

1. Appendix to Fig. 5

In the following we provide some explicit mathematical expressions for the algorithm in Fig. 5. We left them out from the paper for space reasons, but also since they are not necessary for understanding the approach as they can be derived. Here we provide them for convenience, *e.g.* the entries of P in case I and case II, as well as the way how the least squares systems in the M-step are solved.

1.1. Rigid Registration (case I)

- E-step: Compute matrix P for pair X and Y_ℓ with entries

$$p_{mn} = \frac{\exp\left(\frac{\|x_n + V_{x_n} d_n - (R(y_m + V_{y_m} d_m) + t)\|^2}{-2\sigma^2}\right)}{\sum_{k=1}^{N_d} \exp\left(\frac{\|x_n + V_{x_n} d_n - (R(y_k + V_{y_k} d_k) + t)\|^2}{-2\sigma^2}\right)}$$

- M-step: Solve for R_ℓ, t_ℓ , *i.e.* minimize $Q_{rigid}(R_\ell, t_\ell)$ by solving a least squares system, $\mathbf{1}$ being a vector with all entries set to 1, \mathcal{X} and \mathcal{Y} are the matrices obtained by stacking all point coordinates from X and Y_ℓ into combined matrices.

$$\begin{aligned} N_p &= \mathbf{1}^T P \mathbf{1}, \\ \mu_x &= \frac{1}{N_p} \mathcal{X}^T P^T \mathbf{1}, \mu_y = \frac{1}{N_p} \mathcal{Y}^T P \mathbf{1}, \\ \hat{\mathcal{X}} &= \mathcal{X} - \mathbf{1} \mu_x^T, \hat{\mathcal{Y}} = \mathcal{Y} - \mathbf{1} \mu_y^T, \\ A &= \hat{\mathcal{X}}^T P^T \hat{\mathcal{Y}}, \\ (U, V, C) &= svd(A), \\ R_\ell &= U C V^T \\ t_\ell &= \mu_x - R_\ell \mu_y \end{aligned}$$

1.2. Non-rigid Registration (case II)

- E-step: Computer P for all pairs Y_ℓ, X

$$p_{mn} = \frac{\exp\left(\frac{\|x_n + V_{x_n} d_n - (R(y_m + V_{y_m} d_m) + t)\|^2}{-2\sigma^2}\right)}{\sum_{k=1}^{N_d} \exp\left(\frac{\|x_n + V_{x_n} d_n - (R(y_k + V_{y_k} d_k) + t)\|^2}{-2\sigma^2}\right)}$$

The terms d_n, d_m, d_k correspond to the bias values at the respective pixel radius from the center, as stored in (d_1, \dots, d_O) .

- M-step: Solve for (d_1, \dots, d_O) by combining OK linear equations of the form $\frac{\partial(Q_{non-rigid}(d_1, \dots, d_O))}{\partial d_i} = 0$ into a joint linear least squares system (*i.e.* combine the equations for all $\ell = 1, \dots, K$)