1. Prove that the following axiom of ceteris paribus logic is valid (see slide 22 of lecture 21 on 11/16):

\[(\alpha \land (\Gamma) \leq (\alpha \land \varphi)) \rightarrow (\Gamma \cup \{\alpha\}) \leq \varphi\]

2. Let \(X, Y\) be subsets of \(W\) and suppose that \(\leq\) is a reflexive, connected and transitive order over \(W\). Say \(X \leq_{\forall\forall} Y\) provided for all \(x \in X\) and for all \(y \in Y\), we have \(x \leq y\). Assume that \(\leq\) is reflexive, transitive and complete, is \(\leq_{\forall\forall}\) also reflexive, transitive, and complete? If so, prove it and if not, give a counterexample. Can you think of any other interesting principles that \(\leq_{\forall\forall}\) satisfies?

3. Recall the model of knowledge and preference from Lecture 22 (on 11/21): \(\mathcal{M} = \langle W, \sim, \preceq, V \rangle\) where \(\sim\) is an equivalence relation and \(\preceq\) is a reflexive, transitive and total preference relation. Truth is defined as follows:

- \(\mathcal{M}, w \models K\varphi\) iff for all \(v \in W\), if \(w \sim v\) then \(\mathcal{M}, v \models \varphi\)
- \(\mathcal{M}, w \models (\preceq)\varphi\) iff there is a \(v \in W\) with \(w \preceq v\) and \(\mathcal{M}, v \models \varphi\)
- \(\mathcal{M}, w \models A\varphi\) iff for all \(v \in W\), \(\mathcal{M}, v \models \varphi\)
- \(\mathcal{M}, w \models (\sim \cap \preceq)\varphi\) iff there is a \(v \in W\) such that \(w \sim v\) and \(w \preceq v\) with \(\mathcal{M}, v \models \varphi\)

Given an example to show that \(K(\psi \rightarrow (\preceq)\varphi)\) and \(K(\psi \rightarrow (\sim \cap \preceq)\varphi)\) are not equivalent (i.e., find a model and state where one of the formulas is true, but the other is not true). It is easy to see that \(A(\psi \rightarrow (\preceq)\varphi) \rightarrow K(\psi \rightarrow (\preceq)\varphi)\) is valid (this is an instance of the validity \(A\varphi \rightarrow K\varphi\), but what is the relationship between \(A(\psi \rightarrow (\preceq)\varphi)\) and \(K(\psi \rightarrow (\sim \cap \preceq)\varphi)\) (does one imply the other or are the two formulas independent)?

The homework is DUE Monday, December 5th (put your answers in my mailbox).