Motion Planning of Biped Robot Climbing Stairs

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Abstract–The motion control of a biped robot that can climb up and down stairs is studied in this work. The algorithm generates walking patterns with desired stable margin and walking speed. Only the approximate stair height and width are required. This algorithm derives the optimal hip height and uses cubic polynomial to generate the hip and foot trajectory. The control of initial and final speeds in a walking cycle makes continuous dynamic walking up and down stairs possible. Simulation shows that this algorithm can successfully achieve dynamic walking up or down stairs. The RoboSapien, a 12 DOF biped robot is built to apply and verify the walking algorithm.

Keywords Biped robot; climbing stairs; dynamic walking; Zero Moment Point; hip trajectory.

1 Introduction

For traditional manipulators, only the motion of the end-effector is of concern. For a biped robot the scenario is much different as there is no link fixed on ground. In order to achieve stable walking both the swing foot (end-effector) motion and whole body motion are required to control. Stable walking can be achieved by Center of Gravity (CG) control and Zero Moment Point (ZMP) control. The CG trajectory control [1] can guarantee the stability of static walking while the ZMP trajectory control [2], [3] can realize stable dynamic walking.

The ZMP is the point on the ground around which the sum of all the moments of the active forces equals zero. In order to achieve stable dynamic walking, the ZMP must be kept inside the support region. This criterion just ensures that the support foot is stationary on the ground in single support phase. Under such circumstances, the biped robot can be considered as a traditional manipulator for analysis purposes.

In the single support phase, cubic polynomial is used to generate the hip and swing foot trajectories which satisfy the ZMP criterion. The swing foot should follow a desired trajectory to avoid obstacles and to satisfy the landing constraint. In the double support phase, the robot transfers the weight from the support foot to the landing foot. This phase is also called weight acceptance phase[4].

In recent years, a lot of biped robots have been developed [5-8] and many researchers [9-21] have focused their research on the biped walking control.

This paper describes an algorithm for planning motion patterns for the biped robot to ascend and descend stairs. The paper is organized as follows. The biped robot is introduced in Section 2. In Section 3, the algorithm for planning the walking patterns are proposed. Simulation and experimental results are provided in Section 4. Section 5 concludes the work.

2 Biped Robot: RoboSapien

RoboSapien (Fig. 1) is designed and built as a research platform to investigate the humanoid robot walking motion. Although RoboSapien has a small size and light weight, it still has the complex joint configuration as human body and can realize complicate activities. Fig. 2 is the final design of joint configuration of RoboSapien. Table 1 listed the specifications of RoboSapien:

Table 1: Specifications of RoboSapien.

	length	mass
Body	0.18	1.5 kg
Thigh	0.08	0.4 kg
Shank	0.08	0.2 kg

3 Walking Algorithm for Ascending and Descending Stairs

The walking motion of the biped can be determined by the hip and foot trajectory. Cubic polynomial is used to generate the hip and swing foot trajectory. Zero Moment Point (ZMP) criterion is used to ensure the stability. As shown in Fig. 3, if the initial and final state are known, the trajectory can be generated by cubic polynomial. And if the trajectory satisfies the ZMP criterion, the robot can achieve continuous walking. Lateral motion is similar with sagittal motion, only sagittal motion is discussed in this work.

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Fig. 1. RoboSapien

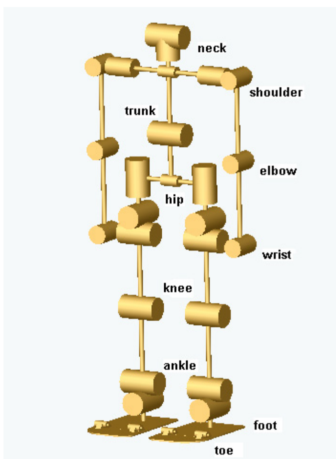

Fig. 2. DOF Configuration of RoboSapien

3.1 *Derivation of the hip height for ascending and descending stairs*

In human walking, the hip is lifted to a maxim height when walking up stairs. And when walking down stairs, the hip is lowered down to enable the foot to reach the lower stair. This is also valid for biped robot. This rule can be represented by:

$$z_{he} = min(max(h_s), max(h_l)) \tag{1}$$

where h_s and h_sm are the possible and maximum hip height respectively for the support leg, h_l and h_lm are the possible and maximum hip height respectively for the land leg as shown in Fig. 4. This rule comes from the observation of human walking. From the viewpoint of the energy, it ensures the minimal energy loss for walking downward and a highest COG position for walking upward. From Fig. 4, for any θ_s, the optimal hip height can be derived. And a suitable θ_s is selected according to the ZMP criterion.

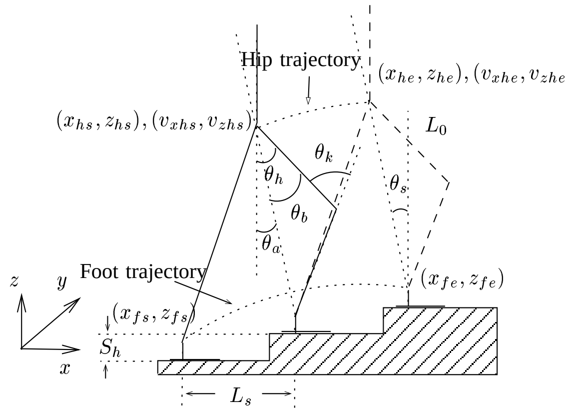

Fig. 3. Hip and foot trajectory

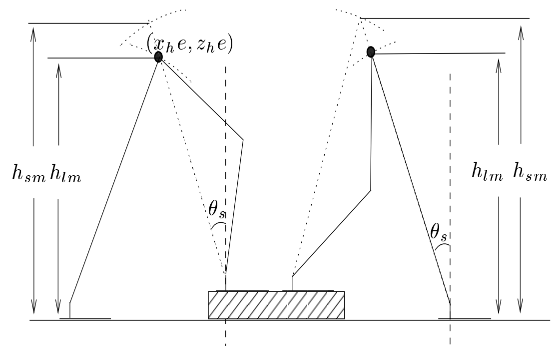

Fig. 4. Determine the height of hip

3.2 *ZMP*

The ZMP is the point on the ground around which the sum of all the moments of the active forces equals zero. Under the assumption that no external force exists the ZMP can be computed by[19]:

$$x_{zmp} = \frac{\sum_i m_i(\ddot{z}_i + g)x_i - \sum_i m_i \ddot{x}_i z_i - \sum_i I_{iy}\ddot{\theta}_{iy}}{\sum_i m_i(\ddot{z}_i + g)} \tag{2}$$

$$y_{zmp} = \frac{\sum_i m_i(\ddot{z}_i + g)y_i - \sum_i m_i \ddot{y}_i z_i - \sum_i I_{ix}\ddot{\theta}_{ix}}{\sum_i m_i(\ddot{z}_i + g)} \tag{3}$$

where $(x_{zmp}, y_{zmp}, 0)$ is the coordinate of the ZMP, (x_i, y_i, z_i) is the mass center of link i on a Cartesian coordinate system. m_i is the mass of link i, g is the gravitational acceleration. I_{ix} and I_{iy} are the initial components, $\ddot{\theta}_{iy}$ and $\ddot{\theta}_{ix}$ are the angular speed around axis y and x at the center of mass of link i. (Fig. 5)

3.3 *Hip trajectory generation*

As shown in Fig. 3, hip trajectory can be generated by cubic polynomial if the initial state and final state are known for single support phase. The initial state is given. The final state includes position $[x_{he}, z_{he}]^T$ and velocity $[v_{hxe}, v_{hze}]^T$. The desired velocity $[v_{hxe}, v_{hze}]^T$ is specified. For any given θ_s,

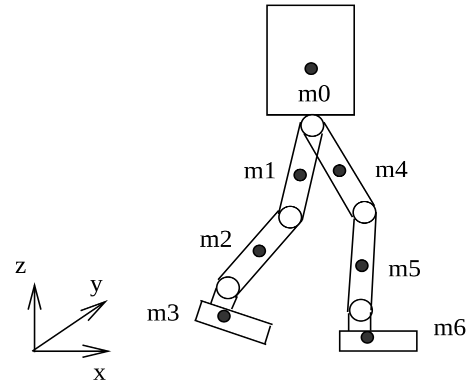

Fig. 5. Biped robot model

z_he can be derived suing (1). Consequently x_he is derived.

Since the initial state is known and the desired final state is achieved, the hip trajectory for single support phase can be generated by cubic polynomial. The initial and final constraint of cubic trajectory for z direction $z_h(t)$ can be described as:

$$z_h(t) = \begin{cases} z_{hs} & \text{if } t = kT \\ z_{he} & \text{if } t = kT + T_s \end{cases} \tag{4}$$

where T is the period for one step and T_s is the period for single support phase.

$$\dot{z_h}(t) = \begin{cases} v_{zhs} & \text{if } t = kT \\ v_{zhe} & \text{if } t = kT + T_s \end{cases} \tag{5}$$

Cubic polynomial

$$z_h(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{6}$$

satisfying the initial and final constraint can be written as:

$$\begin{aligned} z_h(t) = {} & z_{hs} + v_{zhs}(t - kT) \\ & + \frac{3(z_{he} - z_{hs}) - 2v_{zhs}T_s - v_{zhe}T_s}{T_s^2}(t - kT)^2 \\ & + \frac{2(z_{hs} - z_{he}) + (v_{zhs} + v_{zhe})T_s}{T_s^3}(t - kT)^3 \\ & kT < t \leq kT + T_s \end{aligned} \tag{7}$$

$x_h(t)$ is divided into two parts: $x_h(kT)$ to $x_h(kT + T_1)$ and $x_h(kT + T_1)$ to $x_h(kT + T_s)$. The constraint for $x_h(t)$ is:

$$\begin{cases} x_h(t) = x_{hs} & \text{t=kT} \\ x_h(t) = x_{h1} & \text{t=kT+}T_1 \\ x_h(t) = x_{he} & \text{t=kT+}T_s \\ \dot{x}_h(t) = v_{xhs} & \text{t=kT} \\ \dot{x}_h(t^-) = \dot{x}_h(t^+) & \text{t=kT+}T_1 \\ \dot{x}_h(t) = v_{xhe} & \text{t=kT+}T_s \\ \ddot{x}_h(t) = a_0 & \text{t=kT} \end{cases} \tag{8}$$

where a_0 is to be specified to satisfy the initial acceleration. The cubic polynomial trajectory can be derived as:

$$x_h(t) = \begin{cases} x_{hs} + v_{xhs}(t - kT) + \frac{1}{2}a_0(t - kT)^2 \\ + \frac{(x_{h1} - x_{hs} - v_{xhs}T_1 - \frac{1}{2}a_0T_1^2)(t-kT)^3}{T_1^3} \\ kT < t \leq kT + T_1 \\ \\ x_{h1} + v_{xh1}(t - kT - T_1) \\ + \frac{(3(x_{he} - x_{h1}) - 2v_{xh1}(T_2 - T_1))(t - kT - T_1)^2}{(T_2 - T_1)^2} \\ + \frac{(2(x_{h1} - x_{he}) + (v_{xh1} + v_{x2})(T_2 - T_1))(t - kT - T_1)^3}{(T_2 - T_1)^3} \\ kT + T_1 < t \leq kT + T_s \end{cases}$$

3.4 Swing foot trajectory

Cubic polynomial is used to generate the swing foot trajectory of single support phase. At the starting point and ending point the following position and speed constraint must be satisfied:

$$\begin{cases} x_f(t) = x_{fs} & \text{t=kT} \\ x_f(t) = x_{fe} & \text{t=kT+}T_s \\ z_f(t) = z_{fs} & \text{t=kT} \\ z_f(t) = z_{fe} & \text{t=kT+}T_s \end{cases} \tag{9}$$

where $x_{fe} = x_{fs} + 2L_s$, $z_{fe} = z_{fs} + 2S_h$. As shown in Fig. 3, L_s is the step length and S_h is the stair height.

$$\begin{cases} \dot{x}_f(t) = 0 & t = kT \\ \dot{x}_f(t) = 0 & t = kT + T_s \\ \dot{z}_f(t) = 0 & t = kT \\ \dot{z}_f(t) = 0 & t = kT + T_s \end{cases} \tag{10}$$

Assuming that the obstacle's height is H_o and position is at x_o. In order to avoid the collision with the obstacle the height of swing foot should be larger than H_o at $x = x_o$. The constraint can be described as:

$$\begin{cases} z(t) = H_o & t = kT + T_o \\ \dot{z}(t) = 0 & t = kT + T_o \end{cases} \tag{11}$$

Giving the start and end position on x and z direction and the height of the obstacle H_o, a smooth foot trajectory $f(t) = [x_f(t), z_f(t)]^T$ can be generated by

$$\begin{aligned} x_f(t) = {} & x_{fs} + 3(x_{fe} - x_{fs}) \cdot \frac{(t - kT)^2}{{T_s}^2} - 2(x_{fe} \\ & -x_{fs}) \cdot \frac{(t - kT)^3}{{T_s}^3} kT \leq t \leq kT + T_s \end{aligned} \tag{12}$$

$$z_f(t) = \begin{cases} z_{fs} + 3(z_{fm} - z_{fs}) \cdot \frac{(t-kT)^2}{{T_m}^2} - 2(z_{fm} \\ -z_{fs}) \cdot \frac{(t-kT)^3}{{T_m}^3} \quad kT \leq t \leq kT + T_m \\ \\ z_{fm} + 3(z_{fe} - z_{fm}) \cdot \frac{(t-kT-T_m)^2}{(T_s - T_m)^2} - 2(z_{fe} \\ -z_{fm}) \cdot \frac{(t-kT-T_m)^3}{(T_s-T_m)^3} kT + T_m < t \leq kT + T_s \end{cases}$$

The joint position of hip and knee of the swing leg can be derived by inverse kinematics. Suppose the hip and swing foot position in the sagittal plane at time t is $h(t) = [x_h(t), z_h(t)]^T$ and $f(t) = [x_f(t), z_f(t)]^T$. The inverse kinematics of the swing leg can be derived as follows:

$$\begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} = \begin{bmatrix} sin^{-1} \frac{((x_f(t)-x_h(t))}{\sqrt{(x_f(t)-x_h(t))^2+(z_h(t)-z_f(t))^2}} \\ cos^{-1} \frac{\sqrt{(x_f(t)-x_h(t))^2+(z_h(t)-z_f(t))^2}}{2L_o} \end{bmatrix}.$$

As shown in Fig. 3, $\theta_h = \theta_a + \theta_b$ and $\theta_k = 2\theta_b$. θ_h and θ_k are the hip and knee joint angle respectively and L_o is the length of thigh and shank.

For different T_1 and T_s, hip and foot trajectories are generated. And a set of trajectories with desired ZMP margin is selected.

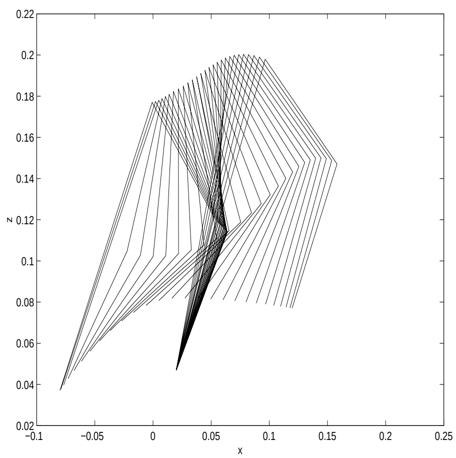

Fig. 7. Gait for single support phase.

4 SIMULATION AND EXPERIMENT

4.1 Simulation

A 12-DOF biped model in Yobotics is used to verify this algorithm. PD controller is used on each joint. The biped model has the same parameters with biped robot RoboSapien. The parameters utilized in the simulation study are listed in Table 2. is listed in table 2:

Table 2: Parameters of walking simulation.

Parameter	value
step length	0.115 m
stair height	0.015 m
walking speed	1.0 sec/step

The walking cycle is divided into six stages. Fig. 6 shows the snap shots of the biped motion in each stage when ascending stairs. First stage is single support stage. Second stage is falling forward stage. This stage is to enable the land foot contact the stair. Third stage is the double support phase. In this phase, the robot transfer the weight from the rear foot to the front foot. Fourth, fifth and sixth are symmetry to the first three respectively. The walking cycle of descending stairs is the same with ascending stairs.

The gait for the single support phase is shown in Fig. 7.

Fig. 8 shows the ZMP trajectory along the walking direction. Fig. 9 shows how the hip joint velocity changes with the hip joint position. The trajectory converging to a limit cycle proves that the locomotion is stable. Fig. 10, 11, 12 show the joint position of ankle, knee and hip. The body angle is controlled by the support hip by a PD controller. Fig. 13 shows the pitch of the body.

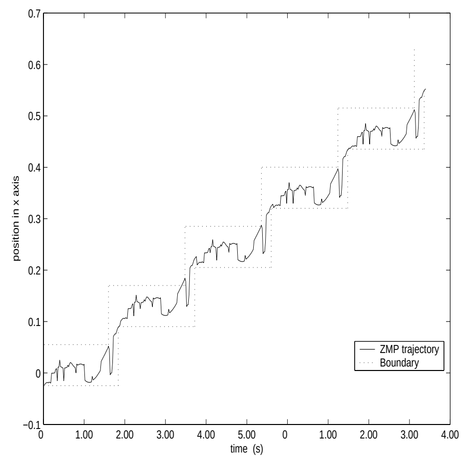

Fig. 8. ZMP trajectory along the walking direction.

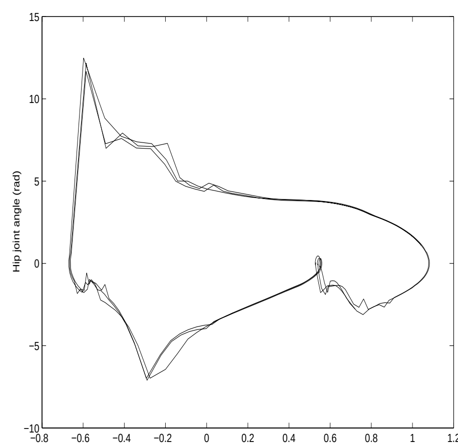

Fig. 9. The position and velocity of hip joint converge to a limit cycle.

4.2 Implementation

The algorithm is implemented on the biped robot RoboSapien. RoboSapien has 12 DC servo motors on the lower body. Eight force sensors are mounted on the two feet and a tilt sensor is fixed on the body. A DSP is used to generate the gait and send the PWM to control the motors. Fig. 14 shows the experiment of RoboSapien walking up stairs with the parameters listed in Table 2.

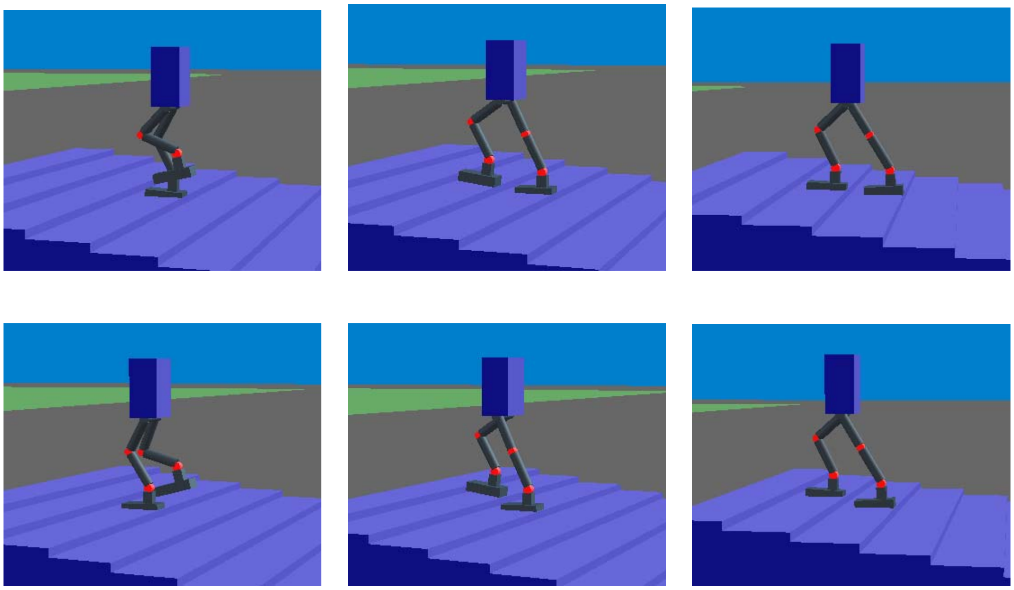

Fig. 6. The simulation snap shots of biped locomotion climbing stairs

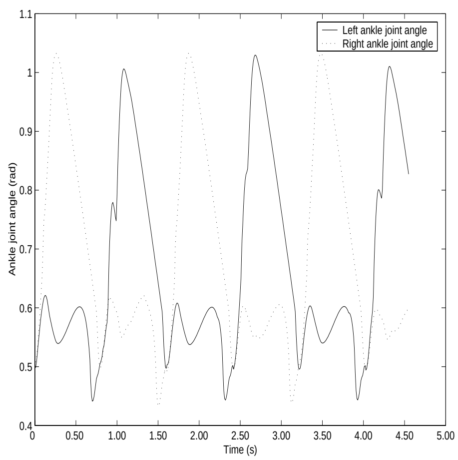

Fig. 10. The ankle joint position.

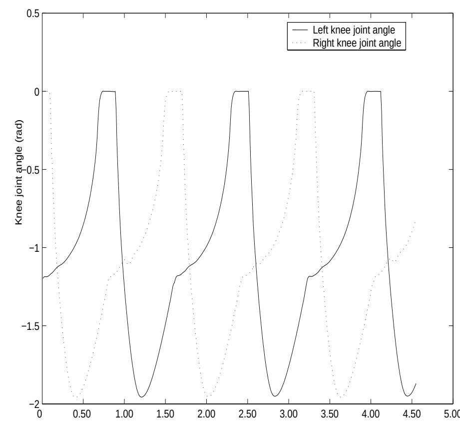

Fig. 11. The knee joint position.

5 Conclusion

In this work, an algorithm for the biped robot to ascend and descend stairs is proposed. A 12-DOF biped robot is simulated in Yobotics to verify the performance of this walking algorithm. The algorithm is validated by implementing on actual robot and dynamic walking up and down stairs is realized.

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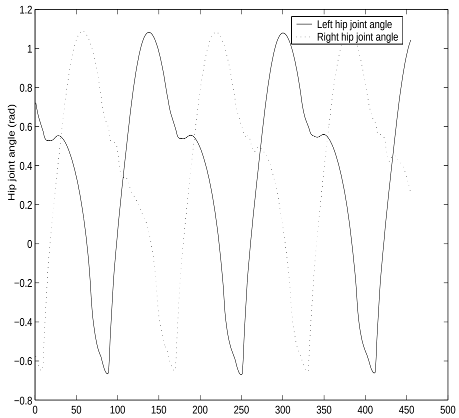

Fig. 12. The hip joint position.

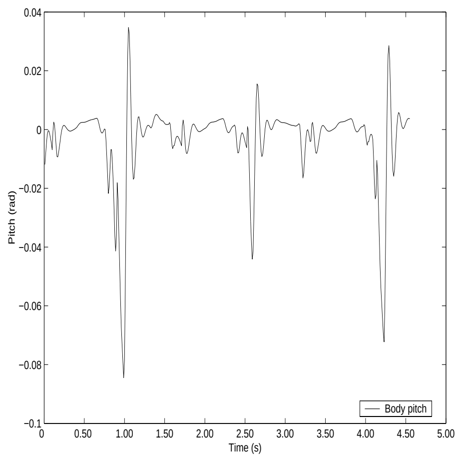

Fig. 13. The pitch of the body.

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Fig. 14. Experiment of RoboSapien walking on stairs.

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