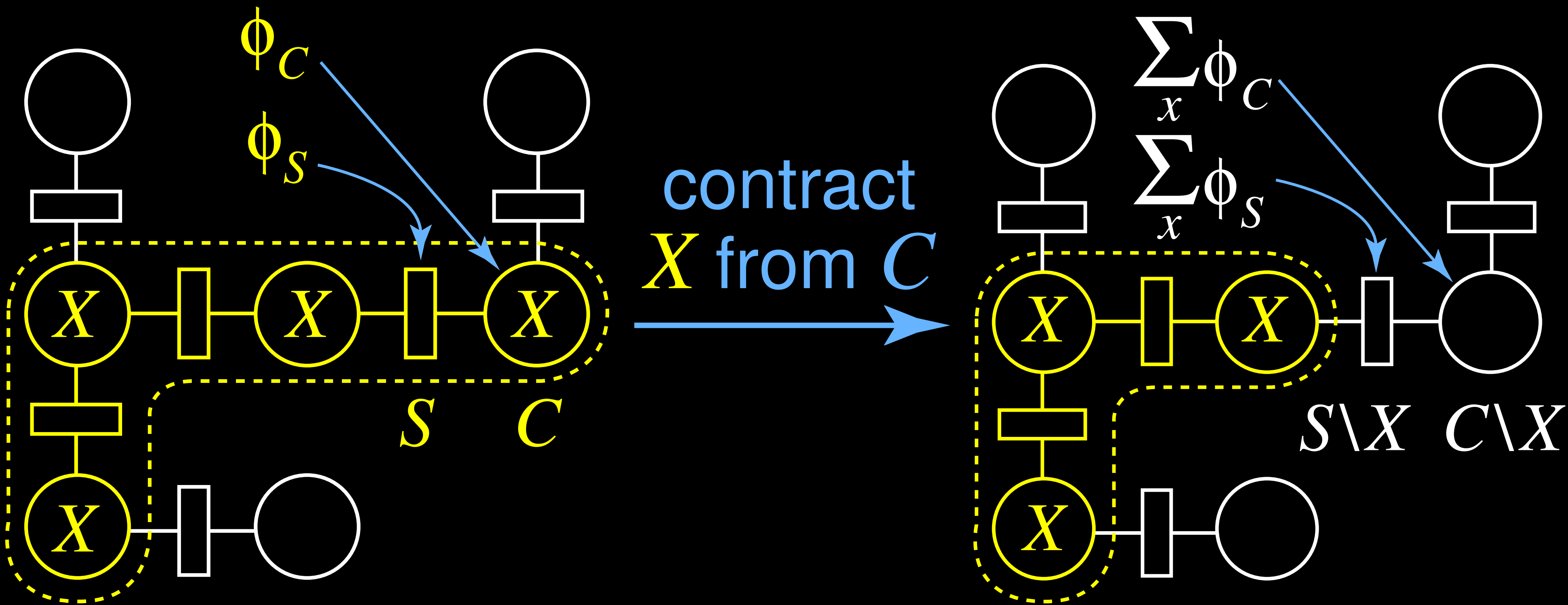

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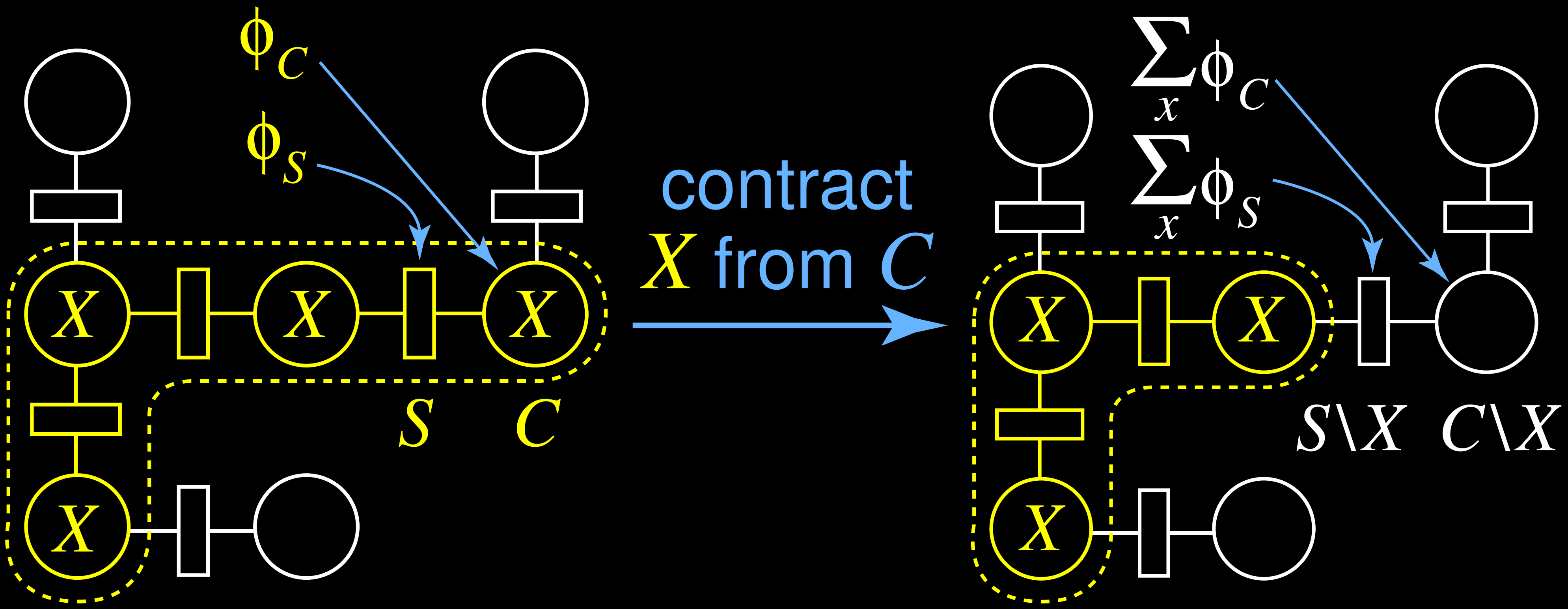

This cuts all edges between X and $C - S$, the variables X no longer resides with.

Variable contraction is an I-projection

Proposition. If $\tilde{p}$ is the density obtained by contracting X from C, then

$$\tilde{p} = \underset{\{q \,:\, X \perp\!\!\!\perp (C - S) \mid (S \setminus X)\}}{\arg\min} D(p \,||\, q)$$

Adaptive approximation

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$$D(p \,||\, \tilde{p}) = I(X; C - S \,|\, S \setminus X)$$

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- This can be computed using $\phi_C \propto p_C$.

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- To thin C, perform the contraction that minimizes this approximation error.

Thin junction tree filters for SLAM

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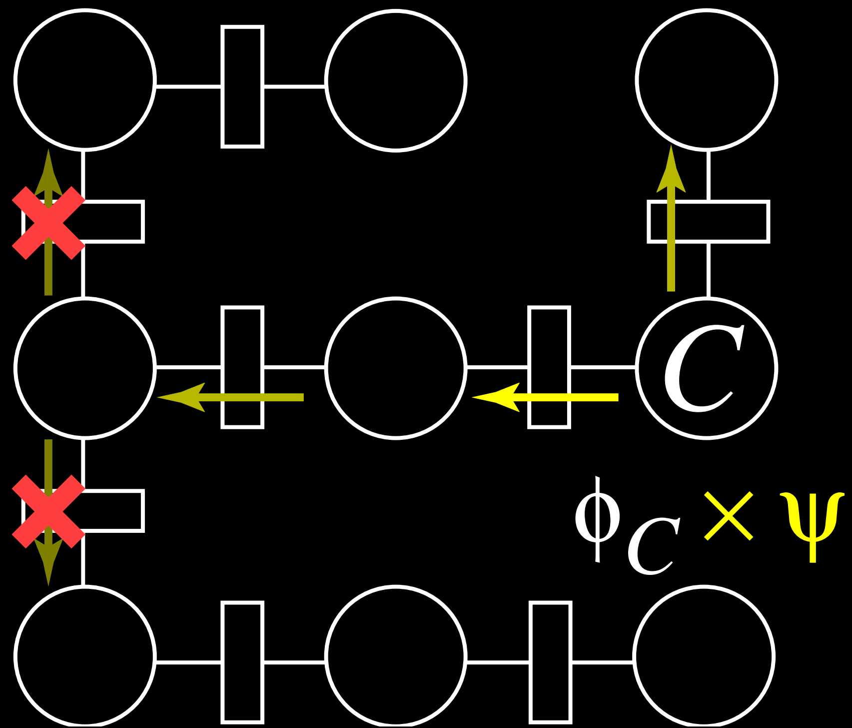
- This $O(N_t)$ time complexity is due (mainly) to message passing in the estimation step.

Adaptive message passing

Propagate messages only as long as they induce significant change in the belief state.



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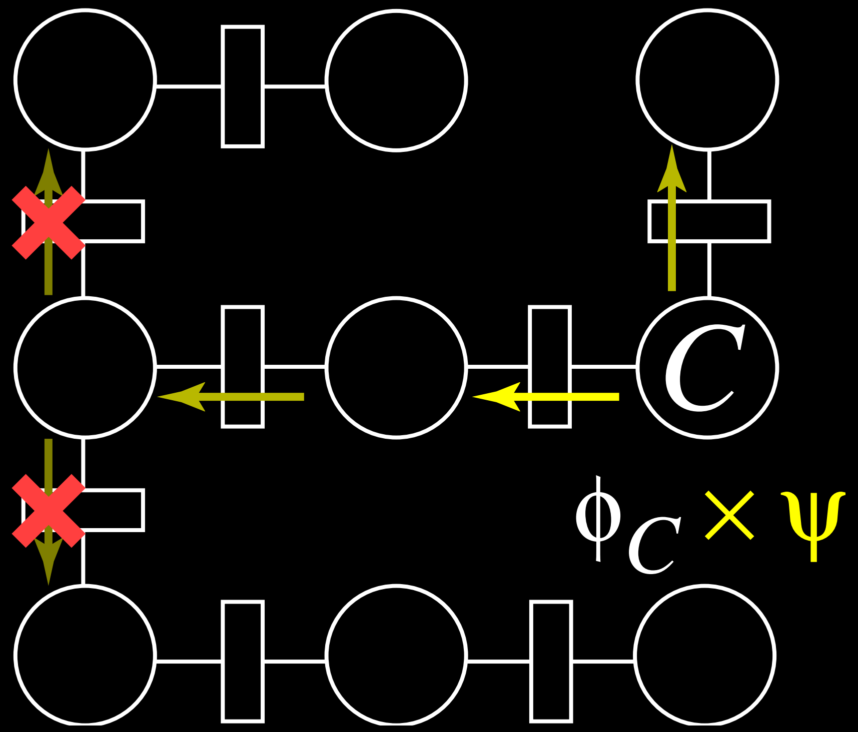
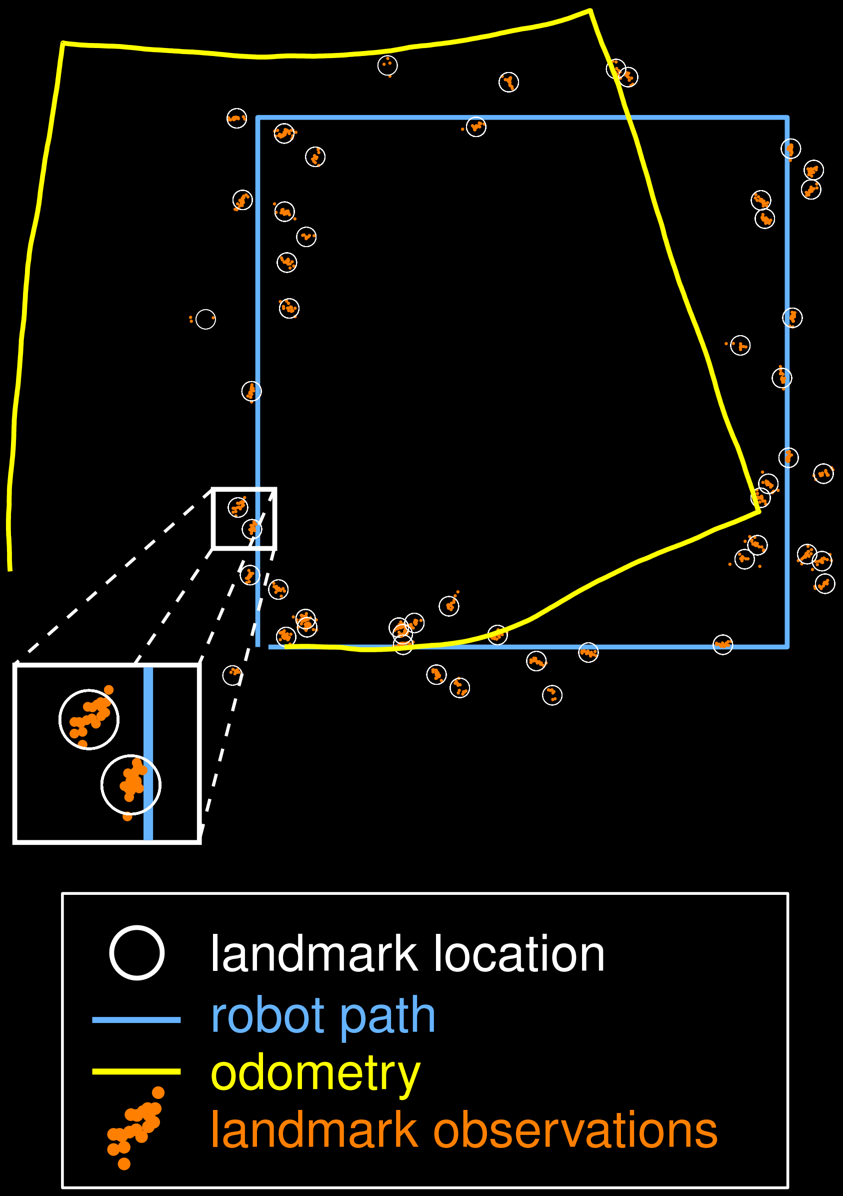
Significance is measured by $D(\phi_S^* \,||\, \phi_S)$, which decreases with distance.



Simulation results



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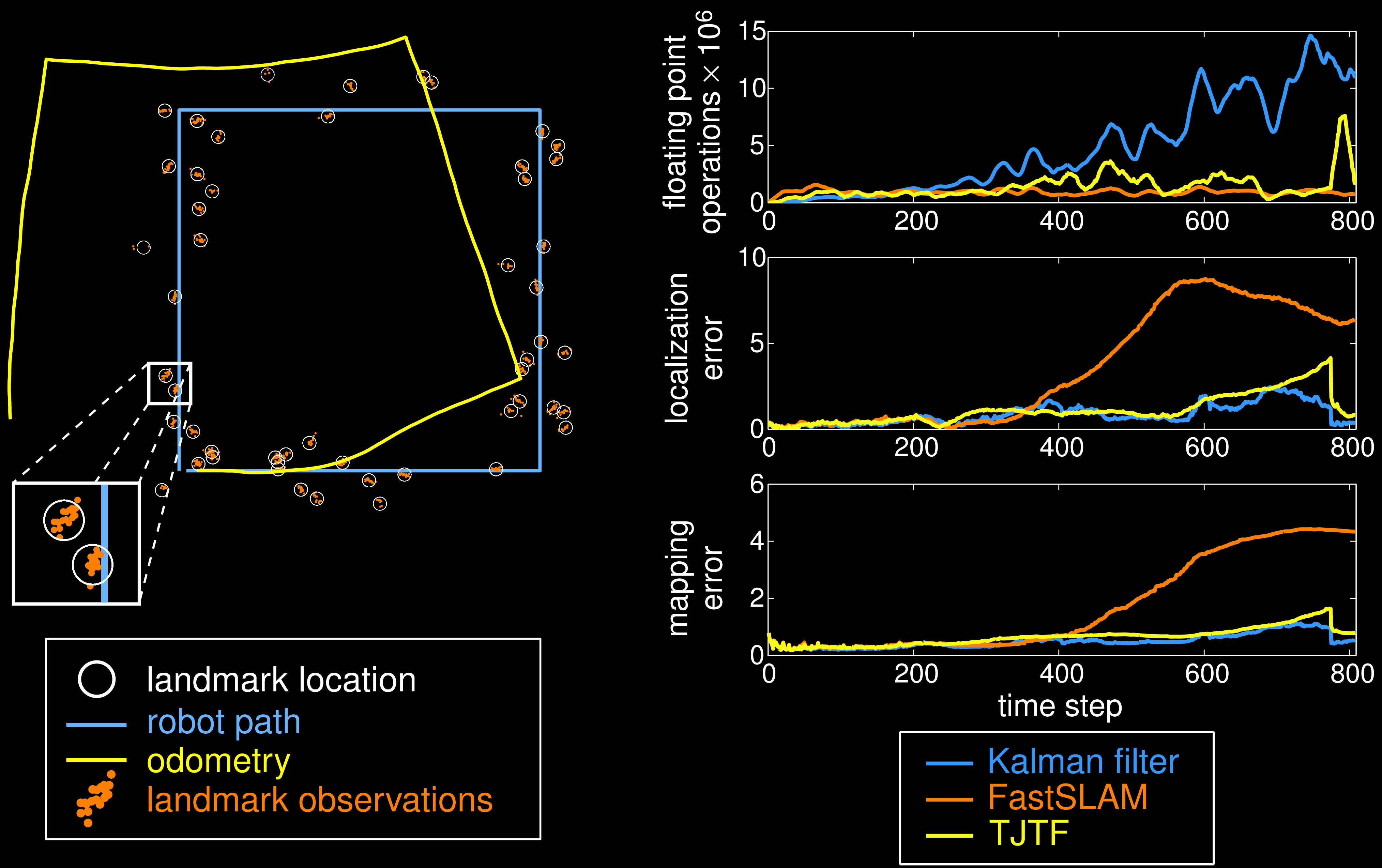

Summary

Thin junction tree filtering:

- a novel algorithm for adaptive approximate filtering in dynamic Bayesian networks
- an elegant solution to the Simultaneous Localization and Mapping problem

More movies and the implementation:

http://www.cs.berkeley.edu/~paskin/slam

Thanks to intel for supporting this research!