Assignment #1
Due: Start of class on Tuesday, September 8th

1. (5 points) We want to generate a random number between 2 and 12 by rolling two dice and adding their values together. We are given two options to do this: (1) roll two 6-sided dice, or (2) roll one 4-sided die and one 8-sided die. Are both these options equivalent in terms of their probability mass functions? Explain why or why not.

2. (15 point) [Based on Carlton & Devore, Chap. 1, Problem #67] 70% of the aircraft that disappear while in flight are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft that are not discovered do not have such a locator. Suppose an aircraft has disappeared. If the missing aircraft has an emergency locator, what is the probability that it will not be discovered?

3. (20 point) [From Carlton & Devore, Chap. 1, Problem #93] 70% of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities:
   a. P(all of the next three vehicles pass inspection)
   b. P(at least one of the next three vehicles pass inspection)
   c. P(exactly one of the next three vehicles pass inspection)
   d. P(at most one of the next three vehicles pass inspection)
   e. Given that at least one of the next three vehicles passes inspection, what is the conditional probability that all three pass?

4. (15 points) Say the Apple store sells iPhones with three different amounts of storage: 16 GB, 64 GB, and 128 GB. Let X = number of GB of storage in the iPhone purchased by a random customer. Say that X has a probability mass function:

<table>
<thead>
<tr>
<th>x</th>
<th>16 GB</th>
<th>64 GB</th>
<th>128GB</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.2</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

   a. Compute E[X]
   b. Compute E[X^2]
   c. Compute Var(X)
   d. If the price of an iPhone is 5X + 100, what is the expected price paid by a customer for an iPhone?

5. (15 points) Recall the game set-up in the “St. Petersburg’s paradox” discussed in class: there is a fair coin which comes up "heads" with probability p = 0.5. The coin is flipped repeatedly until the first "tails" appears. Let N = number of coin flips before the first "tails" appears (i.e., N = the number of consecutive "heads" that appear). Given that no one really has
infinite money to offer as payoff for the game, consider a variant of the game where you win MIN(2^N, $128). In other words, $128 is the maximum amount that the game provider will pay you after playing (if you get 7 or more “heads” before you get the first “tails”). Compute the expected payoff of the game. Show how you derived your answer.

6. (15 points) An urn contains 5 white balls and 5 black balls. Two balls are drawn randomly (without replacement) from the urn. If they are the same color, you win $2.00. If they are different colors, you lose $1.00 (i.e., you win -$1.00). Let X = the amount you win.
   a. What is E[X]?
   b. What is Var(X)?

7. (15 points) [Based on Kreps, Chap. 15, Problem 15.1]
   Consider the three gambles depicted in Figure 1 below and two decision makers, each of whom chooses among gambles based on maximizing expected utility. For each of these two, the utility function argument is the amount of winnings/losses from the gambles being contemplated.

   (i) Student One, whose utility function for the range of prizes in these gambles is given by the function U(x) = 1 - e^{-0.00001x}, where x is the dollar value of the prize.
   (ii) Student Two, who like Student One has constant risk aversion for this range of prizes but who is more risk averse than Student One. Student Two’s utility function is given by the function U(x) = 1 - e^{-0.00002x}.

   For each of these two students, find the student’s utility for each of the three gambles and determine which is the preferred gamble (A, B, or C) for each student. You may find it helpful to use an Excel spreadsheet or calculator to solve this problem. Note that the Excel function EXP(x) can be used to calculate e^x.