Dynamic Epistemic Logic: Literature


Dynamic Epistemic Logic
Dynamic Epistemic Logic

\[ \mathcal{M} \otimes E \]
Dynamic Epistemic Logic

\[ M \otimes E \]

- Initial epistemic model.
- Abstract description of an epistemic event.
Abstract Description of the Event

Ann looks at the card while Bob is looking away.
Abstract Description of the Event

Ann looks at the card while Bob is looking away.
Product Update
Product Update

\[
\begin{align*}
A, B & \xrightarrow{M} A, B \\
\text{P} & \xrightarrow{A, B} \neg \text{P} \\
\text{s} & \xrightarrow{A, B} \text{t} \\
\end{align*}
\]

\[
\begin{align*}
A, B & \xrightarrow{E} A, B \\
P & \xrightarrow{B} \top \\
e_1 & \xrightarrow{A} e_2 \\
\end{align*}
\]
Product Update

\[
M_{A, B} \rightarrow P \rightarrow \neg P \rightarrow M_{A, B}
\]

\[
E_{A, B} \rightarrow P \rightarrow \neg P \rightarrow E_{A, B}
\]

\[
(s, e_1) \rightarrow P \rightarrow \neg P \rightarrow (t, e_1)
\]

\[
P \rightarrow (s, e_2) \rightarrow \neg P \rightarrow (t, e_2)
\]
Product Update

$A, B \rightarrow M \rightarrow A, B$  
$A, B \rightarrow \neg P \rightarrow t$  
$E \rightarrow A \rightarrow B \rightarrow e_1 \rightarrow P \rightarrow e_2 \rightarrow T$  

$(s, e_1) \rightarrow P$  
$(s, e_2) \rightarrow P$  
$(t, e_1) \rightarrow \neg P$  
$(t, e_2) \rightarrow \neg P$
Product Update

\( \text{M} \) \( \text{A, B} \) \( P \) \( s \) \( \neg P \) \( t \) \( \text{A, B} \) \( \times \) \( \text{E} \) \( \text{A, B} \) \( P \) \( e_1 \) \( e_2 \) \( \top \) \( \text{B} \) \( \text{A} \) \( (s, e_1) \) \( P \) \( B \) \( (s, e_2) \) \( \neg P \) \( (t, e_2) \)
Product Update

\[ M \]

\[ P \]
\[ A, B \]
\[ s \]
\[ \neg P \]
\[ A, B \]
\[ t \]

\[ \otimes \]

\[ E \]
\[ P \]
\[ A \]
\[ e_1 \]
\[ B \]
\[ e_2 \]

\[ \neg P \]
\[ (t, e_2) \]

\[ (s, e_1) \]
\[ P \]
\[ B \]
\[ (s, e_2) \]

\[ \neg P \]
\[ (t, e_2) \]

\[ A, B \]

\[ (s, e_1) \]
\[ P \]
\[ B \]
\[ (s, e_2) \]

\[ \neg P \]
\[ (t, e_2) \]

\[ A, B \]
Let $M = \langle W, R, V \rangle$ be a Kripke model.

An **event model** is a tuple $A = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$. 
Product Update Details

Let $\mathcal{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathcal{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$.

The update model $\mathcal{M} \otimes \mathcal{A} = \langle W', R', V' \rangle$ where
Product Update Details

Let $M = \langle W, R, V \rangle$ be a Kripke model.

An **event model** is a tuple $A = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$.

The **update model** $M \otimes A = \langle W', R', V' \rangle$ where

$W' = \{(w, a) \mid w \models Pre(a)\}$
Product Update Details

Let $\mathcal{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathcal{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$.

The update model $\mathcal{M} \otimes \mathcal{A} = \langle W', R', V' \rangle$ where

- $W' = \{ (w, a) \mid w \models Pre(a) \}$
- $(w, a)R'(w', a')$ iff $wRw'$ and $aSa'$
Product Update Details

Let $\mathcal{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathcal{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$.

The update model $\mathcal{M} \otimes \mathcal{A} = \langle W', R', V' \rangle$ where

- $W' = \{ (w, a) \mid w \models Pre(a) \}$
- $(w, a)R'(w', a')$ iff $wRw'$ and $aSa'$
- $(w, a) \in V'(p)$ iff $w \in V(p)$
Let $\mathcal{M} = \langle W, R, V \rangle$ be a Kripke model.

An **event model** is a tuple $\mathcal{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \to \wp(A)$.

The **update model** $\mathcal{M} \otimes \mathcal{A} = \langle W', R', V' \rangle$ where

- $W' = \{(w, a) \mid w \models Pre(a)\}$
- $(w, a)R'(w', a')$ iff $wRw'$ and $aSa'$
- $(w, a) \in V'(p)$ iff $w \in V(p)$

$\mathcal{M}, w \models [A, a] \varphi$ iff $\mathcal{M}, w \models Pre(a)$ implies $\mathcal{M} \otimes \mathcal{A}, (w, a) \models \varphi$. 
Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.

Is this procedure correct?
Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.

Is this procedure correct? Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob does not know that Ann knows that he knows about the talk.
5. And nothing else.
Dynamic Epistemic Logic

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.
Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.
Dynamic Epistemic Logic

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

Ann knows which event took place.
Dynamic Epistemic Logic

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

Bob thinks a different event took place.
Recall the Ann and Bob example: **Charles tells Bob that the talk is at 2PM.**

That is, Bob learns the time of the talk, but Ann learns nothing.
Dynamic Epistemic Logic
Dynamic Epistemic Logic

$$( \mathcal{M} \otimes E_1 ) \otimes E_2$$
Dynamic Epistemic Logic

The initial model (Ann and Bob are ignorant about $P_{2PM}$).

Private announcement to Ann about the talk.
Product Update
Product Update

\[ \mathcal{M} \otimes E_1 \]

\[ M \otimes E_1 \]

\[ P \quad \neg P \quad s \quad t \]

\[ A, B \quad B \]

\[ E_2 \]

\[ e_1 \quad P \quad e_2 \quad T \]

\[ A \quad B \quad \neg P \]

\[ P \quad A, B \]

\[ e_3 \]

\[ A \quad B \quad \neg P \]
Product Update

\[
\begin{align*}
& (s, e_1) \quad P \\
& (s, e_3) \quad P \\
& P \quad (s, e_2) \\
& \neg P \quad (t, e_3)
\end{align*}
\]
Product Update

\[ P, B, A, B, P \]

\[ s \quad P \quad (s, e_1) \quad P \quad (s, e_2) \quad P \quad \neg P \quad (t, e_3) \]

Logic and Artificial Intelligence 10/33
Product Update

\[(s, e_1) \models \neg K_B K_A K_B P\]  

\[(s, e_1) \models P\]  

\[P \models (s, e_2)\]  

\[(s, e_3) \models P\]  

\[\neg P \models (t, e_3)\]
Product Update

\[(s, e_1) \models \neg K_B K_A K_B P\]

\[(s, e_1) \quad B \quad (s, e_2)\]

\[(s, e_3) \quad P \quad \neg P \quad (t, e_3)\]
Product Update

\[(s, e_1) \models \neg K_B K_A K_B P\]

\[(s, e_1) \quad P \quad B \quad P \quad (s, e_2)\]

\[(s, e_3) \quad P \quad A \quad (t, e_3) \quad \neg P\]
Product Update

\[(s, e_1) \models \neg K_B K_A K_B P\]
1. The agents’ *observational* powers.

2. The *type* of change triggered by the event.

3. The underlying *protocol* specifying which events (observations, messages, actions) are available (or permitted) at any given moment.
1. The agents’ *observational* powers.

2. The *type* of change triggered by the event.

3. The underlying *protocol* specifying which events (observations, messages, actions) are available (or permitted) at any given moment.
A recurring issue in any formal model representing agents’ (changing) informational attitudes is how to account for the fact that the agents are limited in their access to the available inference steps, possible observations and available messages.
A recurring issue in any formal model representing agents’ (changing) informational attitudes is how to account for the fact that the agents are limited in their access to the available inference steps, possible observations and available messages.

1. This may be because the agents are not logically omniscient and so do not have unlimited reasoning ability.
A recurring issue in any formal model representing agents’ (changing) informational attitudes is how to account for the fact that the agents are limited in their access to the available inference steps, possible observations and available messages.

1. This may be because the agents are not logically omniscient and so do not have unlimited reasoning ability.

2. But it can also be because the agents are following a predefined protocol that explicitly or implicitly limits statements available for observation and/or communication.
A recurring issue in any formal model representing agents’ (changing) informational attitudes is how to account for the fact that the agents are limited in their access to the available inference steps, possible observations and available messages.

1. This may be because the agents are not logically omniscient and so do not have unlimited reasoning ability.

✓ But it can also be because the agents are following a predefined protocol that explicitly or implicitly limits statements available for observation and/or communication.
Epistemic Temporal Logic


The ‘Playground’

\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]
The ‘Playground’

t = 0

t = 1

t = 2

t = 3
The ‘Playground’

$t = 0$

$t = 1$

$t = 2$

$t = 3$
Formal Languages

- $P\varphi$ ($\varphi$ is true sometime in the past),
- $F\varphi$ ($\varphi$ is true sometime in the future),
- $Y\varphi$ ($\varphi$ is true at the previous moment),
- $N\varphi$ ($\varphi$ is true at the next moment),
- $N_e\varphi$ ($\varphi$ is true after event $e$),
- $K_i\varphi$ (agent $i$ knows $\varphi$) and
- $C_B\varphi$ (the group $B \subseteq A$ commonly knows $\varphi$).
An ETL model is a structure $\langle H, \{\sim_i\}_{i \in A}, V \rangle$ where $\langle H, \{\sim_i\}_{i \in A} \rangle$ is an ETL frame and $V : At \rightarrow 2^{\text{finite}(H)}$ is a valuation function.

Formulas are interpreted at pairs $H, t$:

$$H, t \models \varphi$$
Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t + 1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t - 1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$. 
Truth in a Model

- \( H, t \models P\varphi \iff \text{there exists } t' \leq t \text{ such that } H, t' \models \varphi \)
- \( H, t \models F\varphi \iff \text{there exists } t' \geq t \text{ such that } H, t' \models \varphi \)
- \( H, t \models N\varphi \iff H, t + 1 \models \varphi \)
- \( H, t \models Y\varphi \iff t > 1 \text{ and } H, t - 1 \models \varphi \)
- \( H, t \models K_i\varphi \iff \text{for each } H' \in \mathcal{H} \text{ and } m \geq 0 \text{ if } H_t \sim_i H'_m \text{ then } H', m \models \varphi \)
- \( H, t \models C\varphi \iff \text{for each } H' \in \mathcal{H} \text{ and } m \geq 0 \text{ if } H_t \sim_* H'_m \text{ then } H', m \models \varphi \).

where \( \sim_* \) is the reflexive transitive closure of the union of the \( \sim_i \).
Returning to the Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.
Returning to the Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.
Returning to the Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.

Is this procedure correct?
\( t = 0 \quad \Rightarrow \quad t = 1 \quad \Rightarrow \quad t = 2 \quad \Rightarrow \quad t = 3 \)

\[ m_{A \rightarrow C} \]

\[ m_{C \rightarrow B} \]

\[ m_{2PM} \]

\[ m_{3PM} \]

\[ H, 3 \models \varphi \]
Bob’s uncertainty: $H, 3 \models \neg K_B P_{2PM}$
Bob’s uncertainty + ‘Protocol information’: $H, 3 \models K_B P_{2PM}$
Bob’s uncertainty + ‘Protocol information’:
\[ H, 3 \models \neg K_B K_A K_B P_{2PM} \]
Bob’s uncertainty + ‘Protocol information’:
\[ H, 3 \models \neg K_B K_A K_B P_{2PM} \]
Bob’s uncertainty + ‘Protocol information’:
\[ H, 3 \models \neg K_B K_A K_B P_{2PM} \]
Bob’s uncertainty + ‘Protocol information’:

\[ H, 3 \models -K_B K_A K_B P_{2PM} \]
Parameters of the Logical Framework

1. Expressivity of the formal language. Does the language include a common knowledge operator? A future operator? Both?

2. Structural conditions on the underlying event structure. Do we restrict to protocol frames (finitely branching trees)? Finitely branching forests? Or, arbitrary ETL frames?

3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?
Parameters of the Logical Framework

1. **Expressivity of the formal language.** Does the language include a common knowledge operator? A future operator? Both?
Parameters of the Logical Framework

1. **Expressivity of the formal language.** Does the language include a common knowledge operator? A future operator? Both?

2. **Structural conditions on the underlying event structure.** Do we restrict to protocol frames (finitely branching trees)? Finitely branching forests? Or, arbitrary ETL frames?
Parameters of the Logical Framework

1. **Expressivity of the formal language.** Does the language include a common knowledge operator? A future operator? Both?

2. **Structural conditions on the underlying event structure.** Do we restrict to protocol frames (finitely branching trees)? Finitely branching forests? Or, arbitrary ETL frames?

3. **Conditions on the reasoning abilities of the agents.** Do the agents satisfy perfect recall? No miracles? Do they agents’ know what time it is?
Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.

- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.

- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\text{len}(H) = \text{len}(H')$. 
Perfect Recall

$t = 0$

$e_2$ $e_4$

$t = 1$

$e_1$ $e_5$ $e_2$ $e_3$

$t = 2$

$e_1$ $e_3$ $e_7$ $e_6$

$t = 3$

$e_1$ $e_3$
Perfect Recall

t = 0

t = 1

t = 2

t = 3

$e_1 \ e_5 \ e_2 \ e_3$

$e_2 \ e_4 \ e_3 \ e_6$

$e_1 \ e_2 \ e_7$

$e_1 \ e_3$
Perfect Recall

$t = 0$

$t = 1$

$t = 2$

$t = 3$
No Miracles

t = 0

e_2

e_4

t = 1

e_1

e_5

e_2

e_3

e_4

e_2

e_1

e_2

t = 2

e_1

e_3

e_7

e_6

e_2

e_1

e_3

t = 3
No Miracles

$t = 0$

$t = 1$

$t = 2$

$t = 3$
No Miracles

$t = 0$

$t = 1$

$t = 2$

$t = 3$
Ideal Agents

Assume there are two agents

Theorem
The logic of ideal agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).


Constrained Public Announcement

1. $A \rightarrow \langle A \rangle^T$ vs. $\langle A \rangle^T \rightarrow A$
Constrained Public Announcement

1. $A \rightarrow \langle A \rangle^T$ vs. $\langle A \rangle^T \rightarrow A$

2. $\langle A \rangle K_i P \iff A \land K_i \langle A \rangle P$
Constrained Public Announcement

1. $A \rightarrow \langle A \rangle^\top$ vs. $\langle A \rangle^\top \rightarrow A$

2. $\langle A \rangle K_i P \leftrightarrow A \land K_i \langle A \rangle P$

3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle^\top \land K_i (A \rightarrow \langle A \rangle P)$
Constrained Public Announcement

1. \( A \rightarrow \langle A \rangle^\top \) vs. \( \langle A \rangle^\top \rightarrow A \)

2. \( \langle A \rangle K_i P \leftrightarrow A \land K_i \langle A \rangle P \)

3. \( \langle A \rangle K_i P \leftrightarrow \langle A \rangle^\top \land K_i (A \rightarrow \langle A \rangle P) \)

4. \( \langle A \rangle K_i P \leftrightarrow \langle A \rangle^\top \land K_i (\langle A \rangle^\top \rightarrow \langle A \rangle P) \)
1. The agents’ *observational* powers.

2. The *type* of change triggered by the event.

3. The underlying *protocol* specifying which events (observations, messages, actions) are available (or permitted) at any given moment.
1. The agents’ observational powers.

2. The type of change triggered by the event.

3. The underlying protocol specifying which events (observations, messages, actions) are available (or permitted) at any given moment.
Agents may differ in precisely how they incorporate new information into their epistemic states. These differences are based, in part, on the agents’ perception of the source of the information. For example, an agent may consider a particular source of information *infallible* (not allowing for the possibility that the source is mistaken) or merely *trustworthy* (accepting the information as reliable, though allowing for the possibility of a mistake).
Informative Actions

Public Announcement: Information from an infallible source

Conservative Upgrade: Information from a trusted source

Radical Upgrade: Information from a strongly trusted source
Informative Actions

Incorporate the new information $\varphi$
Informative Actions

Public Announcement: Information from an infallible source (!φ): A ⪯i B
Informative Actions

Public Announcement: Information from an infallible source
(!φ): A ≺; B

Conservative Upgrade: Information from a trusted source
(↑φ): A ≺; C ≺; D ≺; B ∪ E
Informative Actions

Public Announcement: Information from an infallible source (!\(\phi\)): \(A \prec_i B\)

Conservative Upgrade: Information from a trusted source (\(\uparrow\phi\)): \(A \prec_i C \prec_i D \prec_i B \cup E\)

Radical Upgrade: Information from a strongly trusted source (\(\uparrow\uparrow\phi\)): \(A \prec_i B \prec_i C \prec_i D \prec_i E\)