### Acey Deucey

- **Have a standard deck of 52 cards**
  - Ranks of cards: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
  - Three cards drawn (without replacement)
  - What is probability that rank of third card drawn is between the ranks of the first two cards, exclusive?
    - E.g., if ranks of first two cards drawn are 4 and 9, then want probability that third card is a 5, 6, 7 or 8
- **Solution set**
  - Let $X$ = difference between rank of 1st and 2nd card
    - $P(X = 0) = 3/51$
    - After picking first card, there are 3 others with same rank
    - This is not really relevant. Just a warm-up to get you thinking!

### Acey Deucey Solution

- **Solution**
  - $P(X = i) = (13 - i) \cdot \frac{2}{13} \cdot \frac{4}{51}$, where $1 \leq i \leq 12$
    - $(13 - i)$ ways to choose two ranks that differ by $i$
    - First card has $2/13$ chance of being one of those 2 ranks
    - Second card is one of 4 cards (out of 51) that differ in rank by $i$
  - Want: $\sum_{i=1}^{12} P(X = i) P(3\text{rd card between first two}| X = i)$
    - $\sum_{i=1}^{12} \frac{8(13 - i)}{365(364)} \cdot \frac{4(i - 1)}{365(51)}$
    - Of remaining 50 cards, there are 4 cards of each $(i - 1)$ ranks
    - $\sum_{i=1}^{12} \frac{8(13 - i)}{365(364)} \cdot \frac{4(i - 1)}{51}$

### Birthdays Tres Compadres

- **Have a group of 100 people**
  - Let $X$ = number of days of year that are birthdays of exactly 3 people in group
  - **What is $E[X]$?**
    - First, compute probability $p$ that a particular day is the birthday of exactly 3 people in the group
      - Let $A_i$ = number of people that have birthday on day $i$
      - $p = P(A_i = 3) = \left( \frac{100}{365} \right) \left( \frac{1}{365} \right)^3$
    - Let $X_i = 1$ if $A_i = 3$, and 0 otherwise
    - $E[X] = E[\sum_{i=1}^{364} X_i] = \sum_{i=1}^{364} E[X_i] = \sum_{i=1}^{364} P(A_i = 3) = \sum_{i=1}^{364} \frac{100}{365} \left( \frac{1}{365} \right)^3 p = 365 p$

### More Birthdays, More Fun

- **Have a group of 100 people**
  - Let $Y$ = number of distinct birthdays
  - **What is $E[Y]$?**
  - **Solution**
    - Let $Y_i = 1$ if day $i$ is the birthday of at least 1 person, and 0 otherwise
    - $E[Y] = P(Y) = 1 - P(Y') = 1 - \left( \frac{364}{365} \right)^{100}$
    - $E[Y] = E[\sum_{i=1}^{365} Y_i] = \sum_{i=1}^{365} E[Y_i] = 365 \left[ 1 - \left( \frac{364}{365} \right)^{100} \right]$

### MOM Loves the Geometric

- **Consider I.I.D. random variables $X_1, X_2, ..., X_n$**
  - $X_i \sim \text{Geo}(p)$
  - **Estimate $p$ using Method of Moments**
  - **Solution**
    - Recall, for $X_i \sim \text{Geo}(p)$, we know $E[X_i] = 1/p$
    - Rewrite as $p = 1/E[X_i]$
    - Using Method of Moments:
      $$ p = \frac{1}{E[X_1]} = \frac{1}{m_1} = \frac{1}{X} = \frac{1}{n \sum_{i=1}^{n} X_i} = \hat{p} $$