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Dynamics of large networks

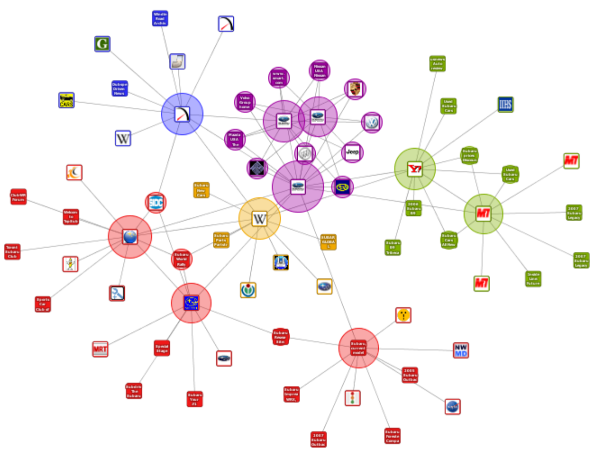
Thesis defense, September 3 2008

Web: Rich data

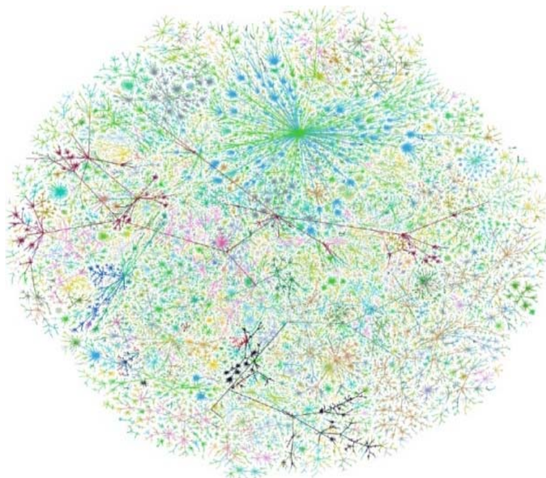
- **Today:** Large on-line systems have detailed records of human activity
  - On-line communities:
    - Facebook (64 million users, billion dollar business)
    - MySpace (300 million users)
  - Communication:
    - Instant Messenger (~1 billion users)
  - News and Social media:
    - Blogging (250 million blogs world-wide, presidential candidates run blogs)
  - On-line worlds:
    - World of Warcraft (internal economy 1 billion USD)
    - Second Life (GDP of 700 million USD in '07)

Can study phenomena and behaviors at scales that before were never possible

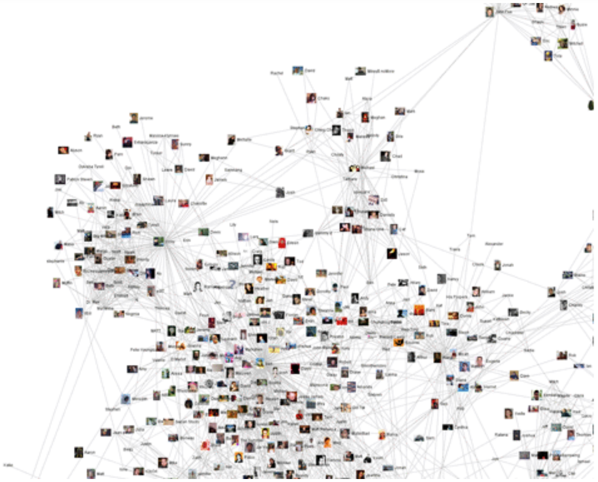
Rich data: Networks



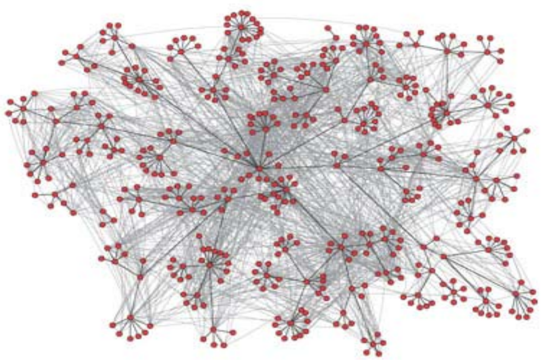
a) World wide web



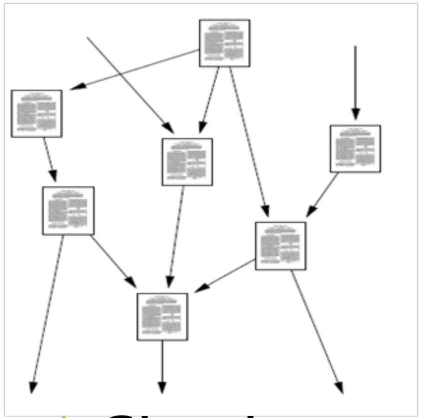
b) Internet (AS)



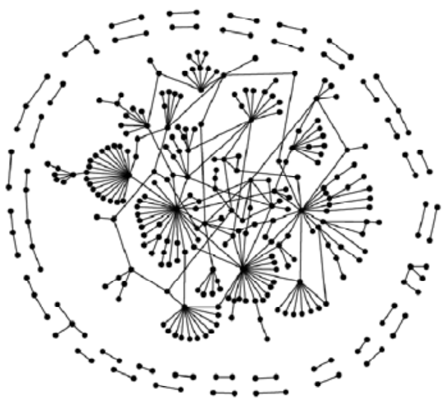
c) Social networks



d) Communication



e) Citations



f) Biological networks

Networks: What do we know?

- We know lots about the network structure:
  - Properties: Scale free [Barabasi '99], Clustering [Watts-Strogatz '98], Navigation [Adamic-Adar '03, LibenNowell '05], Bipartite cores [Kumar et al. '99], Network motifs

We know much less about **processes** and **dynamics** of **networks**

  - Models: Preferential attachment [Barabasi '99], Small-world [Watts-Strogatz '98], Copying model [Kleinberg el al. '99], Heuristically optimized tradeoffs [Fabrikant et al. '02], Congestion [Mihail et al. '03], Searchability [Kleinberg '00], Bowtie [Broder et al. '00], Transit-stub [Zegura '97], Jellyfish [Tauro et al. '01]

This thesis: Network dynamics

- Network evolution
  - How network structure changes as the network grows and evolves?
- Diffusion and cascading behavior
  - How do rumors and diseases spread over networks?
- Large data
  - Observe phenomena that is “invisible” at smaller scales

Data size matters

- We need massive network data for the patterns to emerge
  - MSN Messenger network [WWW '08] (the largest social network ever analyzed)
    - 240M people, 255B messages, 4.5 TB data
  - Product recommendations [EC '06]
    - 4M people, 16M recommendations
  - Blogosphere [in progress]
    - 164M posts, 127M links

This thesis: The structure

	Network Evolution	Network Cascades	Large Data
Observations	**Q1:** How does network structure evolve over time?	**Q4:** What are patterns of diffusion in networks?	**Q7:** What are the properties of a social network of the whole planet?
Models	**Q2:** How to model individual edge attachment?	**Q5:** How do we model influence propagation?	**Q8:** What is community structure of large networks?
Algorithms (applications)	**Q3:** How to generate realistic looking networks?	**Q6:** How to identify influential nodes and epidemics?	**Q9:** How to predict search result quality from the web graph?

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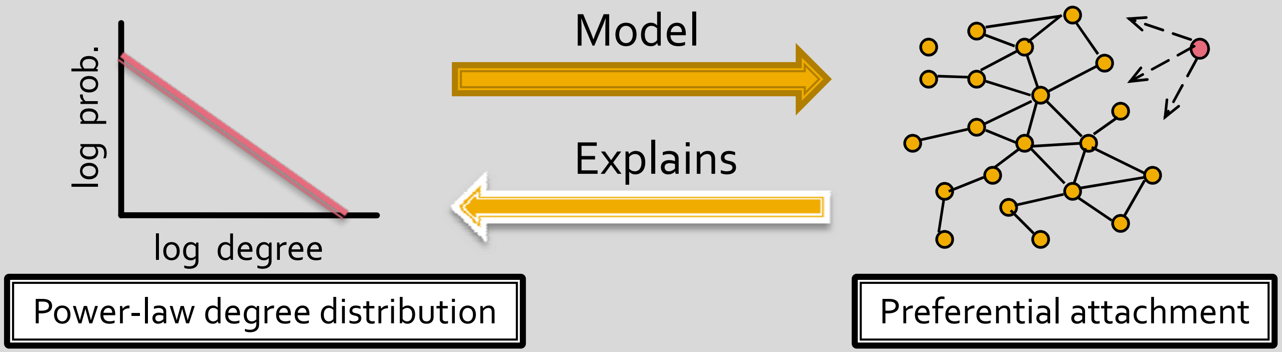
Background: Network models

- Empirical findings on real graphs led to new network models



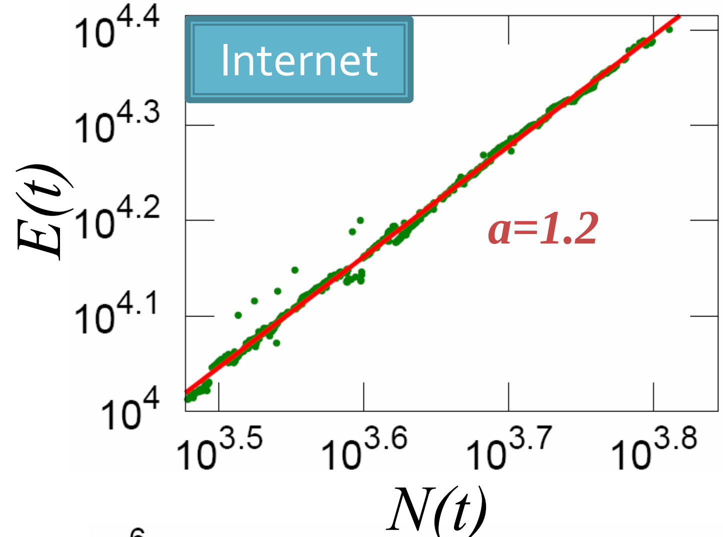
- Such models make assumptions/predictions about other network properties
- What about network evolution?

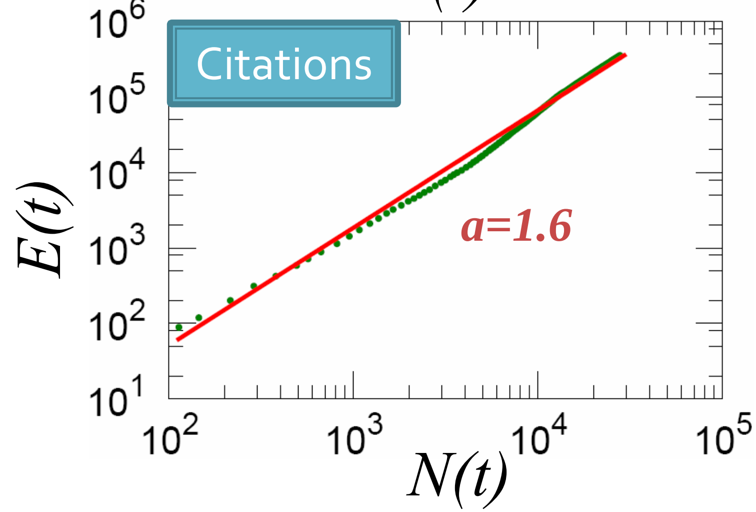
Q1) Network evolution

- What is the relation between the number of nodes and the edges over time?
- ~~Prior work assumes: constant average degree over time~~
- Networks are denser over time
- **Densification Power Law:**

$$E(t) \propto N(t)^a$$

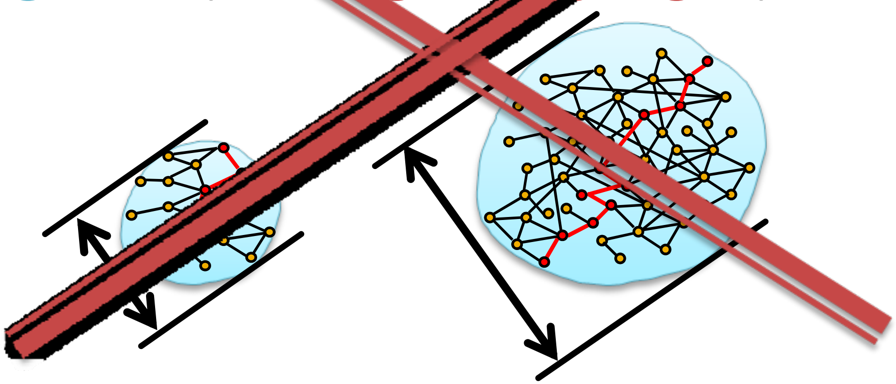
a … densification exponent (1 ≤ a ≤ 2)



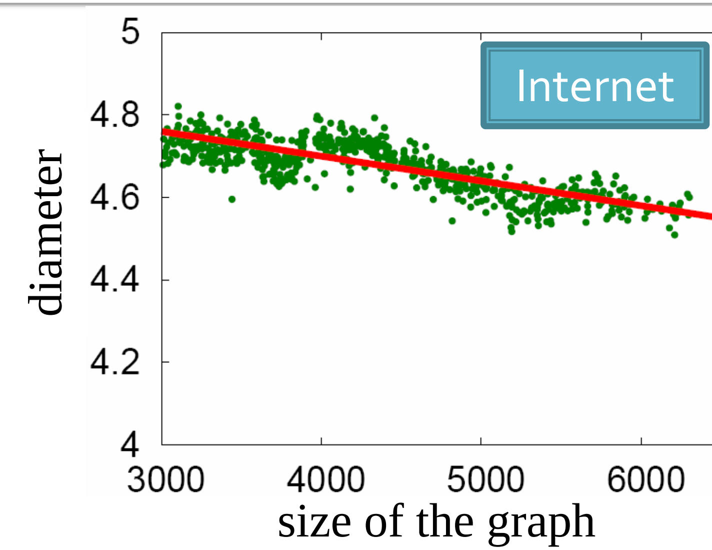


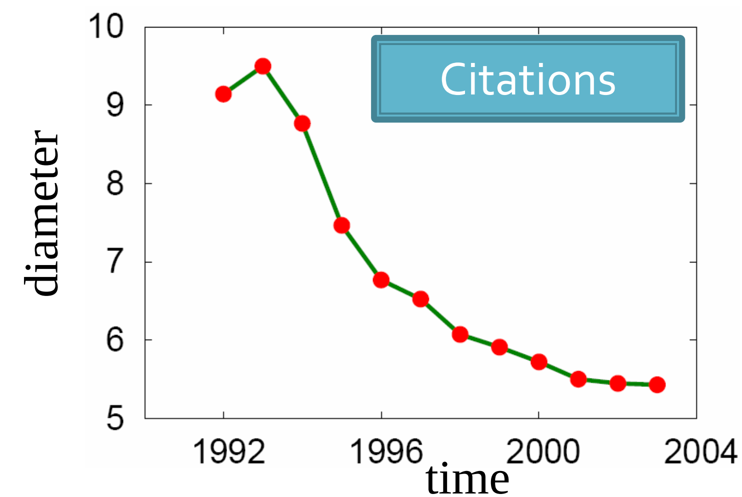
Q1) Network evolution

- ~~Prior models and intuition say that the network diameter slowly grows (like log N, log log N)~~



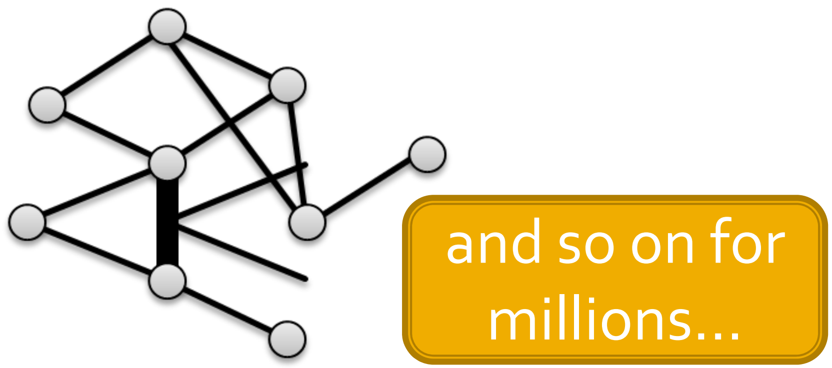
- **Diameter shrinks over time**
  - as the network grows the distances between the nodes slowly **decrease**





Q2) Modeling edge attachment

- We directly observe atomic events of network evolution (and not only network snapshots)



We can model evolution at finest scale

- Test individual edge attachment
  - Directly observe events leading to network properties
- Compare network models by likelihood (and not by just summary network statistics)

[w/ Backstrom-Kumar-Tomkins, KDD ’08]

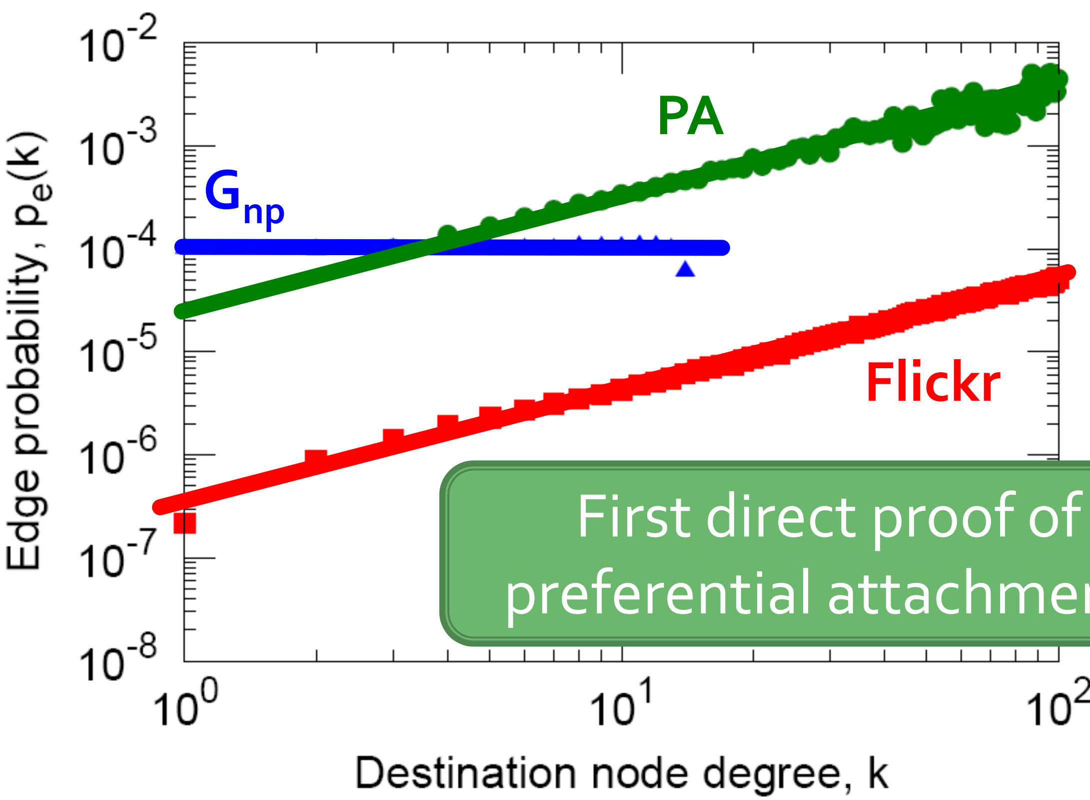
Setting: Edge-by-edge evolution

- Network datasets
  - Full temporal information from the first edge onwards
  - LinkedIn (N=7m, E=30m), Flickr (N=600k, E=3m), Delicious (N=200k, E=430k), Answers (N=600k, E=2m)
- We model 3 processes governing the evolution
  - P1) Node arrival: node enters the network
  - P2) Edge initiation: node wakes up, initiates an edge, goes to sleep
  - P3) Edge destination: where to attach a new edge
    - Are edges more likely to attach to high degree nodes?
    - Are edges more likely to attach to nodes that are close?

[w/ Backstrom-Kumar-Tomkins, KDD ’08]

Edge attachment degree bias

- Are edges more likely to connect to higher degree nodes?



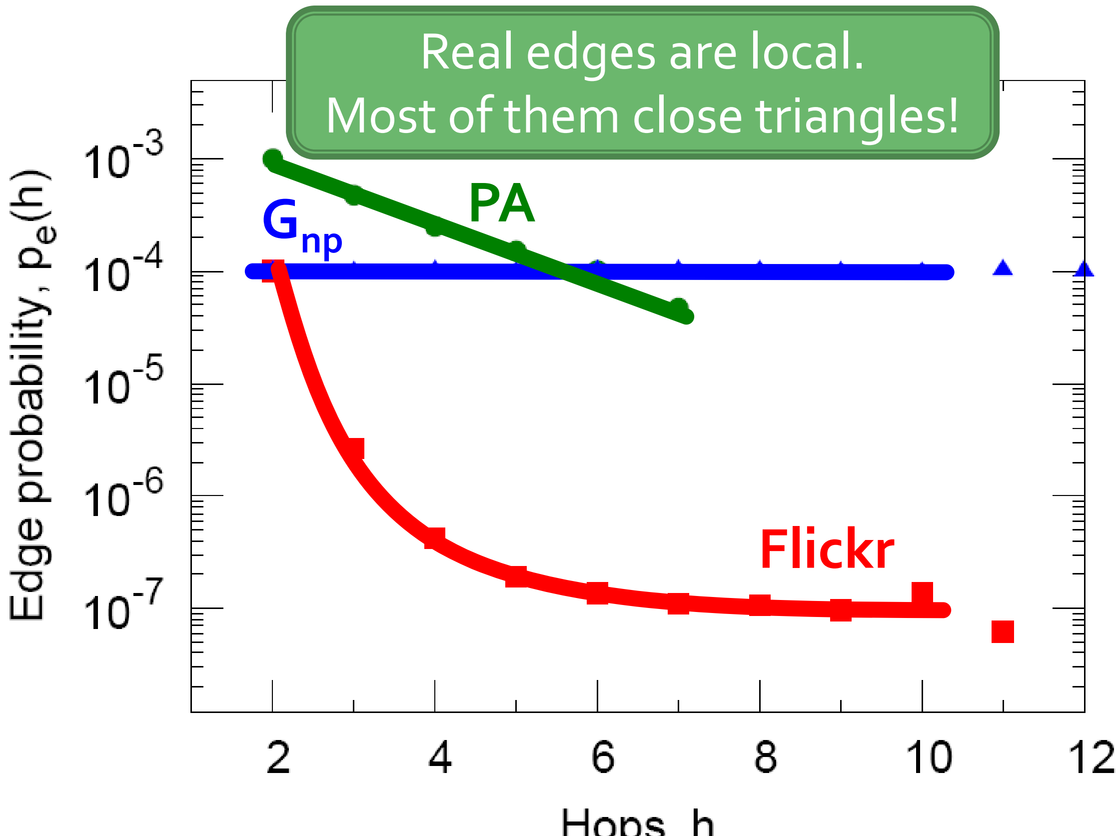
First direct proof of preferential attachment!

$$p_e(k) \propto k^{\tau}$$

Network	τ
G_{np}	0
PA	1
Flickr	1
Delicious	1
Answers	0.9
LinkedIn	0.6

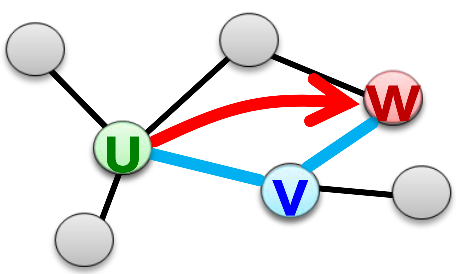
But, edges also attach locally

- Just before the edge *(u,w)* is placed how many hops is between *u* and *w*?



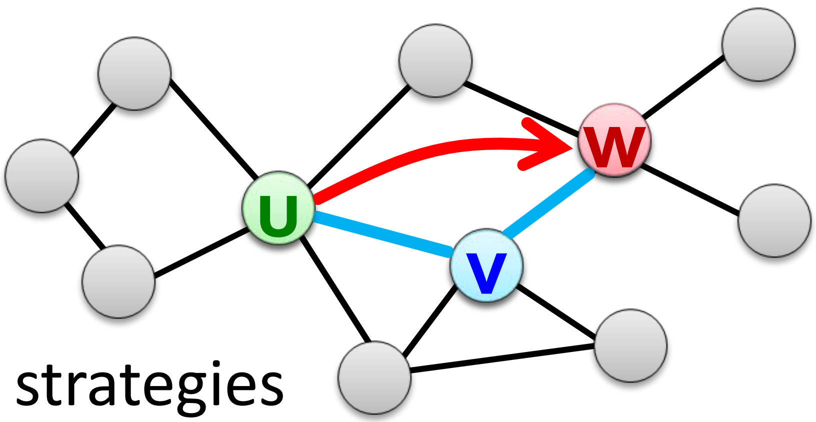
Fraction of triad closing edges

Network	% Δ
Flickr	66%
Delicious	28%
Answers	23%
LinkedIn	50%



How to best close a triangle?

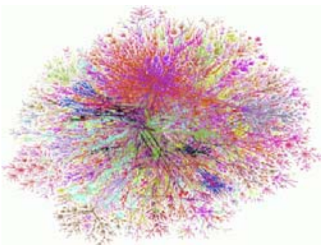
- New triad-closing edge *(u,w)* appears next
- We model this as:
  1. *u* chooses neighbor *v*
  2. v chooses neighbor *w*
  3. Connect *(u,w)*
  - We consider 25 triad closing strategies
  - and compute their log-likelihood



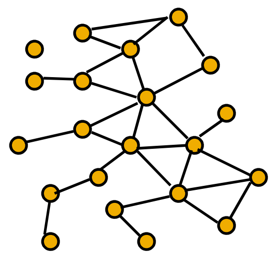
- Triad closing is best explained by
  - choosing a node based on the number of common friends and time since last activity
  - (just choosing random neighbor also works well)

Q3) Generating realistic graphs

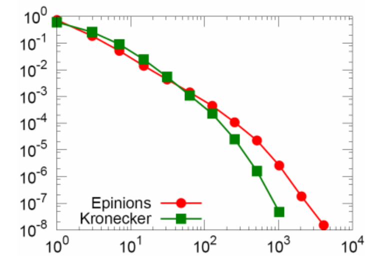
Problem: generate a realistic looking synthetic network



Given a real network



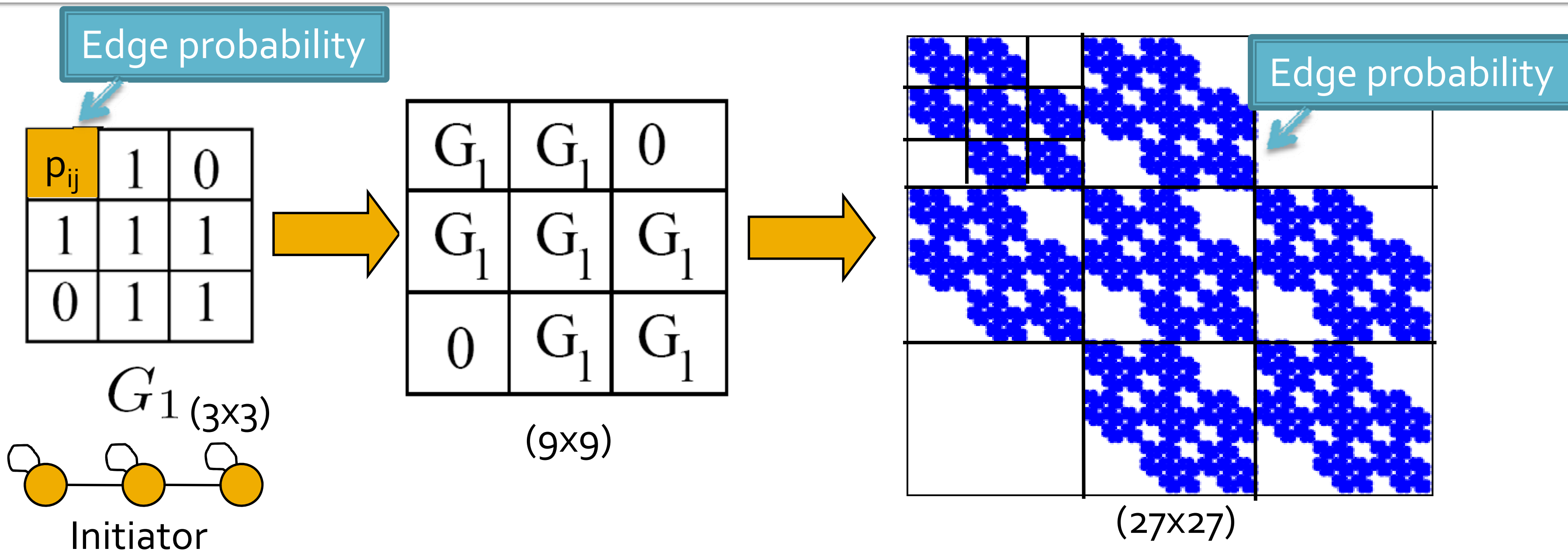
Generate a synthetic network



Compare graph properties, *e.g.*, degree distribution

- Why synthetic graphs?
  - Anomaly detection, Simulations, Predictions, Null-model, Sharing privacy sensitive graphs, …
- **Q:** Which network properties do we care about?
- **Q:** What is a good model and how do we **fit** it?

Q3) The model: Kronecker graphs

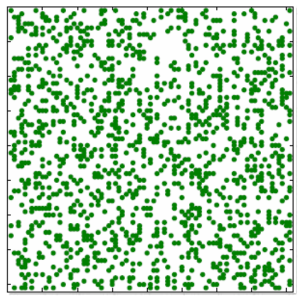
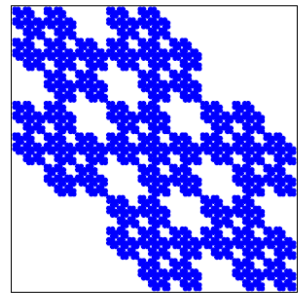


Kronecker product of graph adjacency matrices

- We prove Kronecker graphs mimic real graphs:
  - Power-law degree distribution, Densification, Shrinking/stabilizing diameter, Spectral properties

Q5) Kronecker graphs: Estimation

- Maximum likelihood estimation

$\arg\max_{\Theta} P($  $|$  $\leftarrow$ Kronecker $\Theta)$

$$\Theta = \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$$

- Naïve estimation takes *$O(N!N^2)$*:
  - *$N!$* for different node labelings:
    - **Our solution:** Metropolis sampling: *$N!$* → ***(big) const***
  - *N^2* for traversing graph adjacency matrix
    - **Our solution:** Kronecker product ($E << N^2$): *N^2* → ***E***
- Do stochastic gradient descent

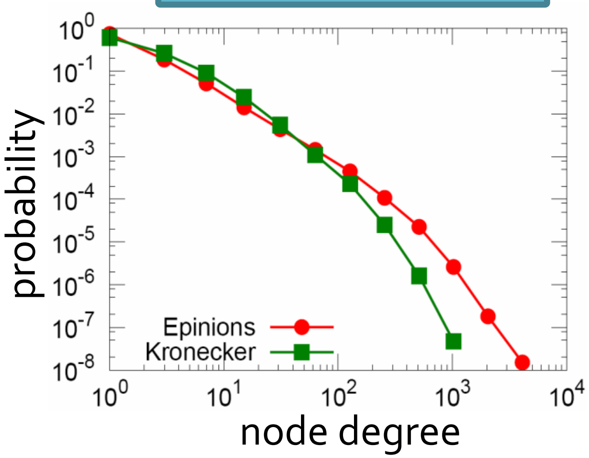
We estimate the model in O(E)

Estimation: Epinions (N=76k, E=510k)

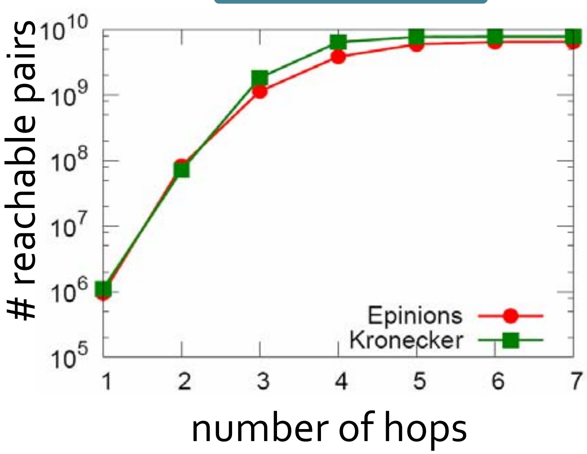
- We search the space of ~$10^{1,000,000}$ permutations
- Fitting takes 2 hours
- Real and Kronecker are very close

$$\hat{\Theta} = \begin{array}{|c|c|}\hline 0.99 & 0.54 \\ \hline 0.49 & 0.13 \\ \hline\end{array}$$

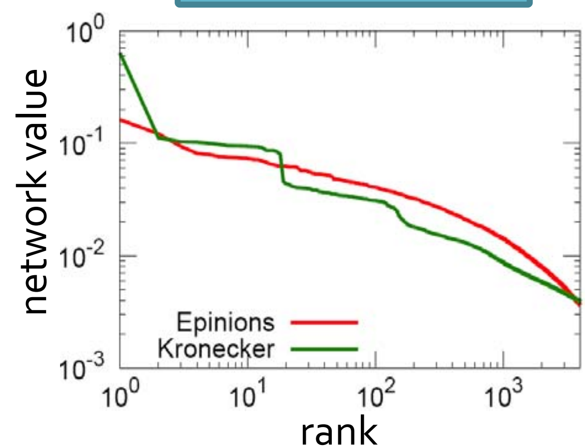
Degree distribution



Path lengths



"Network" values



Thesis: The structure

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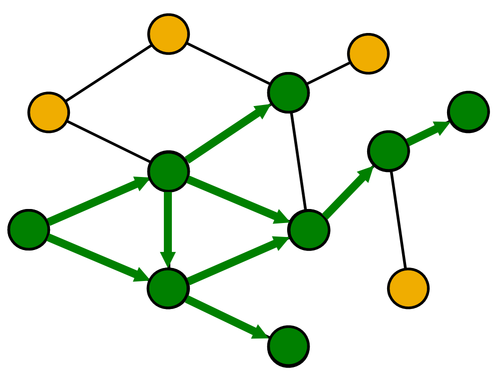
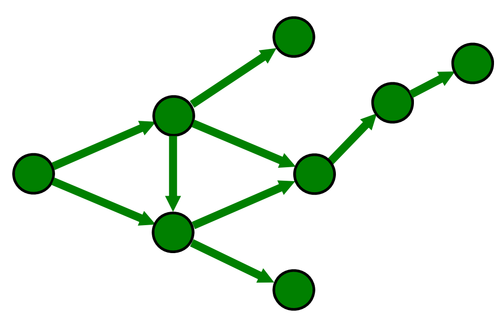
Part 2: Diffusion and Cascades

- Behavior that cascades from node to node like an epidemic
  - News, opinions, rumors
  - Word-of-mouth in marketing
  - Infectious diseases
- As activations spread through the network they leave a trace – **a cascade**

We observe cascading behavior in large networks



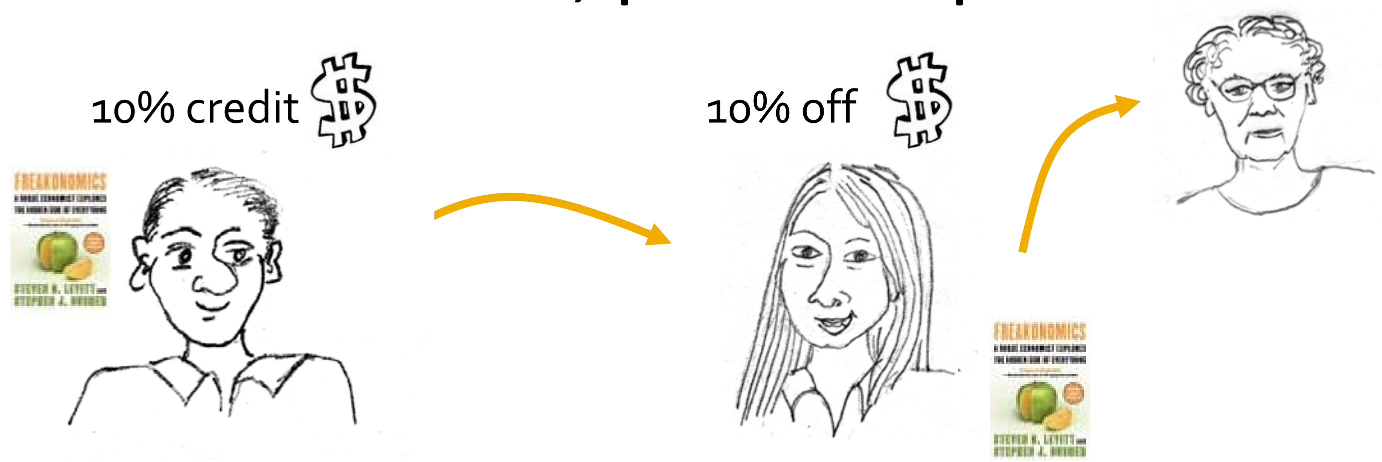
Network



Cascade
(propagation graph)

Setting 1: Viral marketing

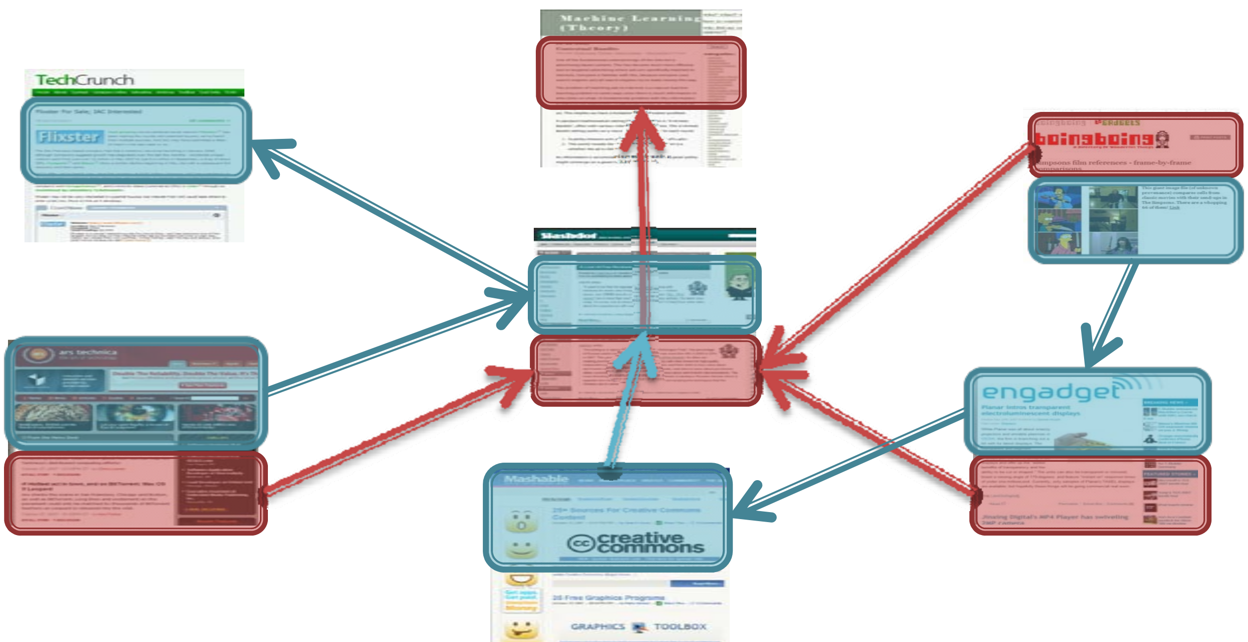
- People send and receive product recommendations, purchase products



- Data: Large online retailer: 4 million people, 16 million recommendations, 500k products

Setting 2: Blogosphere

- Bloggers write posts and refer (link) to other posts and the information propagates

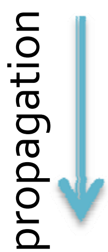


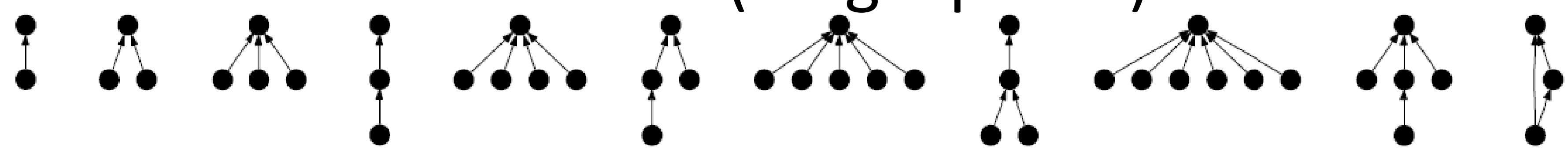
- Data: 10.5 million posts, 16 million links

[w/ Kleinberg-Singh, PAKDD ’06]

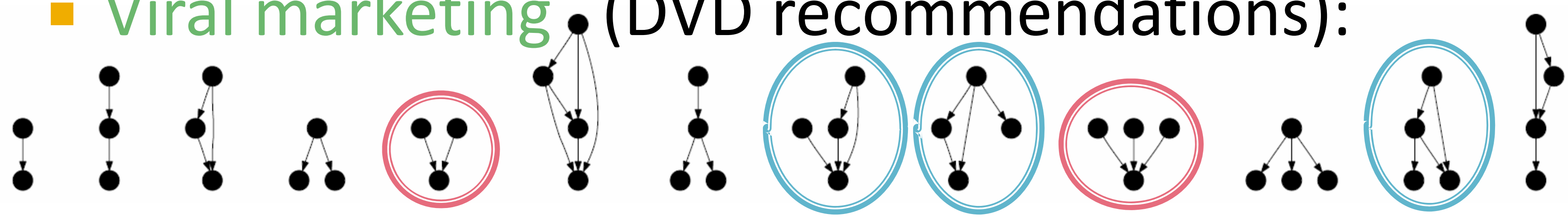
Q4) What do cascades look like?

- Are they stars? Chains? Trees?
- Information cascades (blogosphere):





- Viral marketing (DVD recommendations):



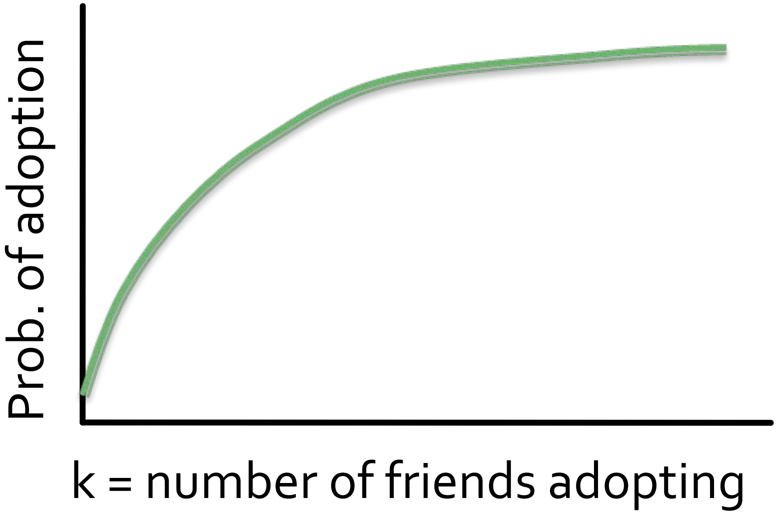
(ordered by frequency)

- Viral marketing cascades are more social:
  - Collisions (no summarizers)
  - Richer non-tree structures

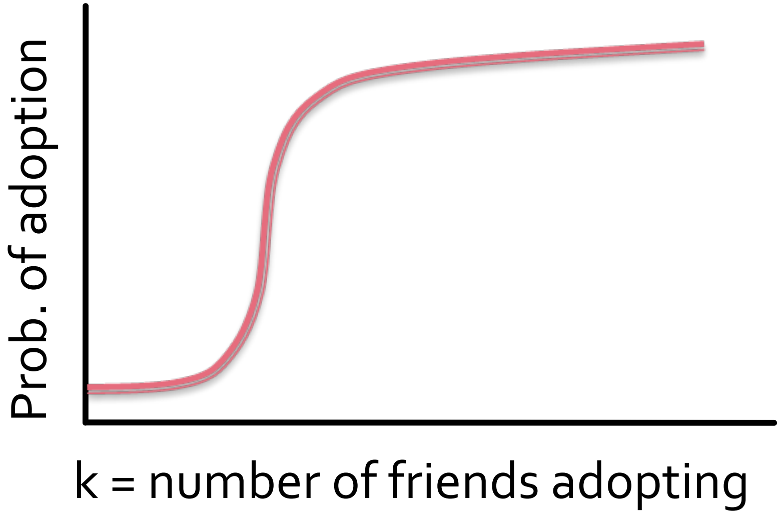
Q5) Human adoption curves

- Prob. of adoption depends on the number of friends who have adopted [Bass '69, Granovetter '78]
- **What is the shape?**

To find the answer we need lots of data

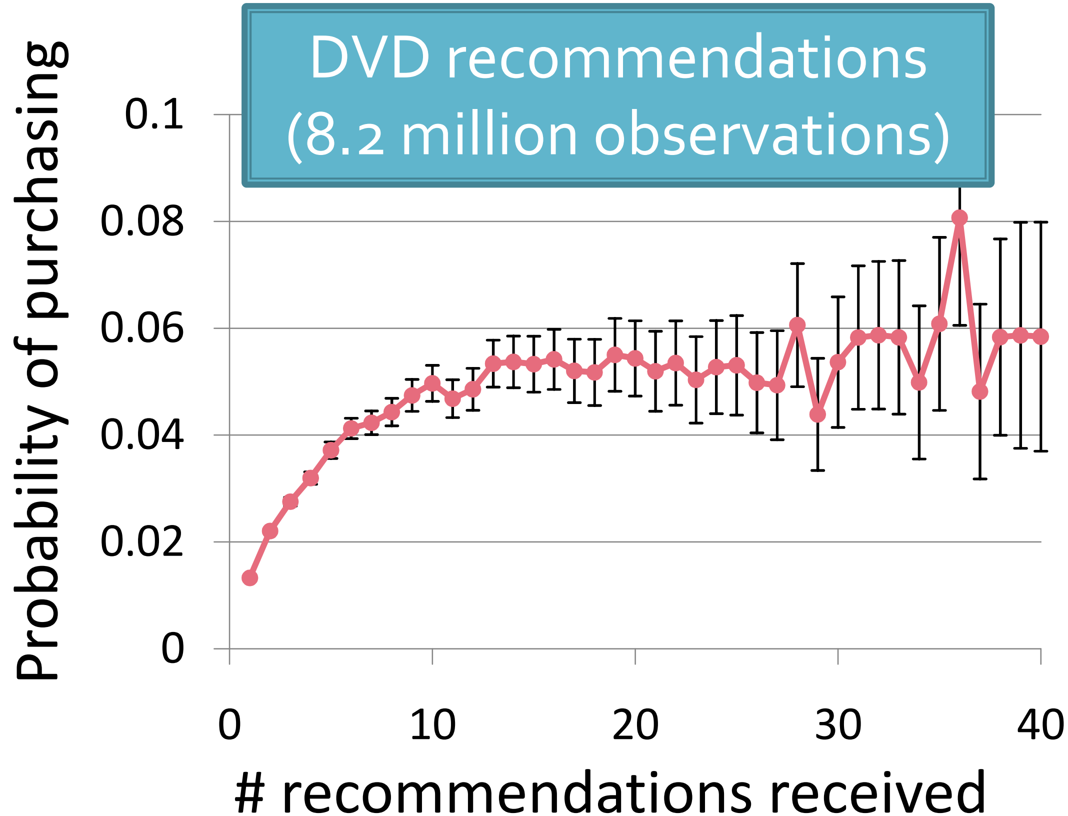


Diminishing returns?



Critical mass?

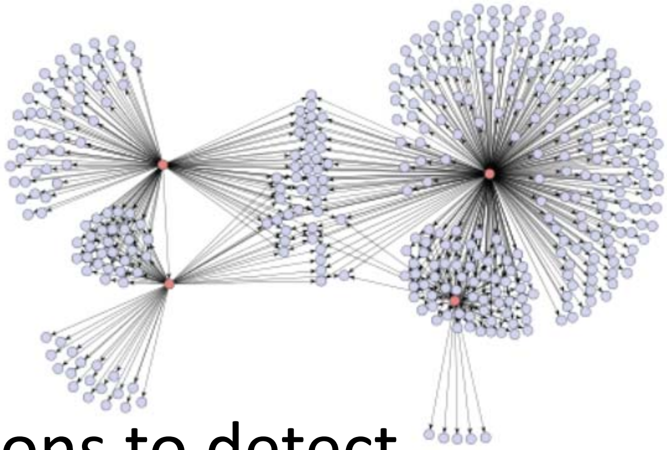
Q5) Adoption curve: Validation



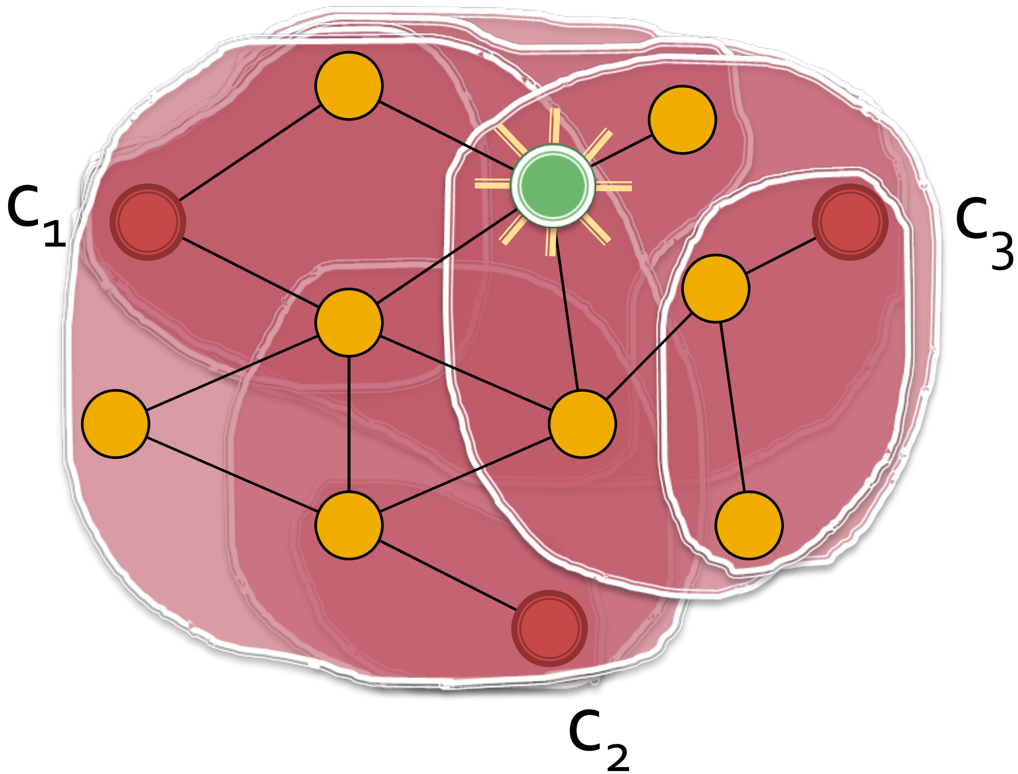
Adoption curve follows the **diminishing returns.**

Q6) Cascade & outbreak detection

- Blogs – information epidemics
  - Which are the influential/infectious blogs?
- Viral marketing
  - Who are the trendsetters?
  - Influential people?
- Disease spreading
  - Where to place monitoring stations to detect epidemics?



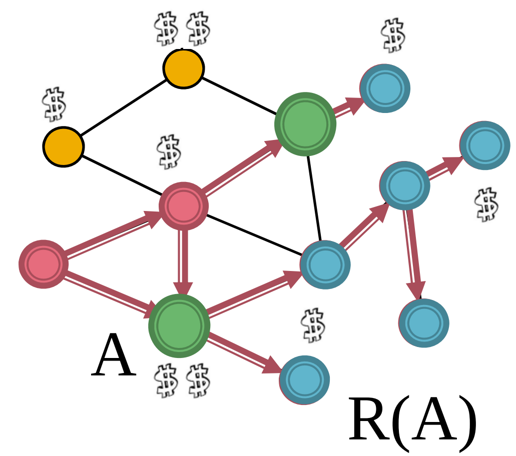
Q6) The problem: Detecting cascades



How to quickly detect epidemics as they spread?

Two parts to the problem

- Cost:
  - Cost of monitoring is node dependent
- Reward:
  - Minimize the number of affected nodes:
    - If A are the monitored nodes, let R(A) denote the number of nodes we save



(We also consider other rewards:
- Minimize time to detection
- Maximize number of detected outbreaks)

Optimization problem

- Given:
  - Graph G(V,E), budget M
  - Data on how cascades $C_1, \ldots, C_i, \ldots, C_K$ spread over time
- Select a set of nodes A maximizing the reward

$$\max_{A \subseteq V} \sum_i \text{Prob}(i) \underbrace{R_i(A)}_{\text{Reward for detecting cascade } i}$$

subject to $cost(A) \leq M$

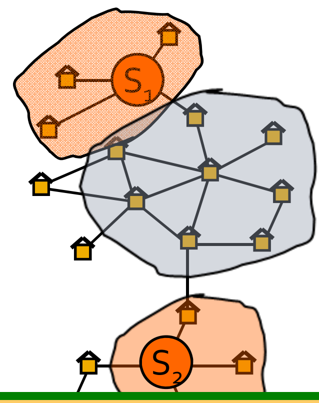
- Solving the problem exactly is NP-hard
  - Max-cover [Khuller et al. '99]

Solution: CELF Algorithm

- We develop CELF (cost-effective lazy forward-selection) algorithm:
  - Two independent runs of a modified greedy
    - Solution set A': ignore cost, greedily optimize reward
    - Solution set A'': greedily optimize reward/cost ratio
  - Pick best of the two: *arg max(R(A'), R(A''))*
- Theorem: If R is submodular then CELF is near optimal
  - CELF achieves *½(1-1/e)* factor approximation

[w/ Krause-Guestrin et al., KDD '07]

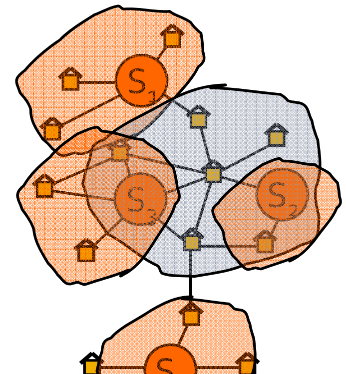
Problem structure: Submodularity



Adding S' helps a lot

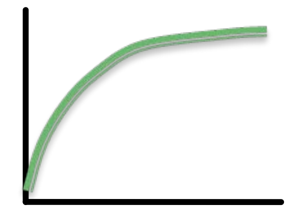
Placement A={S_1, S_2}

New monitored node:



Adding S' helps very little

Placement B={S_1, S_2, S_3, S_4}

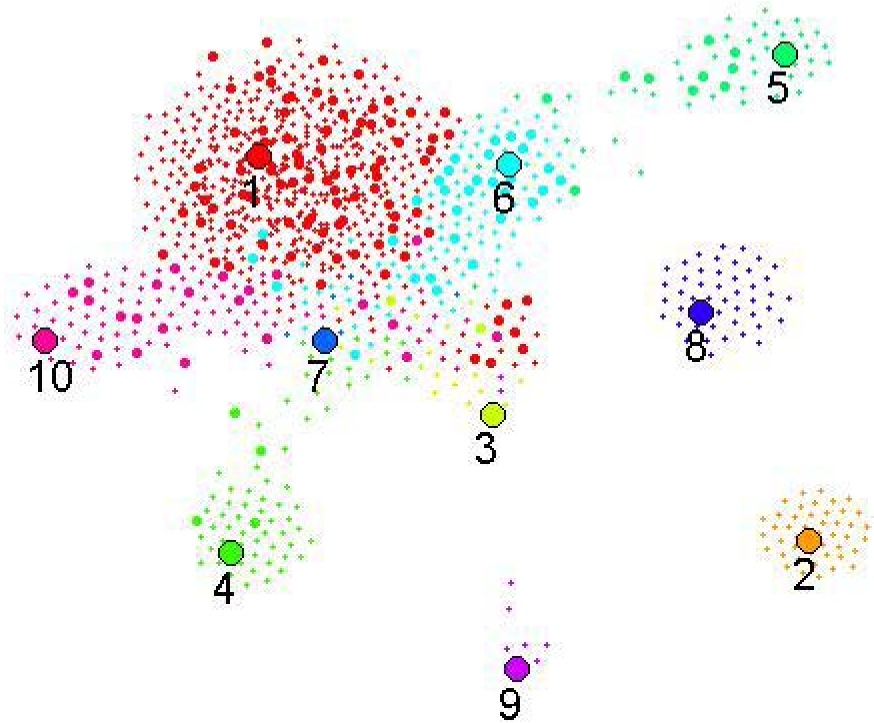


- Theorem: Reward function *R* is submodular (diminishing returns, think of it as "concavity")

$$\underbrace{R(A \cup \{u\}) - R(A)}_{\text{Gain of adding a node to a small set}} \geq \underbrace{R(B \cup \{u\}) - R(B)}_{\text{Gain of adding a node to a large set}} \qquad A \subseteq B$$

Blogs: Information epidemics

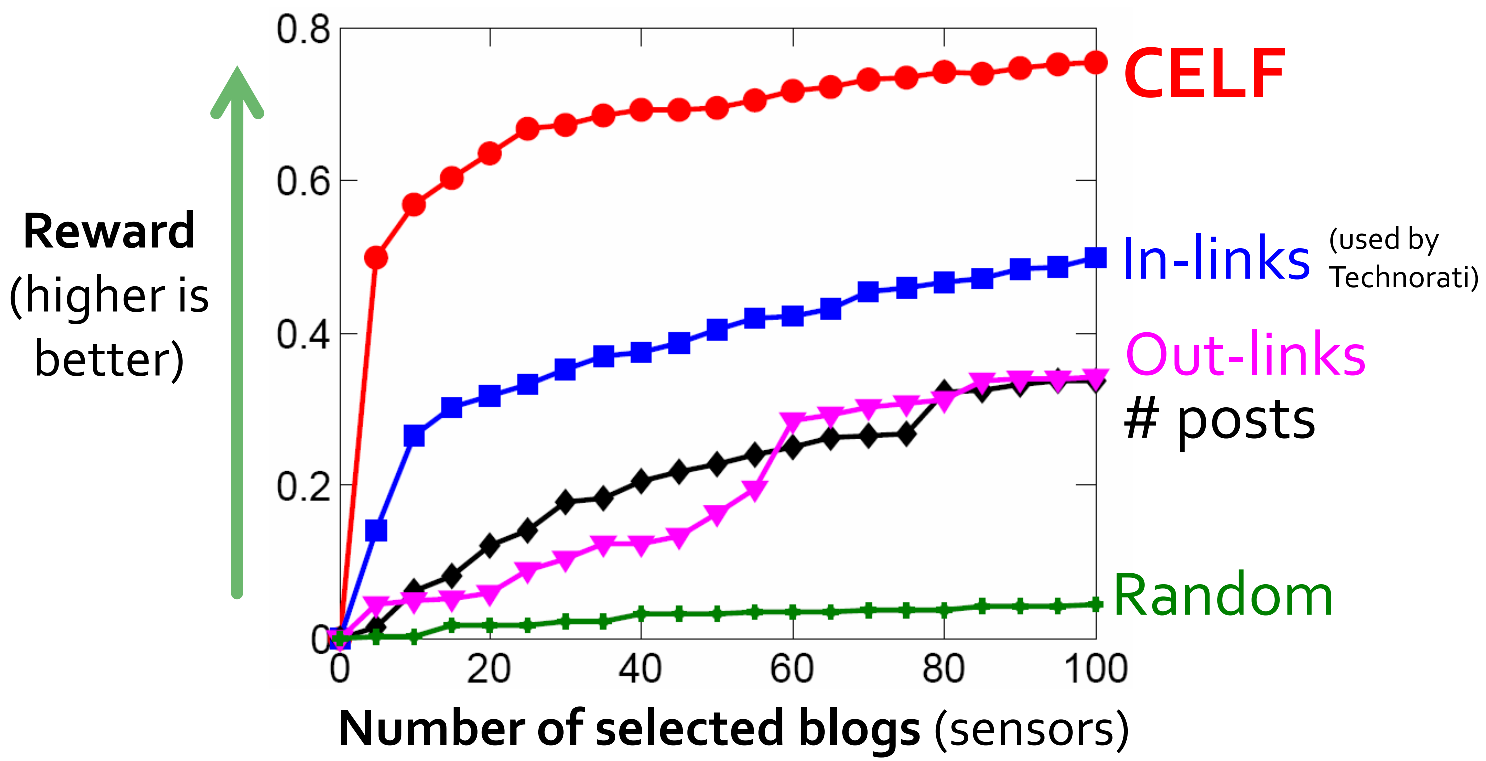
- **Question:** Which blogs should one read to catch big stories?
- Idea: Each blog covers part of the blogosphere



- Each dot is a blog
- Proximity is based on the number of common cascades

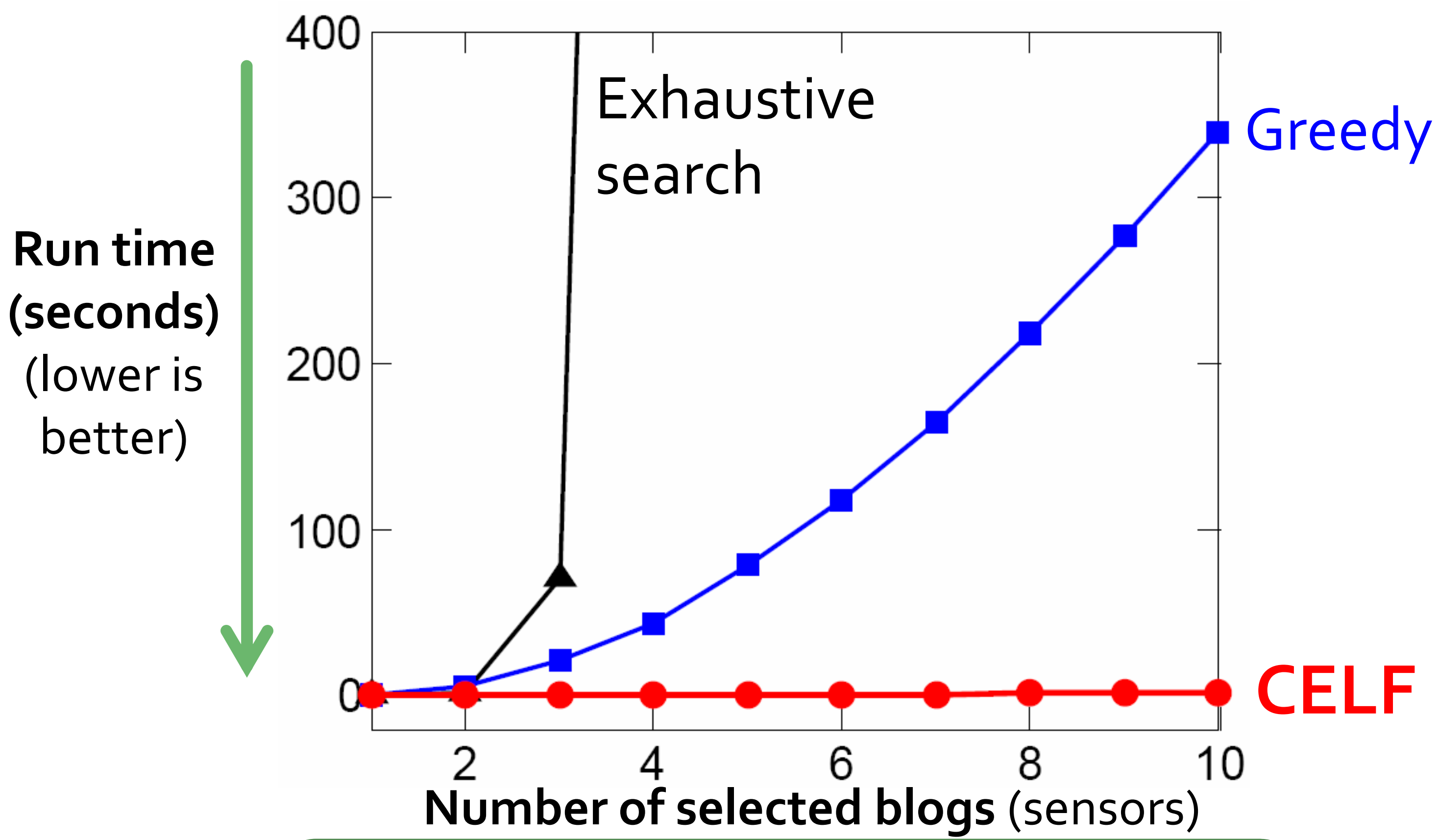
Blogs: Information epidemics

- Which blogs should one read to catch big stories?



For more info see our website: www.blogcascade.org

CELF: Scalability

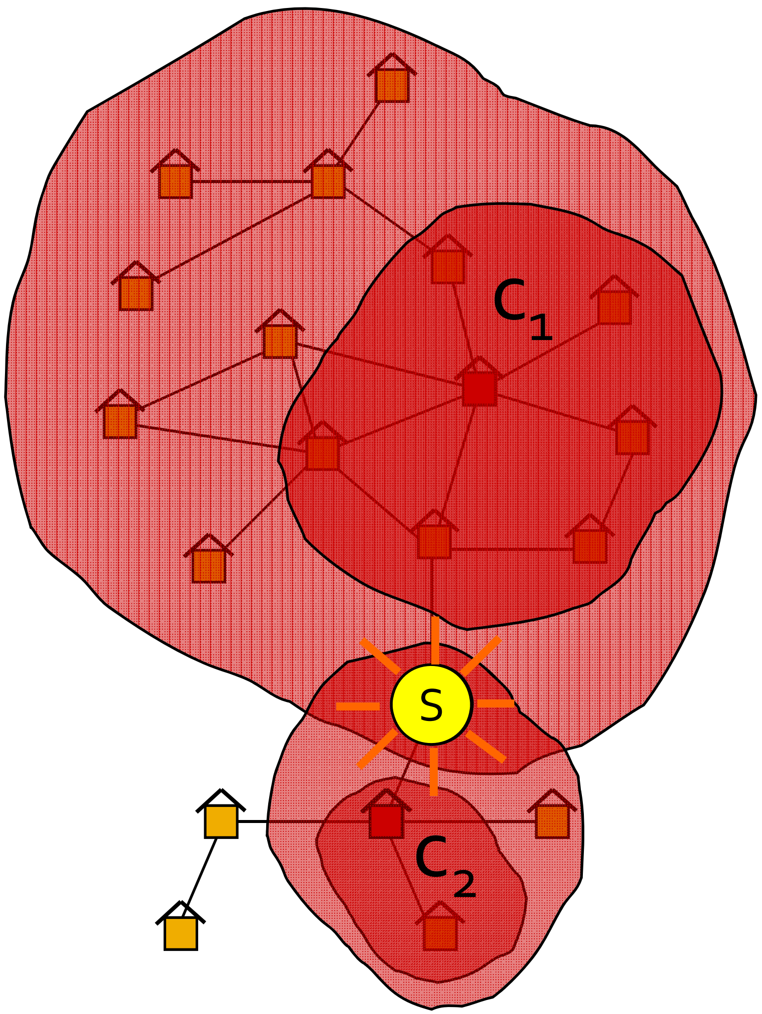


CELF runs **700x** faster than simple greedy algorithm

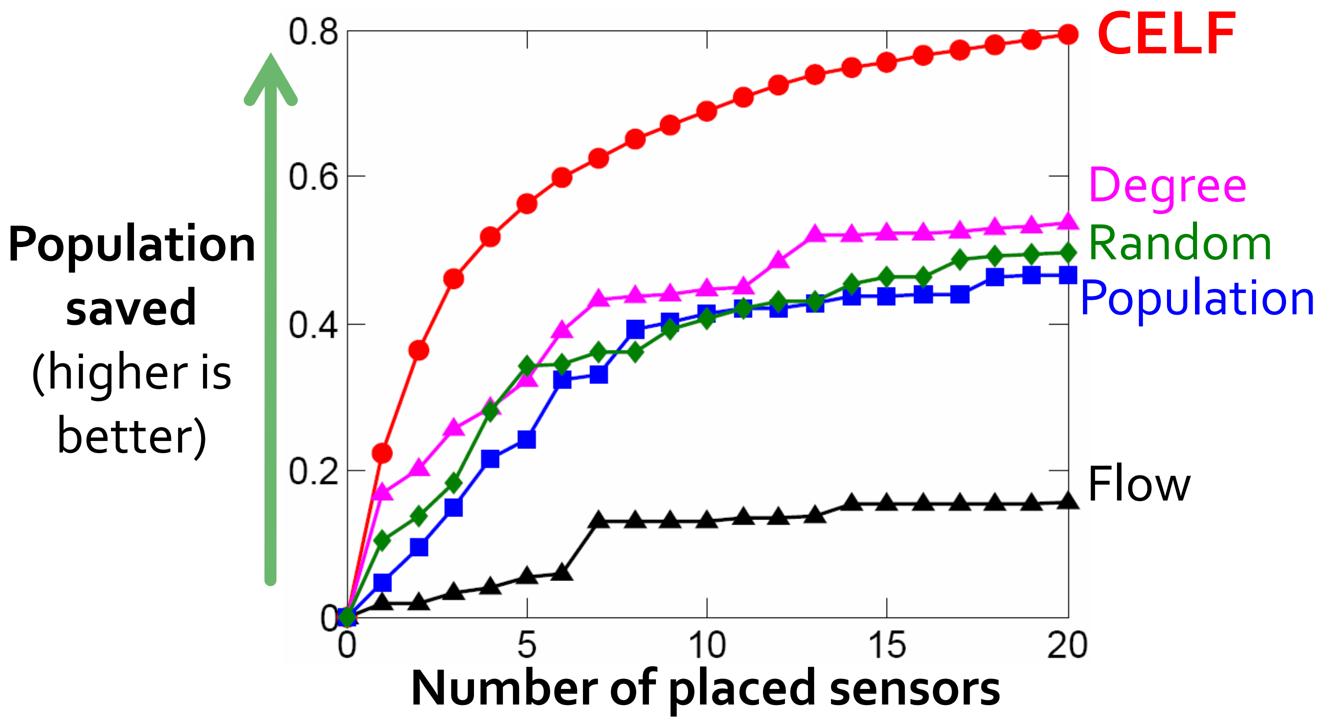
[w/ Krause et al., J. of Water Resource Planning]

Same problem: Water Network

- Given:
  - a real city water distribution network
  - data on how contaminants spread over time
- Place sensors (to save lives)
- Problem posed by the *US Environmental Protection Agency*



Water network: Results



Author	Score
CMU (CELF)	**26**
Sandia	21
U Exter	20
Bentley systems	19
Technion (1)	14
Bordeaux	12
U Cyprus	11
U Guelph	7
U Michigan	4
Michigan Tech U	3
Malcolm	2
Proteo	2
Technion (2)	1

- Our approach performed best at the Battle of Water Sensor Networks competition

Thesis: The structure

	Network Evolution	Network Cascades	Large Data
Observations	**Q1:** How does network structure evolve over time?	**Q4:** What are patterns of diffusion in networks?	**Q7:** What are the properties of a social network of the whole planet?
Models	**Q2:** How to model individual edge attachment?	**Q5:** How do we model influence propagation?	**Q8:** What is community structure of large networks?
Algorithms (applications)	**Q3:** How to generate realistic looking networks?	**Q6:** How to identify influential nodes and epidemics?	**Q9:** How to predict search result quality from the web graph?

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3 case studies on large data

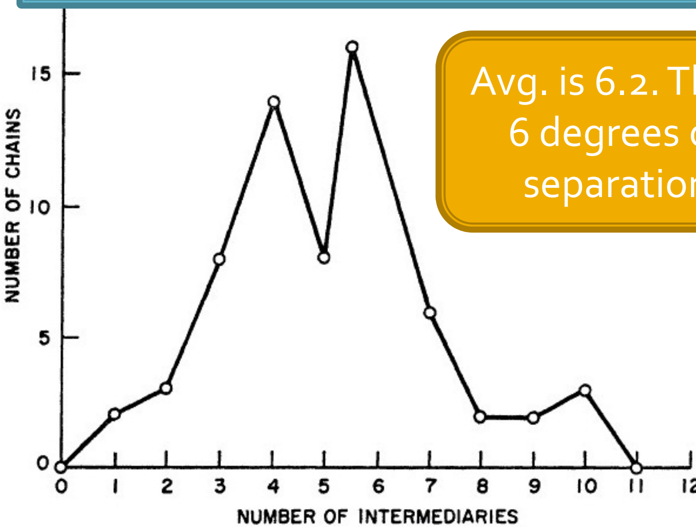
Benefits from working with large data:

- **Q7)** Can test hypothesis at planetary scale
  - 6 degrees of separation
- **Q8)** Observe phenomena previously invisible
  - Network community structure
- **Q9)** Making global predictions from local network structure
  - Web search

Q7) Planetary look on a small-world

- Small-world experiment [Milgram '67]
  - People send letters from Nebraska to Boston
- How many steps does it take?
- **Messenger social network** – largest network analyzed
  - 240M people, 255B messages, 4.5TB data

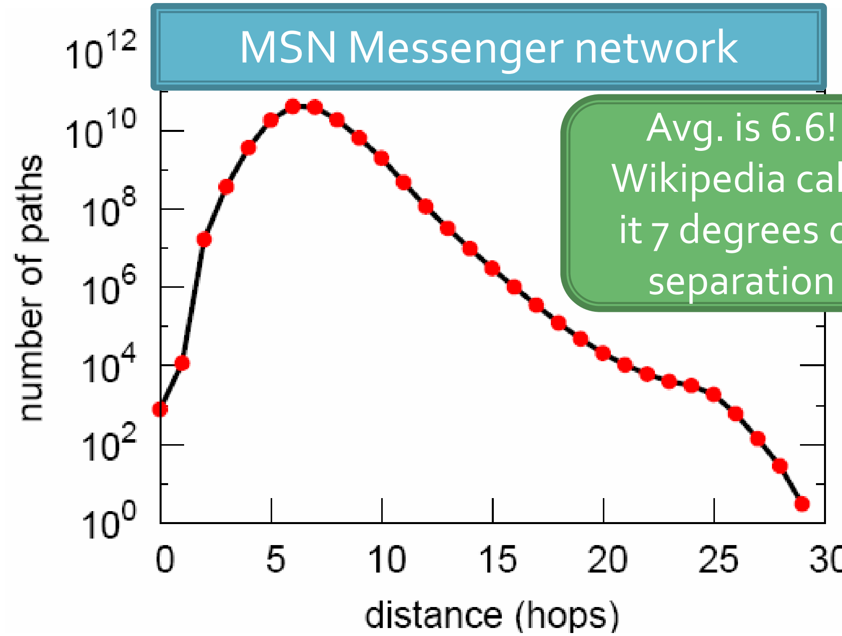
Milgram's small world experiment



Avg. is 6.2. Thus, 6 degrees of separation

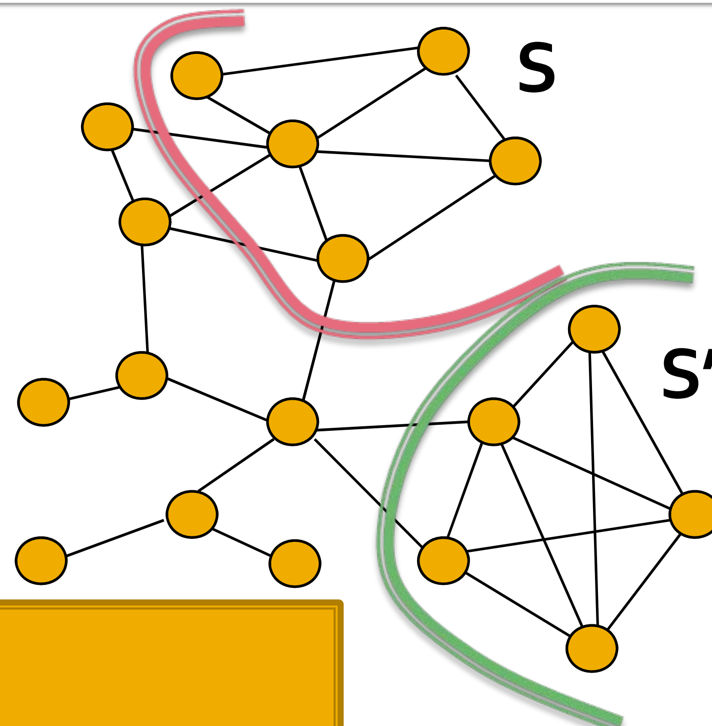
(i.e., hops + 1) [Milgram '67]

MSN Messenger network



Avg. is 6.6! Wikipedia calls it 7 degrees of separation

Q8) Network community structure

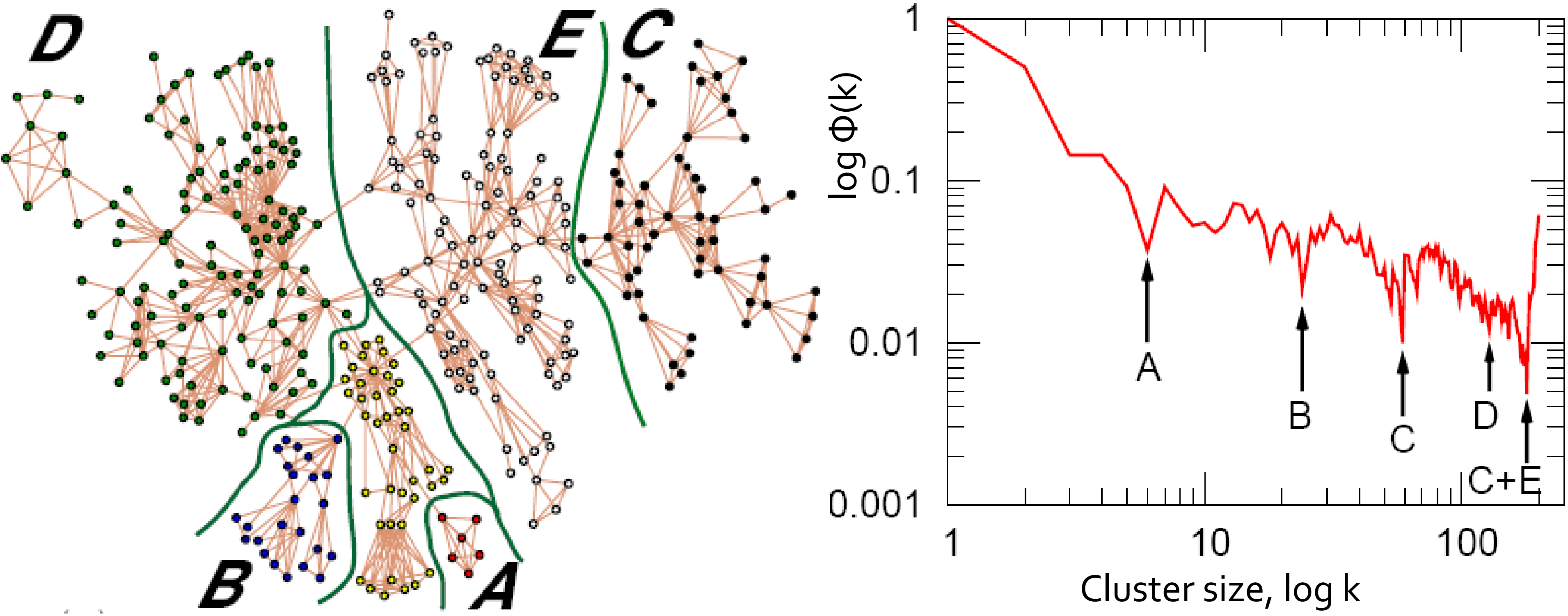


- How community like is a set of nodes?
- Need a natural intuitive measure

Conductance:
Φ(S) = # edges cut / # edges inside

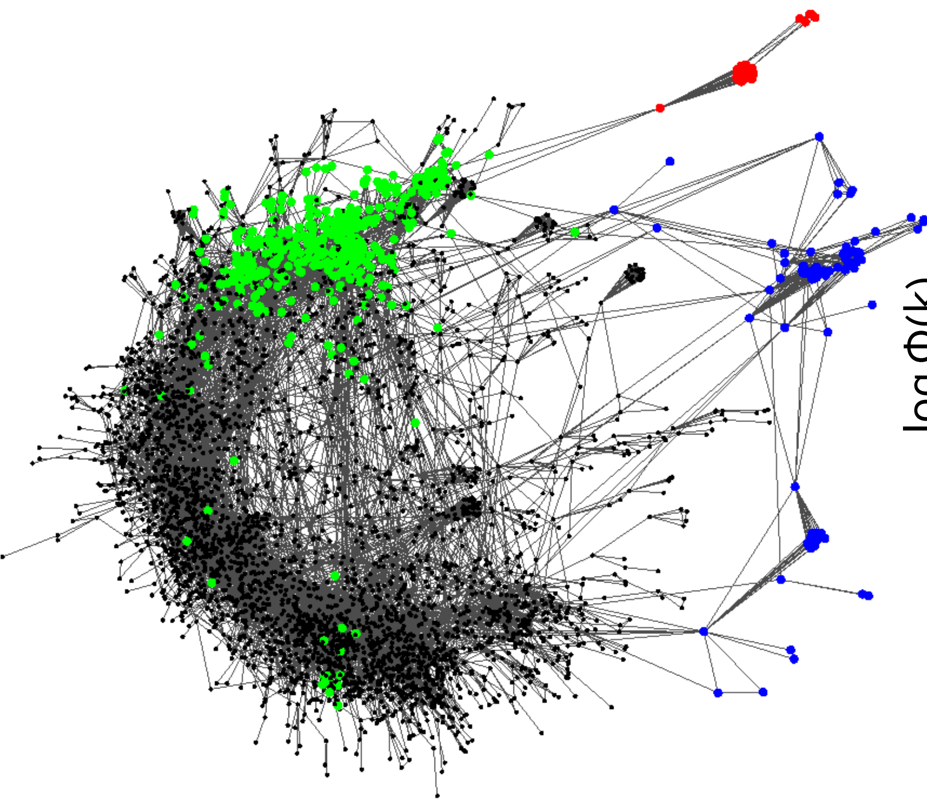
- **Plot:** Score of best cut of volume *k*=|*S*|

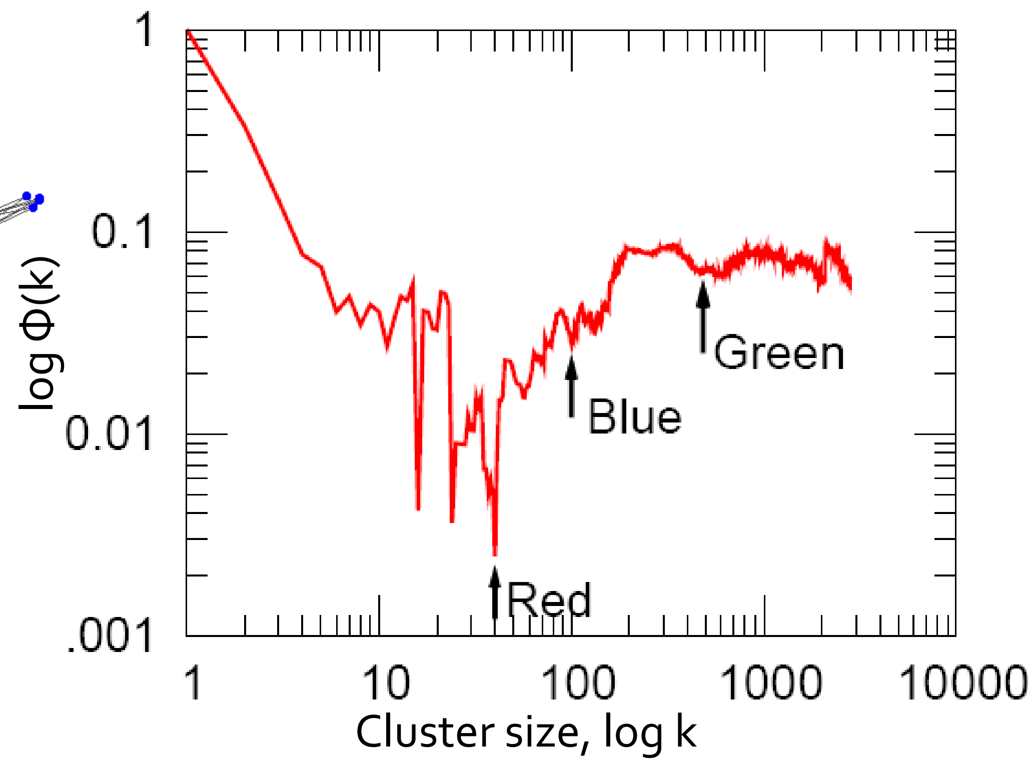
Example: Small network



Collaborations between scientists (N=397, E=914)

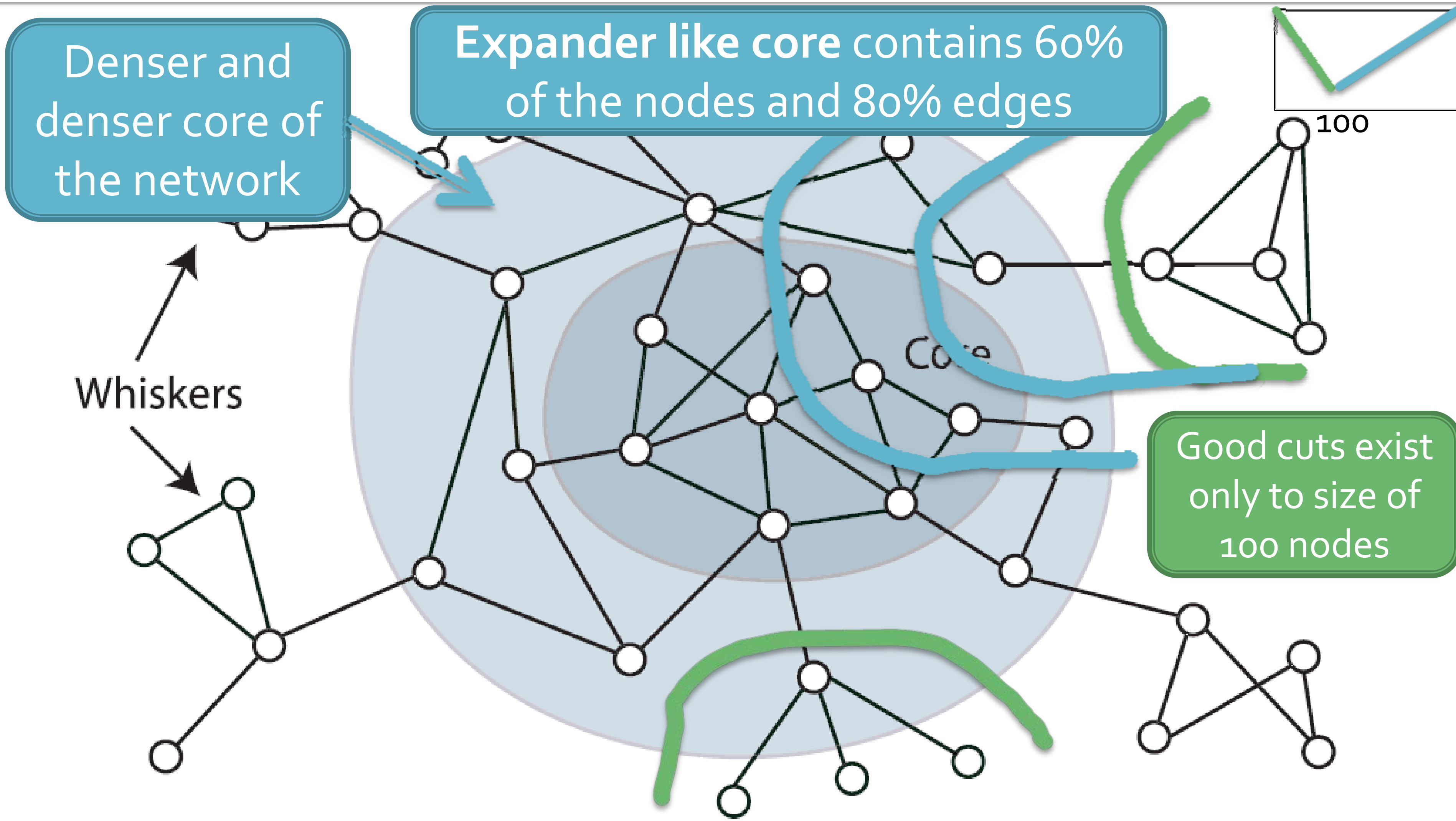
Example: Large network





Collaboration network (N=4,158, E=13,422)

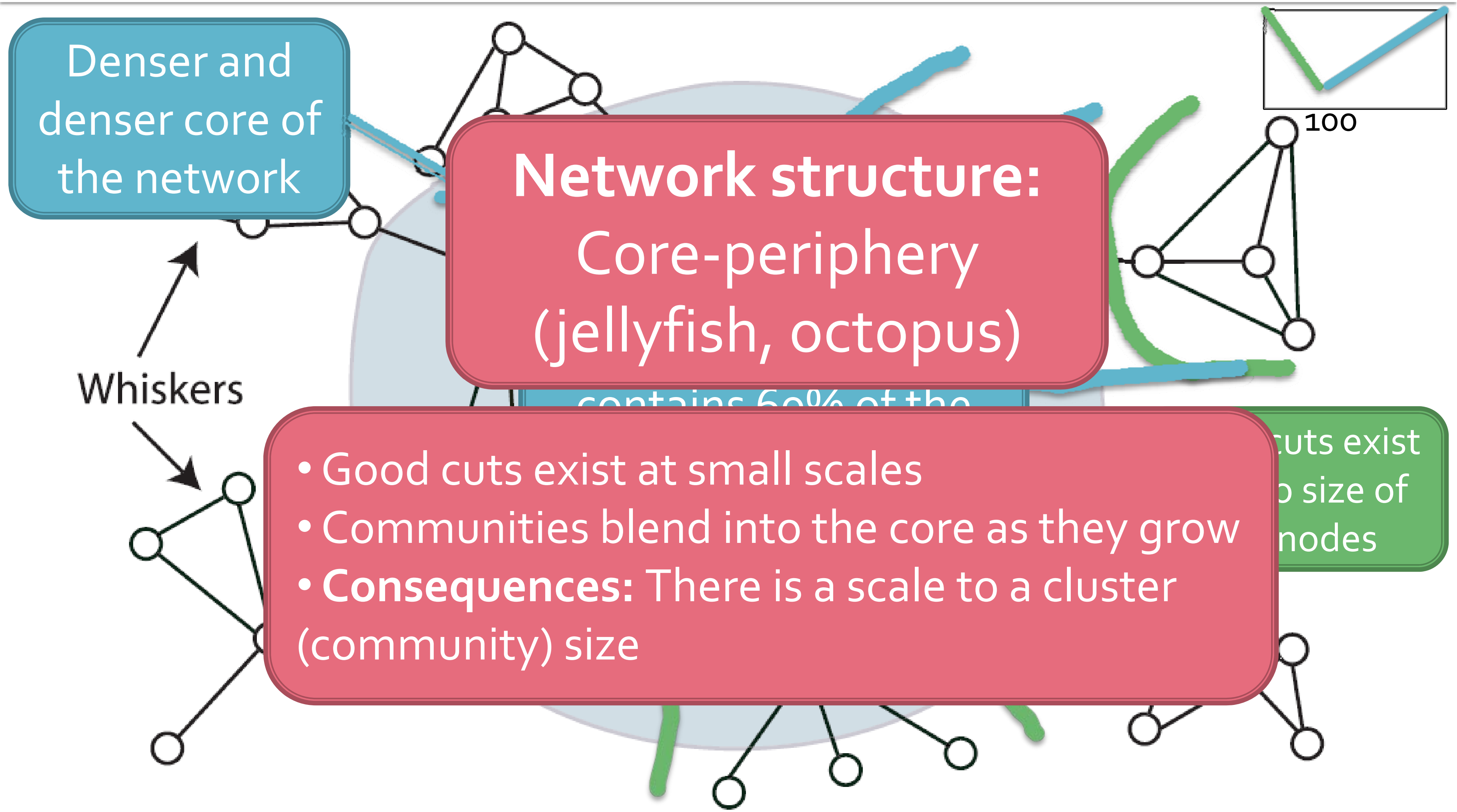
Q8) Suggested network structure



Q8) Suggested network structure

Denser and denser core of the network

Network structure:
Core-periphery
(jellyfish, octopus)

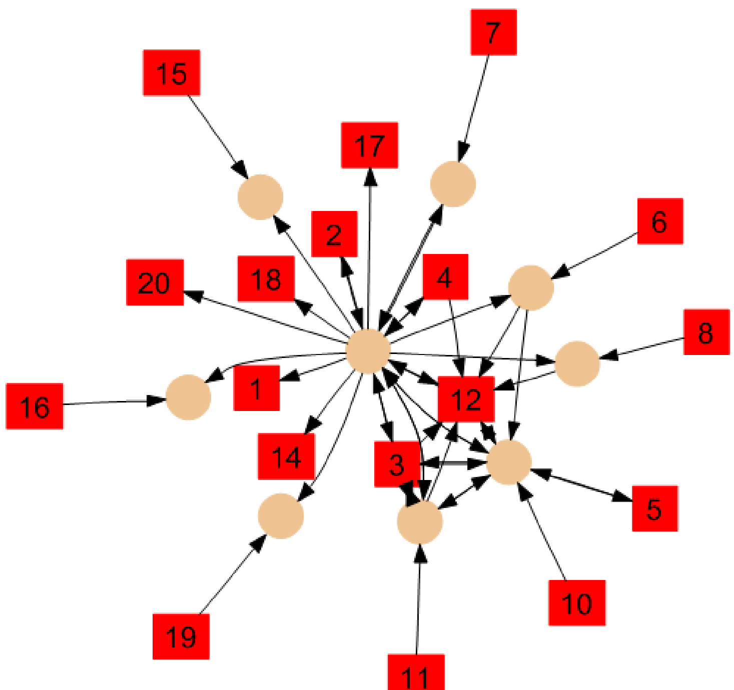


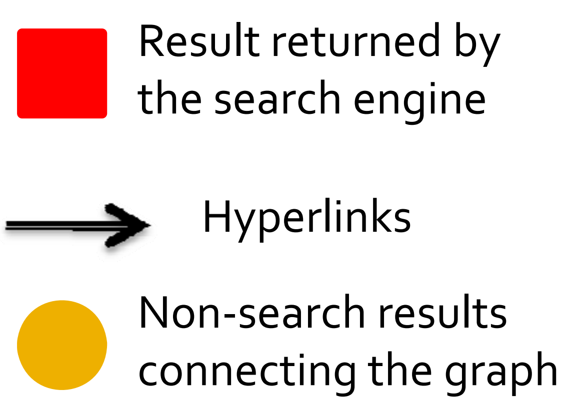
- Good cuts exist at small scales
- Communities blend into the core as they grow
- **Consequences:** There is a scale to a cluster (community) size

[w/ Dumais-Horvitz, WWW '07]

Q9) Web Projections

- User types in a query to a search engine
- Search engine returns results:



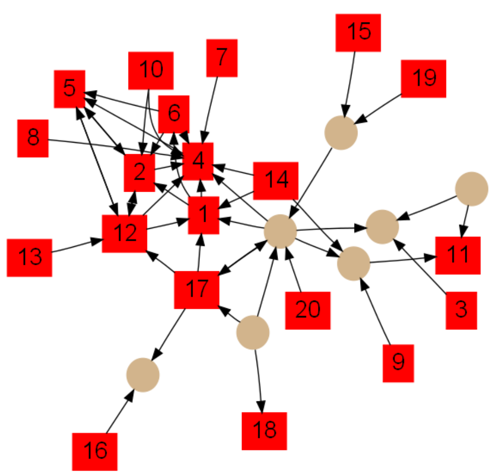


Is this a good set of search results?

[w/ Dumais-Horvitz, WWW '07]

Q9) Web Projections: Results

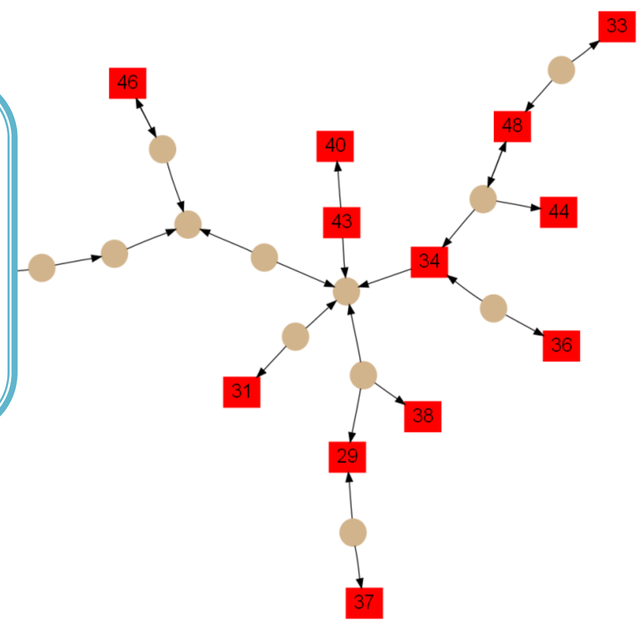
- We can predict search result quality with **80% accuracy** just from the connection patterns between the results



Predict "**Good**"

Good?

Poor?



Predict "**Poor**"

Thesis: The structure

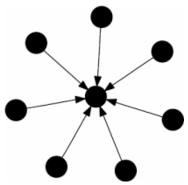
	Network Evolution	Network Cascades	Large Data
Observations	Densification and shrinking diameter	Cascade shapes	7 degrees of separation of MSN
Models	Triangle closing model	Diminishing returns of human adoption	Network community structure
Algorithms (applications)	Kronecker graphs and fitting	Cascade and outbreak detection	Web projections

Future directions: Evolution

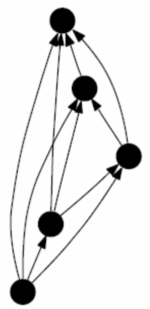
- Why are networks the way they are?
- Health of a social network
  - Steer the network evolution
  - Better design networked services
- Predictive modeling of large communities
  - Online massively multi-player games are closed worlds with detailed traces of activity

Future directions: Diffusion

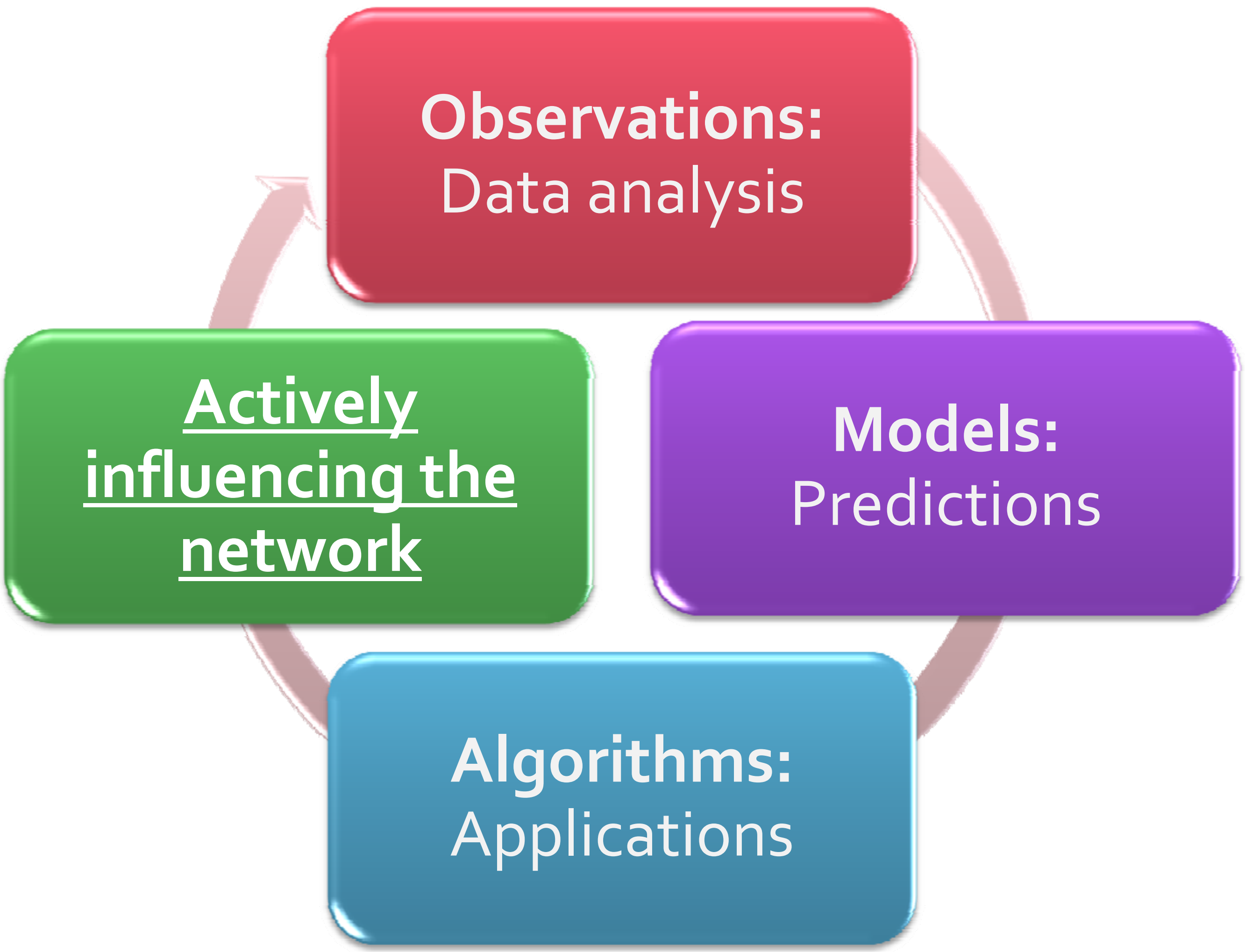
- Predictive models of information diffusion
  - When, where and what post will create a cascade?
  - Where should one tap the network to get the effect they want?
  - Social Media Marketing



- How do news and information spread
  - New ranking and influence measures for blogs
  - Sentiment analysis from cascade structure



What's next?


Observations:
Data analysis
Models:
Predictions
Algorithms:
Applications
Actively influencing the network