Dynamically Eliciting Unobservable Information

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We answer the following question: At $t = 1$, an expert has (probabilistic) information about a random outcome X. In addition, the expert will obtain further information about X as time passes, up to some time $t = T + 1$ at which X will be publicly revealed. (How) Can a protocol be devised that induces the expert, as a strict best response, to reveal at the outset his prior assessment of both X and the information flows he anticipates and, subsequently, what information he privately receives? (The protocol can provide the expert with payoffs that depend only on the realization of X, as well as any decisions he may take.) We show that this can be done with the following sort of protocol: At the penultimate time $t = T$, the expert chooses a payoff function from a menu of such functions, where the menu available to him was chosen by him at time $t = T - 1$ from a menu of such menus, and so forth. We show that any protocol that affirmatively answers our question can be approximated by a protocol of the form described. We show how these results can be extended from discrete time to continuous time problems of this sort.

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Extended Abstract

The classic literature on proper scoring rules answers the following question: An (expected-value maximizing) expert has a probability assessment concerning the outcome of a random variable. Can one devise a payoff function, or scoring rule, that rewards the expert based on a report he makes about his prior and the eventually realized outcome, which induces him to report his true prior as a strict best response?

In this paper, we answer the following generalization of the classic question. The random variable will be realized at some future time $T + 1$, and between now ($t = 1$) and time $T + 1$, the expert will receive information that may change his probability assessment. *Knowing nothing about the nature of this information—what it will tell the expert; when it will be received by the expert—can we design a protocol that induces the expert, as a* **strict best response***, to reveal at the outset his prior and the structure of information he anticipates receiving, and then to provide truthful updates about information he receives as he receives it*? This protocol must provide the expert with a payoff at time $T + 1$ that depends only on the observable outcome of the random variable and the announcements of the expert prior to time $T + 1$.

When can such information be relevant? To set ideas, let us consider the following story. On this Sunday, Alice, an expected value maximizer, wants to plan a ski trip for the following Saturday. She must decide whether to book the hotel. The price is \$200 a night, but it is the last day she can benefit from such a price. On Monday, the price will raise to \$250 a night, and will remain fixed at that price until the end of the week. Alice values a day of snow at \$500, and a day of no snow at \$0. (Staying

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home also gives her a value of \$0.) To help her with her decision, she consults Bob, a weather specialist. If Alice's last chance to book the hotel was this Sunday, then the only information relevant for her decision—booking or not booking—would be Bob's current estimate of the likelihood of snow. Let us suppose Bob gathers all information he has at the current time and, given this information, asserts the likelihood of snow to be .5. With this information alone, Alice's best decision is to book the hotel now to receive an expected payoff of \$50/day.

Now, if Alice has the possibility to book the hotel later in the week—at the increased price of \$250 a night—then the information Bob might learn during the week matters to Alice. If Bob expects to learn nothing new during the week, Alice is still better off booking the hotel on Sunday. But suppose Bob tells Alice that, on Wednesday, he will be able to study again atmospheric conditions. At that time, he will be able to revise his estimate to either .2 or .8. At the current time, he believes that either of these revisions will occur with .5 probability. In that case, Alice optimizes by postponing her decision, and she books the hotel on Wednesday only if Bob revises his assessment upwards, which allows her to raise her expected payoff by an additional \$25/day. In this example, as in most dynamic decision problems, it matters to know both the uncertainty of a future outcome and how uncertainty will unravel over time.

Many real-world decisions have a similar dynamic structure. A standard example is classical futures trading: A decision must be made through time as to how much of a commodity future to buy at a given price, whose payoff obviously depends on the value of the commodity at the time the future matures. Other examples include revenue management, production and inventory planning, energy markets, insurance markets or irreversible investments. This sort of decision problem is extensively studied in the literature on *real options*.

If the individual who makes the decision is informed as to the relevant probabilities, then her problem reduces to solving a dynamic optimization problem. However, in many situations, the individual who makes the decision is only partially informed and may consult a better-informed individual—an expert. The classical scoring rule literature constructs payoff functions that can be used to motivate the expert to reveal all the information he knows that can be relevant in solving a *static* decision problem. In this paper, we want to motivate the expert to reveal all the information he knows that can be relevant in solving a *dynamic* decision problem. It includes (probabilistic) information about the future observable outcomes, (probabilistic) information about future private information that the expert anticipates to receive over time, and what the expert privately observes.

We show how to motivate the expert to reveal all relevant information with a carefully designed protocol of the following sort. At time T, the expert must choose a security (a function from publicly observable outcomes to payoffs) from a set or menu of such securities, where the menu available to him at time T was chosen by him at time $T-1$ from a menu of menus. The menu of menus available to him at time $T-1$ was chosen from a menu of menus of menus available to him at time $T-2$, and so forth. We show that any protocol that induces the expert to reveal all information of interest can be approximated by the protocols just described, and we show to generalize the protocol to continuous time environments.