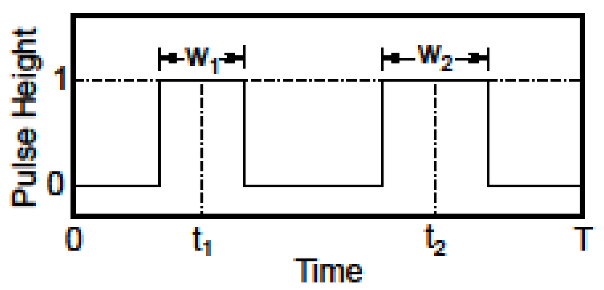
Feasibility of FRI-based square-wave reconstruction with quantization error and integrator noise

Bryan He*‡ Alexander Wein†‡ Lakshminarayan Srinivasan‡

*California Institute of Technology †Massachusetts Institute of Technology ‡Stanford University

Introduction

Conventional Nyquist sampling and reconstruction of square waves at a finite rate will always result in aliasing because square waves are not band limited. Based on methods for signals with finite rate of innovation (FRI), generalized Analog Thresholding (gAT-n) is able to sample square waves at a much lower rate under ideal conditions. The target application is efficient, real-time, implantable neurotechnology that extracts spiking neural signals from the brain.



An example signal for the sampling interval from $t = 0$ to $t = T$. This signals contains two square waves. The first pulse has a duration of w_1 and is centered at time t_1, and the second pulse has a duration of w_2 and is centered at time t_2.

We study the effect of integrator noise and quantization error on the accuracy of reconstructed square waves.

Generalized Analog Thresholding

Generalized Analog Thresholding (gAT) is one emerging class of method for sampling and reconstructing square waves. Methods from the gAT class are denoted gAT-n, where integer $n \geq 1$ is the maximum number of square wave pulses within the analysis interval. gAT uses an analog preprocessing step to allow slower sampling rates. The analog preprocessing stage for gAT-n computes the first $2n$ repeated integrals of the signals, evaluated from the start to the end of the sampling interval. The continuous-time behavior of this stage is evaluated with exact integration, rather than by numerical integration in discrete time. Under noise-free conditions, the gAT samples are denoted $y_1, \ldots, y_{2n}$, where

$$x_1(t) = \int_0^t x(\tau)\, d\tau$$

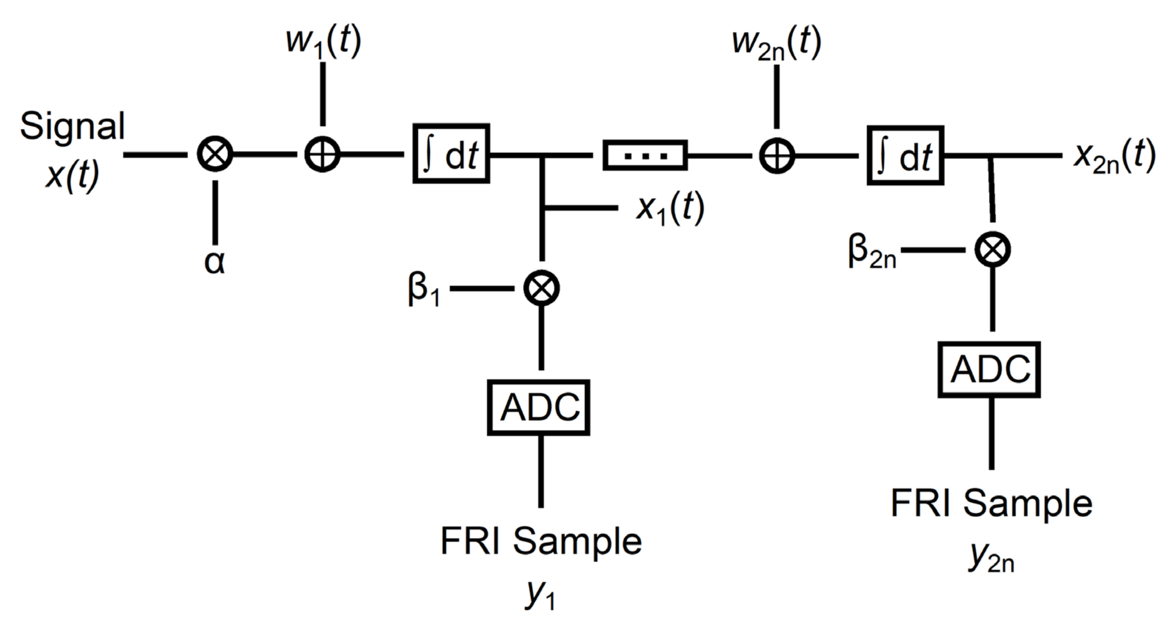
$$x_{k+1}(t) = \int_0^t x_k(\tau)\, d\tau$$

$$y_k = x_k(T)$$

The y_k are then quantized through an ADC to allow reconstruction of pulse times $t_1, \ldots, t_n$ and pulse widths $w_1, \ldots, w_n$ on a digital system.

Simulation Model

The full gAT sampling stage including integrator noise and quantization error is shown below. First, the α scales the continuous-time square-wave input $x(t)$. Next, a serial bank of noisy integrators introduces $w_1(t), \ldots, w_{2n}(t)$, which are independent, identically-distributed zero-mean continuous-time white Gaussian noise processes. Integrator outputs are scaled by β_i to maximally utilize the dynamic range of the ADCs. The ADC-sampled values $y_1, \ldots, y_{2n}$ are passed to the gAT reconstruction stage.



gAT sampling stage, modeled with integrator noise and quantization error.

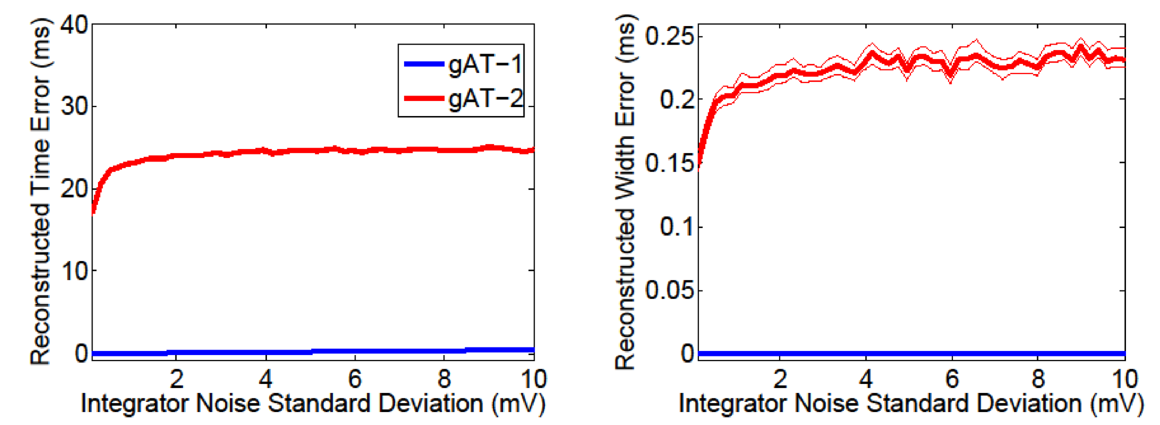
After integration, gAT samples are quantized through an ADC, and our model for the ADC includes the quantization error.

A corresponding noise model for analog integration consists of continuous-time white Gaussian noise $w_1(t), \ldots, w_{2n}(t)$ added to the input of each stage in a sequence of ideal integrators.

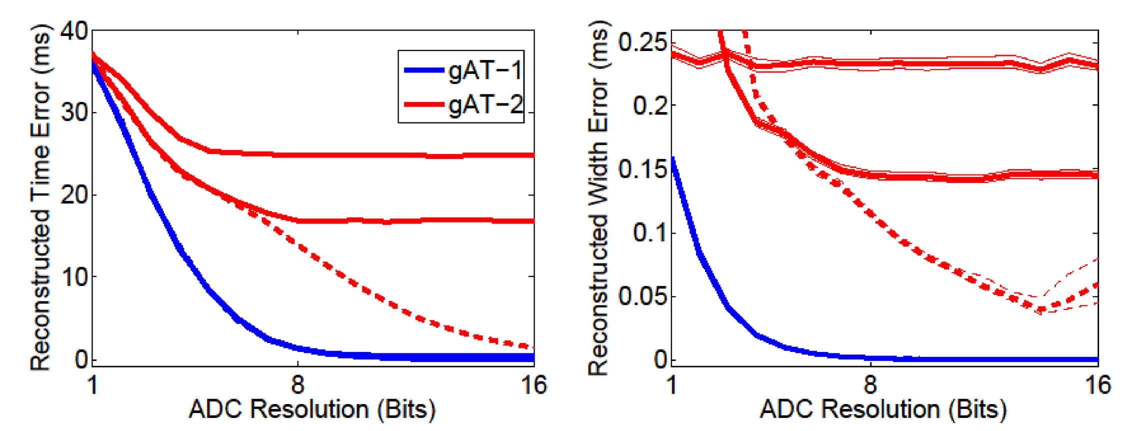
We explore realistic values for integrator noise and input signal amplitude, using specifications from the Texas Instruments IVC102 integrator chip as a first-pass example because of its readily-available data sheet.

Results

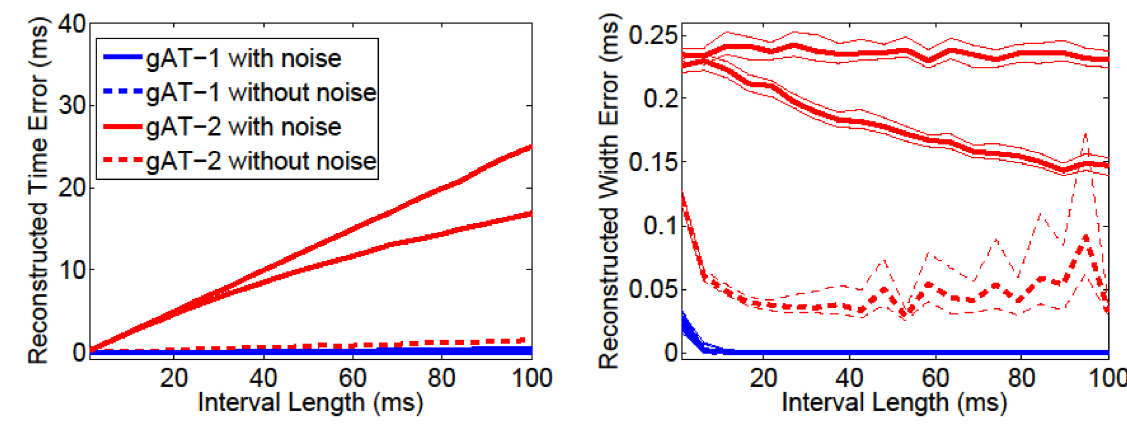
We examined gAT performance on the basis of mean unsigned error in the reconstructed pulse time and width. In the following plots, the solid lines show the expected error for a low level of integrator noise ($\sigma = 0.1\ mV$) and a high level of integrator noise ($\sigma = 10\ mV$) based on the IVC102. The dashed lines show performance with no integrator noise ($\sigma = 0\ mV$). The confidence intervals (95%) are based on 100 bootstrapped averages from 10,000 samples.



First, we assessed performance versus integrator noise for a high resolution (16-bit) ADC and 100 ms sampling interval. Errors grow with integrator noise for both methods as the noise level increases, but much more rapidly for gAT-2 versus gAT-1. This is likely because gAT-2 depends on higher order integrals that accumulate larger variability than lower order integrals in the presence of integrator noise. The gAT-2 performance degredation asymptotes because pulse time and width reconstruction errors are limited by the values represented by the ADC and the length of the sampling interval.



We then determined performance versus ADC bit resolution for a 100 ms sampling interval, based on the IVC102 integrator. As the number of bits used by the ADC increases, the error decreases because the values received from the ADC become more accurate. In general, gAT-2 has larger errors than gAT-1, suggesting its reconstruction process is more sensitive to errors in the samples. The greatest benefit in ADC resolution is achieved within 8 bits. For gAT-2 curves, unusual behavior is seen at resolution below 3 bits, which likely relates to the relationship between reconstruction errors and the specific conditions of our performance testing.



Finally, we examined performance versus sampling interval at varying levels of integrator noise using a 16-bit ADC. As the sampling interval increases in duration, the error in reconstructed time increases, due to accumulated integrator noise. Paradoxically, the reconstructed width error decreases with increasing sampling interval. The reason for this is not immediately clear.

Conclusion

Our simulated analysis examines the feasibility of gAT, a class of FRI-based sampling and reconstruction methods for square-wave signals, under the hardware-induced non-idealities of integrator noise and quantization error. Under ideal conditions, FRI-based gAT methods are capable of reconstructing signals precisely. By simulating these hardware non-idealities, we show that gAT-1 reconstruction could be robust to these hardware non-idealities, where gAT-2 reconstruction is expected to be significantly more brittle in terms of reliably estimating pulse time and width. The brittle performance of gAT-2 is likely in part because gAT-2 reconstruction method requires higher order integrals that accumulate noise from lower-order integration stages. Based on these results, gAT-1 appears to be the preferred candidate over gAT-2 for initial efforts to develop FRI sampling hardware for square-wave signals.

Acknowledgements

The authors would like to thank Julius Kusuma (Schlumberger-Doll Research), Lav Varshney (Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign), and Juhwan Yoo (Broadcom) for discussions on FRI and hardware implementation. B.D.H. and A.W. were supported by the Caltech Summer Undergraduate Research Fellowship. L.S. was supported by the American Heart Association Scientist Development Grant (11SDG7550015).