On Finding Narrow Passages with Probabilistic Roadmap Planners

◇◇

David Hsu, Lydia E. Kavraki, Jean-Claude Latombe
Rajeev Motwani, Stephen Sorkin

◇◇

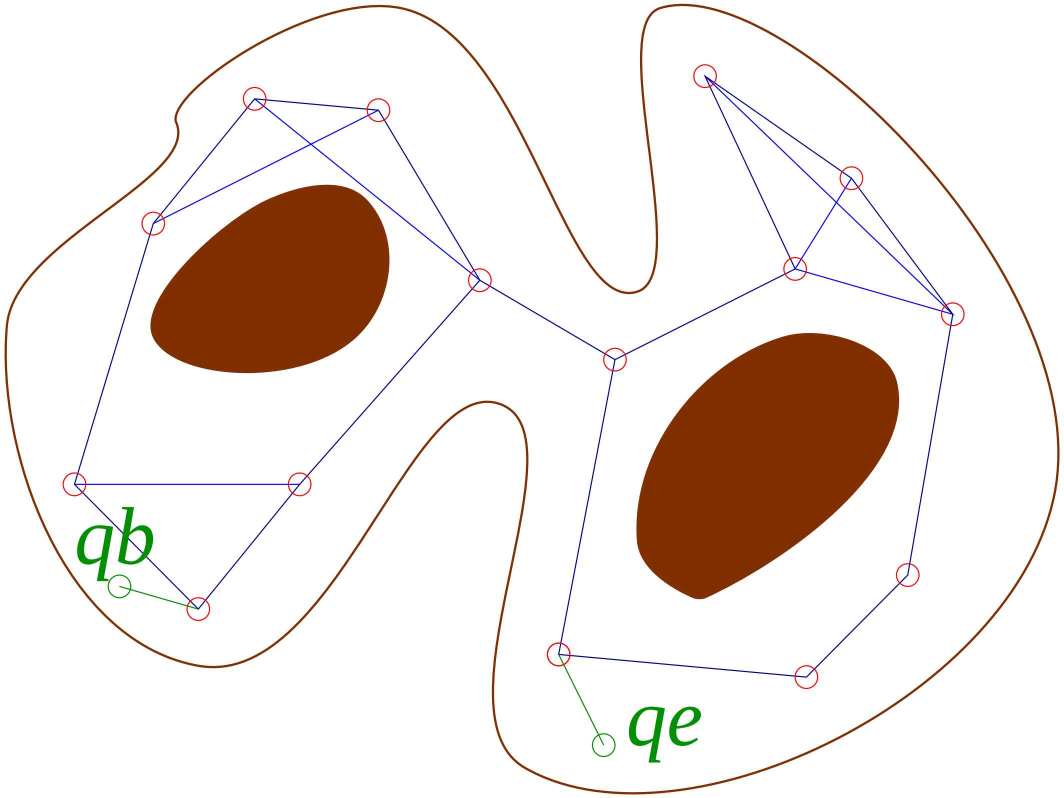
Path Planing Problem

Given:

- A geometric and kinematic model of a rigid or articulated object, called the robot,
- A geometric model of the obstacles in the physical space where the robot operates,

Find a path, *i.e.*, a continuous sequence of collision-free configurations of the robot, connecting two arbitrary input configurations q_b and q_e, called the query configurations, whenever such a path exists; otherwise indicate that no such path exists.

Planning with Probabilistic Roadmap



Probabilistic Roadmap Planners (PRMs)

- Amato and Wu; Amato, Bayazit, Dale, Jones, and Vallejo.
- Barraquand and Latombe.
- Horsch, Schwarz, and Tolle.
- Hsu, Latombe, and Motwani.
- Kavraki and Latombe; Kavraki, Latombe, Motwani, and Raghavan.
- Overmars and Švestka.

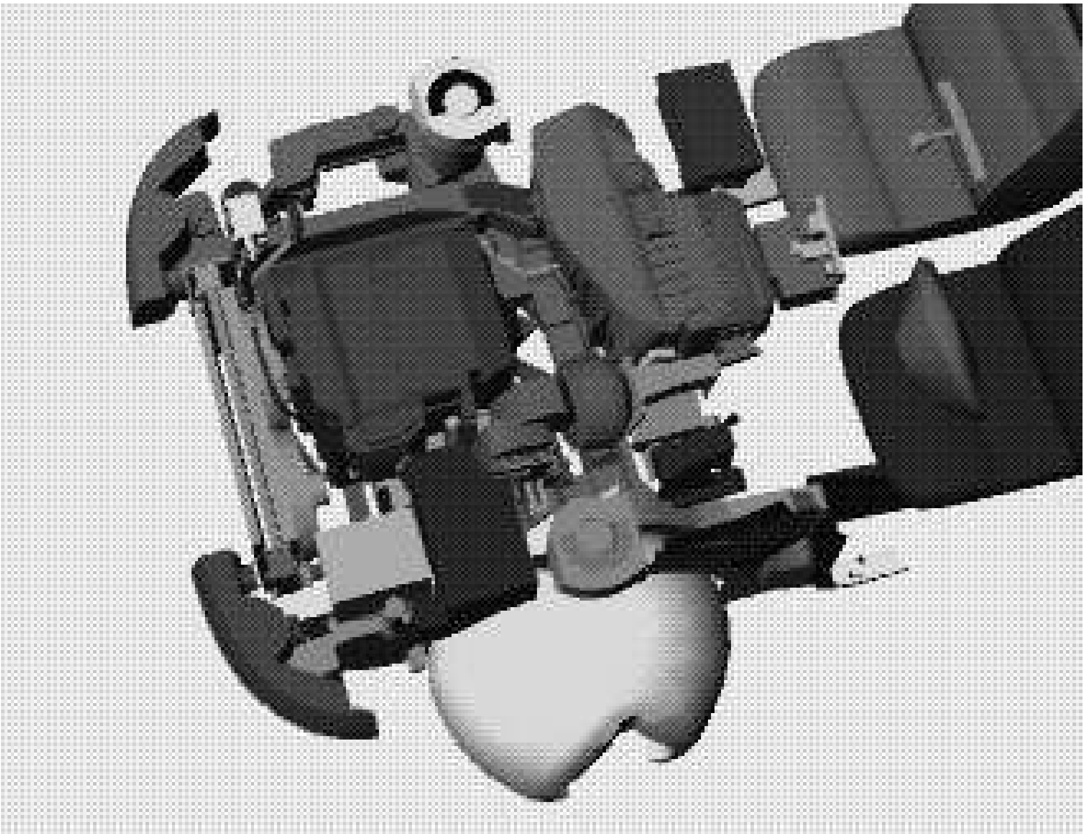
Rationale of PRMs

- In a configuration space of dimension n greater than 4-5 it is in general impractical to compute an explicit representation of the robot's free space.
- Instead, one can use a function dist(q) that computes the Euclidean distance between the robot at configuration q and the obstacles to check configurations and paths for collision.

Function dist

- Gilbert, Johnson, and Keerthi
- Lin and Canny
- Lin, Manocha, Cohen, and Gottschalk
- Mirtich
- Quinlan

Application Example 1

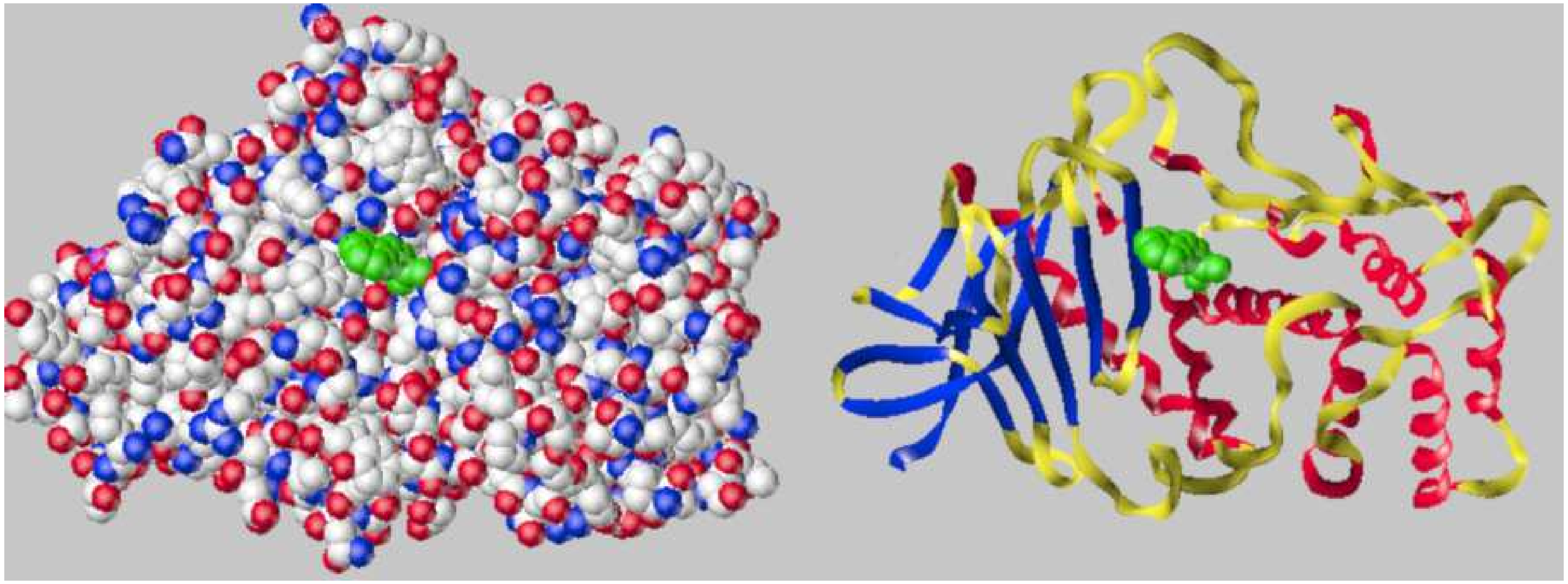


Design for manufacturing/maintainance/servicing

Application Example 2

Animation of digital actors

Application Example 3



Generation of plausible docking motions for drug molecule selection

Outline of the Talk

1. Basic PRM

2. Analysis of basic PRM

3. New sampling strategy

4. Relation to previous work

5. Current work

Notations and Conventions

- The configuration space is the hyper-cube $[0.1]^n$.
- The free space is an open subset of $[0,1]^n$ denoted by F.
- Two configurations q and q' see each other if the straight line segment connecting them lies entirely in F.
- The set of points seen by a configuration q is denoted by $\mathcal{V}(q)$ and is called the visibility set of q.
- The volume of a subset S of the configuration space is denoted by $\mu(S)$.

Probabilistic Roadmap Construction

Procedure roadmap:

1. Pick s configurations uniformly at random in F.

   Call them milestones and let M be the set of milestones.

2. Construct the graph $R = (M, L)$ in which L consists of every pair of milestones that see each other.

   Call R the roadmap.

Query Processing

Procedure query:

1. For $i = \{b, e\}$ do:
   (a) If there exists a milestone m that sees q_i then $m_i \leftarrow m$.
   (b) Else
      i. Repeat t times: Pick a configuration q in F uniformly at random in a neighborhood of q_i,

         until q sees both q_i and a milestone m.
      ii. If all t trials failed then return failure, else $m_i \leftarrow m$.
2. If m_b and m_e are in the same component of the roadmap, then return a path connecting them, else return no-path.

“Good” Roadmap

- Coverage:

  Collectively the milestones see a large portion of the free space F.

- Connectivity:

  There is a one-to-one map between the components of the roadmap and those of F.

Coverage (Definitions)

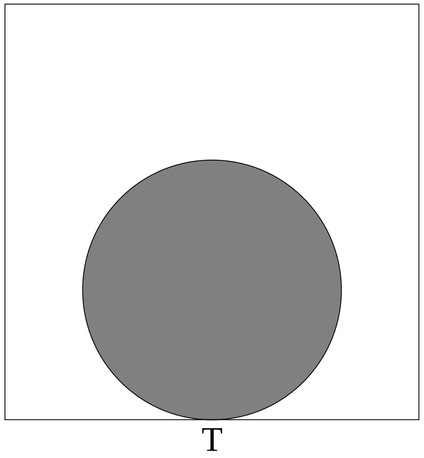
- ϵ-Goodness:

  Let ϵ be a constant in $(0, 1]$. A free configuration q is ϵ-good if $\mu(\mathcal{V}(q)) \geq \epsilon\mu(F)$. The free space F is ϵ-good if every $q \in F$ is ϵ-good.

- Adequate coverage:

  A set of milestones provides adequate coverage of an ϵ-good free space F if the volume of the subset of F not visible from any of these milestones is at most $(\epsilon/2)\mu(F)$.

Example of a Space that is not ϵ-Good



For any given $\epsilon > 0$, there exist points close to T that are ϵ'-good for $\epsilon' < \epsilon$. But any subset of this free space obtained by removing a small neighborhood of T is ϵ-good.

Adequate Coverage Guarantee

Theorem 1:

Let F be an ϵ-good free space. Let ϕ be a constant in $(0, 1]$.

For some constant K, if s is chosen such that:

$$s \geq \frac{K}{\epsilon}(\log \frac{2}{\epsilon\phi}),$$

then roadmap generates a set of milestones that adequately covers F, with probability at least $1 - \phi$.

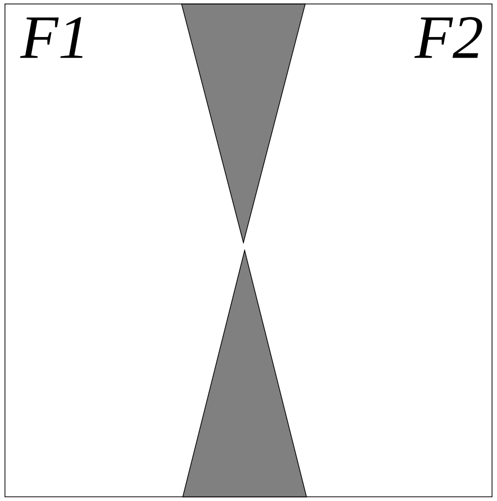
Performance guarantee for query

Theorem 2:

Let the maximum number of iterations t at Step 1(b)i of query be set to $\log(2/\psi)$, where ψ is a constant in $(0, 1]$.

If the milestones adequately cover F, then the probability that query outputs failure is at most ψ. The expected number of iterations is at most 2.

Lookout of a Set



This free space is ϵ-good for $\epsilon \approx .5$

However, the lookout of F_1, *i.e.*, the subset of points of F_1 that see a large portion of F_2, is small.

Expansive Free Space

- Lookout of a subset: Let β be a constant in $(0, 1]$ and S be any subset of any connected component E of the free space F. The β-lookout of S is the set:

$$\beta\text{-lookout}(S) = \{q \in S \,|\, \mu(\mathcal{V}(q)\backslash S) \geq \beta\mu(E\backslash S)\}.$$

- Expansiveness: Let ϵ, α, and β be constants in $(0, 1]$. The free space F is $(\epsilon, \alpha, \beta)$-expansive if it is ϵ-good and, for every connected subset S that is a finite union of visibility sets, we have:

$$\mu(\beta\text{-lookout}(S)) \geq \alpha\mu(S).$$

Connectivity Guarantee

Theorem 3:

Assume that F is $(\epsilon, \alpha, \beta)$-expansive. Let ξ be a constant in $(0, 1]$. If s is chosen such that:

$$s \geq \frac{16}{\epsilon\alpha} \log \frac{8}{\epsilon\alpha\xi} + \frac{6}{\beta} + 2,$$

then with probability at least $1 - \xi$, roadmap generates a roadmap such that no two of its components lie in the same component of F.

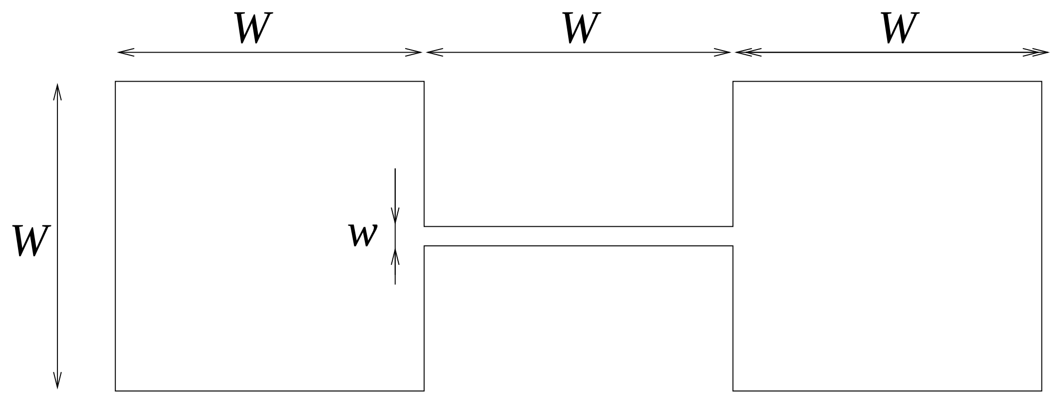
Performance Guarantee for roadmap

Theorems 1 and 3 combined guarantee that in an expansive free space, roadmap generates a good roadmap with high probability.

$$s \sim \frac{1}{\epsilon\alpha} \log \frac{1}{\epsilon\alpha\xi} + \frac{1}{\beta}$$

For a given free space, they do not allow us to estimate s, but they tell us that the probability that roadmap does not generate a good roadmap decreases exponentially with s, and that s increases moderately as ϵ, α, and β become smaller.

Dependance on Space Dimension



This free space is $(\epsilon, \alpha, \beta)$-expansive for $\epsilon, \alpha, \beta \sim w/W$.

n-D version: The narrow passage is narrow along $k \in [1, n-1]$ dimensions. Then: $\epsilon, \alpha, \beta \sim (w/W)^k$.

Path-Clearance Hypothesis

Let q and q' be two configurations in F such that there exists a path τ connecting them. Let ℓ be the length of τ and σ be its clearance.

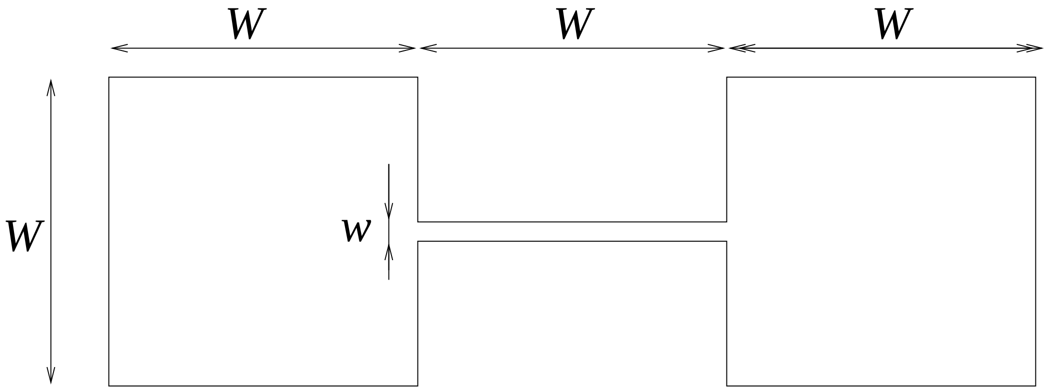
Theorem 4:

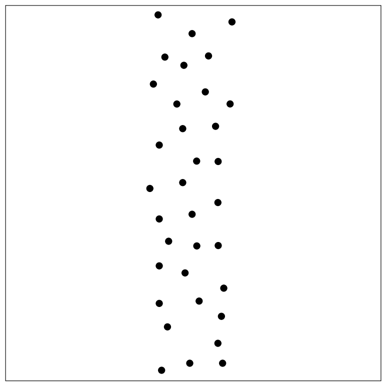
Let ζ be a constant in $(0,1]$ and $a = 2^{-n}\mu(\mathcal{B}_1)/\mu(F)$, where $\mathcal{B}_1$ denotes the unit ball in $\mathbf{R}^n$. If s is chosen such that:

$$s \geq \frac{1}{a\sigma^n} \log \frac{2\ell}{\sigma\zeta},$$

then with probability at least $1-\zeta$ roadmap generates a roadmap in which one component contains two milestones m and m' such that q sees m and q' sees m'.

Comparison of Theorems $1+3$ and Theorem 4

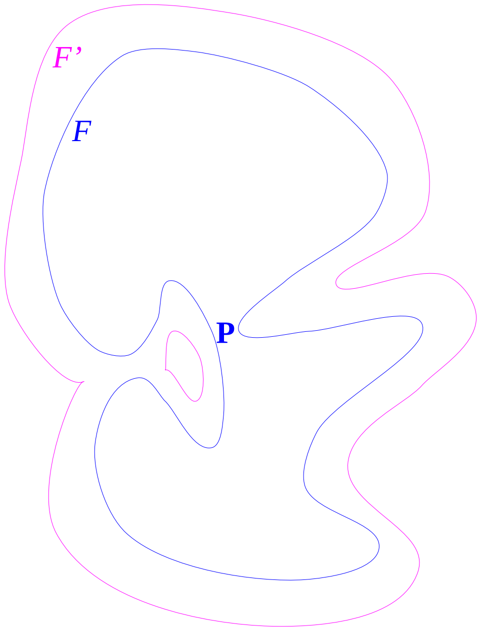




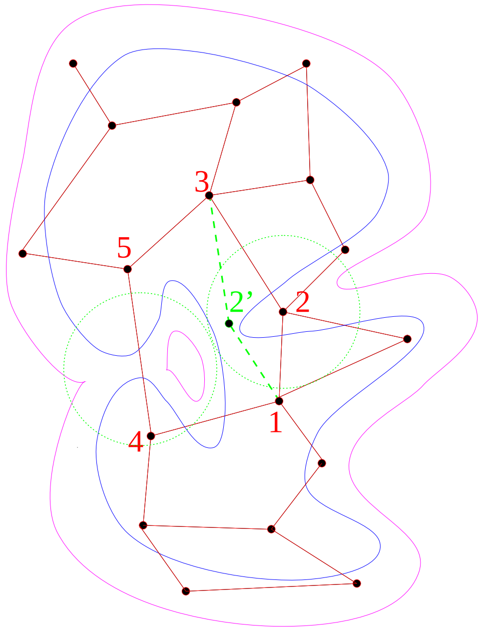
Th. $1+3$	$s \sim w^k$	large ϵ, α, β
Th. 4	$s \sim w^n$	small σ

But Theorem 4 may be easier to use in practice.

New Sampling Strategy Based on Dilatation and Retraction of Free Space

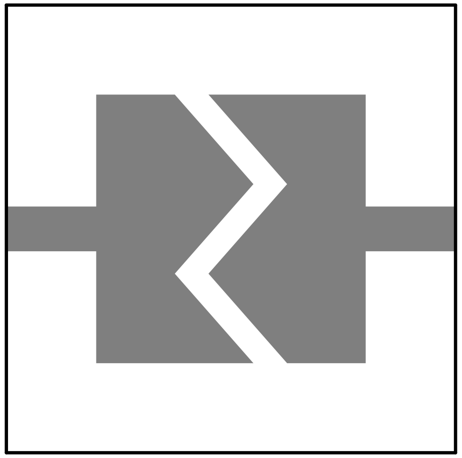
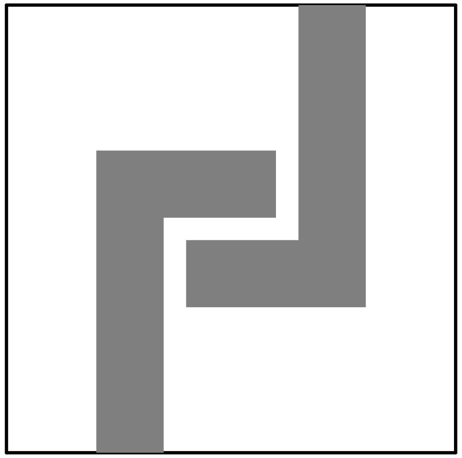


Dilatation of free space
by allowing some penetration distance
of the robot into the obstacles



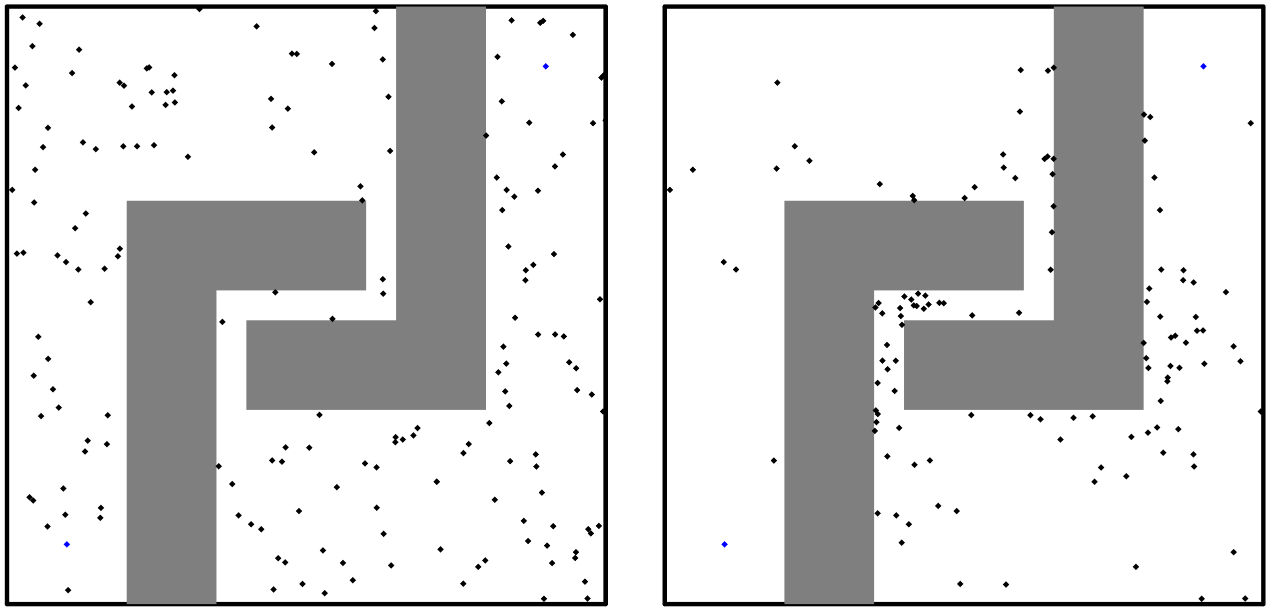
Retraction of free space
by locally resampling around
the milestones and links not in F

Experimental Results on Two Examples

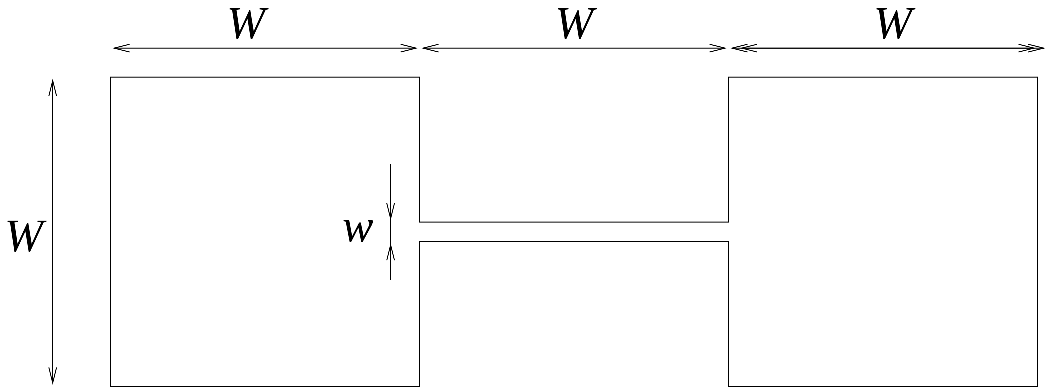
Algorithm	#mil.	#cdist	#mil.	#cdist
new-roadmap	65	16,812	112	11,035
roadmap	281	67,894	290	70,971

Examples of Milestone Distributions



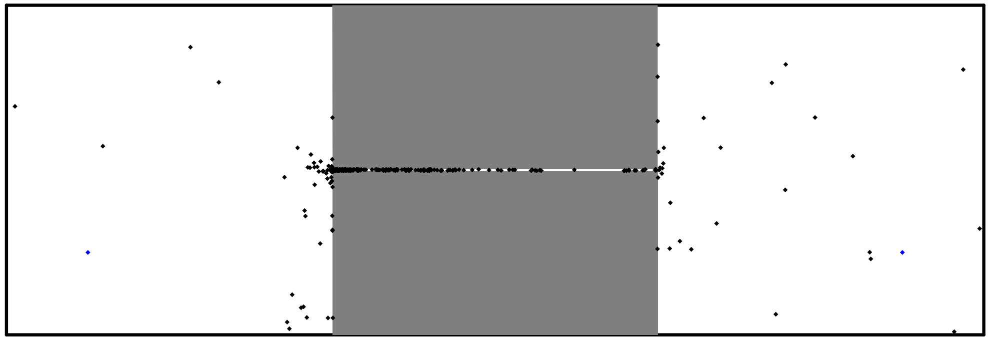
roadmap new-roadmap

Results in a Very Narrow Passage



w/W	r	#mil. (initial)	#mil. (final)	#cdist
.1	1	33	95	9,320
.01	1	38	118	32,942
.001	4	34	200	79,931
.0001	4	35	193	72,944
.00001	5	34	191	82,958

Milestone Distribution



$w/W = .001$

Results in 6-D

k	average #mil. (`new-roadmap`)	estimated #mil. (`roadmap`)
1	61	40
2	105	800
3	1656	16,000

(Results obtained with $w/W = .05$)

Discussion

- Dilatation of the free space increases path clearance. It also increases ϵ-goodness and expansiveness through F'.
- Resampling operations tend to create a greater density of milestones near the boundary of F and in narrow passages, where are the regions of F that are the most responsible for poor ϵ-goodness and expansiveness.
- F' may contain passages not present in F. The corresponding resampling operations fail.

Comparison to Other Strategies

- Two-stage strategies [Kavraki; Kavraki, Latombe, Overmars, and Švetska]: The first stage samples uniformly, while the second stage adds more milestones in areas chosen on the basis of information revealed by the first stage.

- Ray-casting strategies [Amato and Wu; Horsch, Schwarz, and Tolle; Overmars]: Rays are shot from within obstacles or within the free space to generate additional milestones near the boundary of the free space.

Computation of Penetration Distance

- Cameron.
- Dobkin, Hershberger, Kirkpatrick, and Suri.
- Keerthi and Shidharan; Sridharan, Stephanou, Craig, and Keerthi.
- Mirtich.
- Ong; Ong and Gilbert.

Current Work

Implementation and experimentation of a the dilatation/ retraction strategy for a 3-D rigid object with a 6-D configuration space, using V-Clip [Mirtich] to compute penetration distances.