Factors affecting emergence of sensitivity to numerosity through unsupervised learning
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A considerable body of empirical work demonstrates (a) that discrimination of different numerosities is consistent with Weber’s law and (b) that the Weber fraction for numerosity discrimination decreases over the course of development. Since (a) holds for different species and across the life span some have suggested there may be a phylogenetically primitive and innate number sense. At the same time (b), together with findings suggesting that culture and education may affect the Weber fraction, have led to the view that this innate sense may be susceptible to refinement through experience. Here we build on earlier work by Stoianov and Zorzi (2012, *Nature Neuroscience*, 15, 194–196; hereafter, S&Z) to explore the emergence of sensitivity to numerosity in a deep neural network. Of interest is that the neural network is not trained to represent numerosity per se: Its only task is to learn to reconstruct the inputs it receives, which consist of arrays of rectangular blobs varying in number and size. The representations the network forms on its second hidden layer in the course of doing this task are then analyzed or used as inputs to a logistic classifier. S&Z demonstrated that the classifier exhibited signatures of Weber’s law and a Weber fraction similar to that exhibited by educated adults. We considered (1) the time course of the developmental progression and (2) factors affecting the network’s tendency to exhibit Weber’s law-like sensitivity to numerosity. We developed a progressive method for training the two-layer deep network. With this learning algorithm, we illustrate a continuous, non-stage-wise developmental trajectory of number discrimination acuity similar to that seen developmentally. Importantly, even before any unsupervised training the network’s representations already produce approximate Weber’s law-like sensitivity, indicating that neither experience nor design for number may be necessary to produce this signature of approximate number representation. We then examined the characteristics of the training set that may contribute to the network’s numerosity sensitivity as this is emerges through learning. The training set used by S&Z was characterized by (i) uniform distribution of number of blobs in the range from 1-32 (ii) uniform distribution of total area covered by blobs (32 to 256 pixels) so that the number of pixels per blob is inversely proportional to the number of blobs (iii) relatively low variability of blob sizes within arrays and (iv) uniform distribution of blobs across the array. We analyzed a corpus of natural images and found (i) the frequency of occurrence of images containing a given number of objects is Zipfian, i.e. frequency is approximately inversely proportional to the number of objects in the image; (ii) object size in the image does fall off with number of blobs; (iii) there is greater variability in object size within an image and (iv) objects tend to be more clustered in natural images. Simulations with synthetic corpora exploring several of these factors revealed several interesting points. First, if S&Z’s corpus is altered so that the mean area per blob is equal for different blob sizes, sensitivity to numerosity difference decreases gradually as numerosity increases, no longer adhering to Weber’s law, indicating that the smaller area per blob in their corpus was a major factor in producing Weber’s law-like numerosity sensitivity. Second, either Zipfian frequency distribution or reduction in blob size can result in a Weber’s law-like sensitivity profile. Third, when the two are combined to be consistent with natural image statistics, sensitivity for large numbers is too degraded. Taken together, the findings begin to enrich our understanding of what factors may be necessary and sufficient to produce the numerosity sensitivity performance profiles exhibited by natural biological systems, and call into question the need to think that Weber-like sensitivity to numerosity requires any special preparation for representation of numerosity per se.