# vGraph: A Generative Model for Joint Community Detection and Node Representation Learning

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# Agenda

- Motivation
- vGraph
- Experiments

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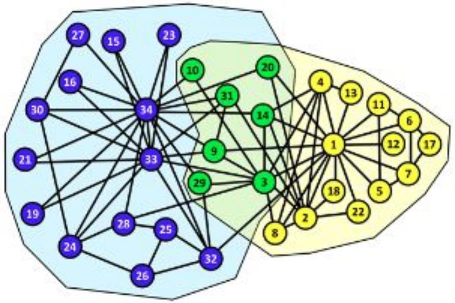
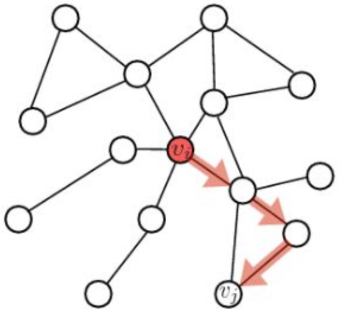
# Motivation

(Non)Overlapping Community Detection




Node Representation Learning

- MF
- LINE
- DeepWalk
- Node2vec

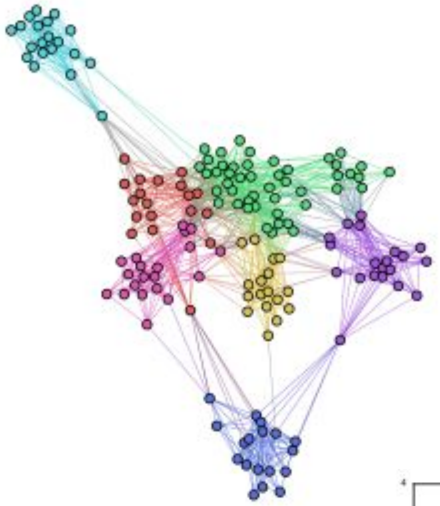
# Motivation

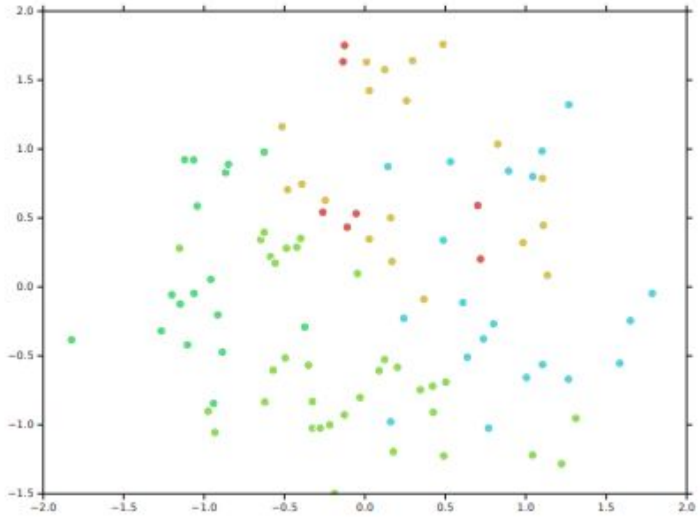
(Overlapping) Community Detection

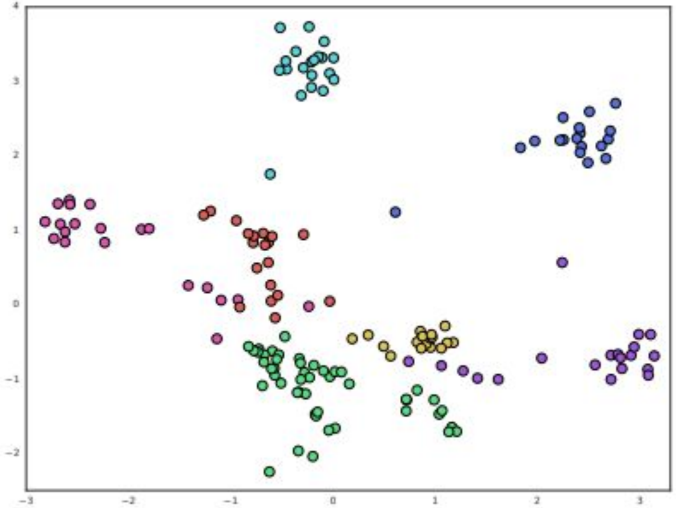
Community Preserving
Network Embedding

Clustering (i.e. K-Means)
Using Node Embeddings
as feature

Node Representation Learning
- MF
- LINE
- DeepWalk
- Node2vec






https://epasto.org/papers/www2019splitter.pdf

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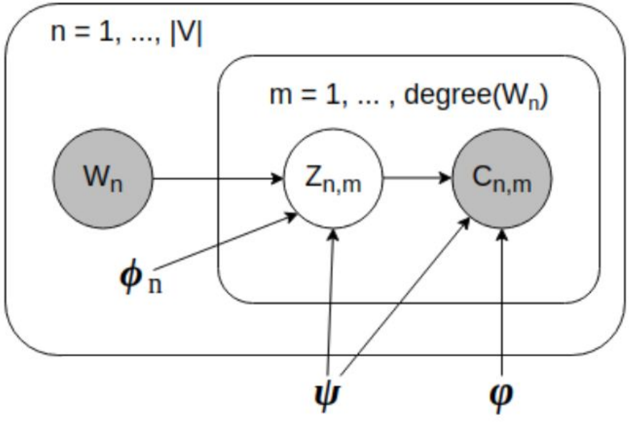
- Motivation
- vGraph
    - Hierarchical vGraph
- Experiments

# vGraph - probabilistic generative model

For each node w, draw a latent variable (community assignment) z ~ p(z|w)

based on the latent variable, Generate linked neighbor c ~ p(c|z)

$$\boxed{p(c|w) = \sum_z p(c|z)p(z|w).}$$




(a) vGraph

# Variational Inference Recipe

- Start with data ***X*** and a model ***p(z, x)***, we are interested in ***p(z | x)***
- Choose a variational approximation ***q(z | x; v)*** **(approximate posteriors)**
- By maximizing likelihood ***log p(X)***, we derive ELBO

$$\mathscr{L}(\boldsymbol{\nu}) = \mathbb{E}_{q(\mathbf{z};\boldsymbol{\nu})}[\log p(\mathbf{x},\mathbf{z}) - \log q(\mathbf{z};\boldsymbol{\nu})]$$

- How to obtain gradients? **(variational optimization)**

**Example Using Gaussian mixture Model:** [https://zhiyzuo.github.io/VI/#general-situation](https://zhiyzuo.github.io/VI/#general-situation)

# vGraph - Variational Inference

- Approximate posterior:

$$q(z|c, w)$$

- ELBO (evidence lower bound):

$$\mathcal{L} = E_{z \sim q(z|c,w)}[\log p_{\boldsymbol{\psi},\boldsymbol{\varphi}}(c|z)] - \mathrm{KL}(q(z|c, w)||p_{\boldsymbol{\phi},\boldsymbol{\psi}}(z|w))$$

# vGraph - Variational Inference

- Approximate posterior:

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- Variational optimization (obtain gradients) → **z** is discrete →

Gumbel Softmax (straight-through gradient estimator) !

# Implementation

$$\mathcal{L} = E_{\underline{z\sim q(z|c,w)}}[\log \boxed{p_{\psi,\varphi}(c|z)}] - \mathrm{KL}(\underline{q(z|c,w)}\|\boxed{p_{\phi,\psi}(z|w)})$$

$$\underline{q_{\phi,\psi}(z=j|w,c)} = \frac{\exp((\phi_w \odot \phi_c)^T \psi_j)}{\sum_{i=1}^{K} \exp((\phi_w \odot \phi_c)^T \psi_i)}.$$

$$\boxed{p_{\phi,\psi}(z=j|w)} = \frac{\exp(\phi_w^T \psi_j)}{\sum_{i=1}^{K} \exp(\phi_w^T \psi_i)},$$

$$\boxed{p_{\psi,\varphi}(c|z=j)} = \frac{\exp(\psi_j^T \varphi_c)}{\sum_{c' \in \mathcal{V}} \exp(\psi_j^T \varphi_{c'})}.$$

$\phi_i$ $\varphi_i$ : Two set of node embeddings

$\psi_j$ : Community embeddings

# Implementation

$$\mathcal{L} = E_{\underline{z \sim q(z|c,w)}}[\log \boxed{p_{\psi,\varphi}(c|z)}] - \mathrm{KL}(\underline{q(z|c,w)}\|\boxed{p_{\phi,\psi}(z|w)})$$

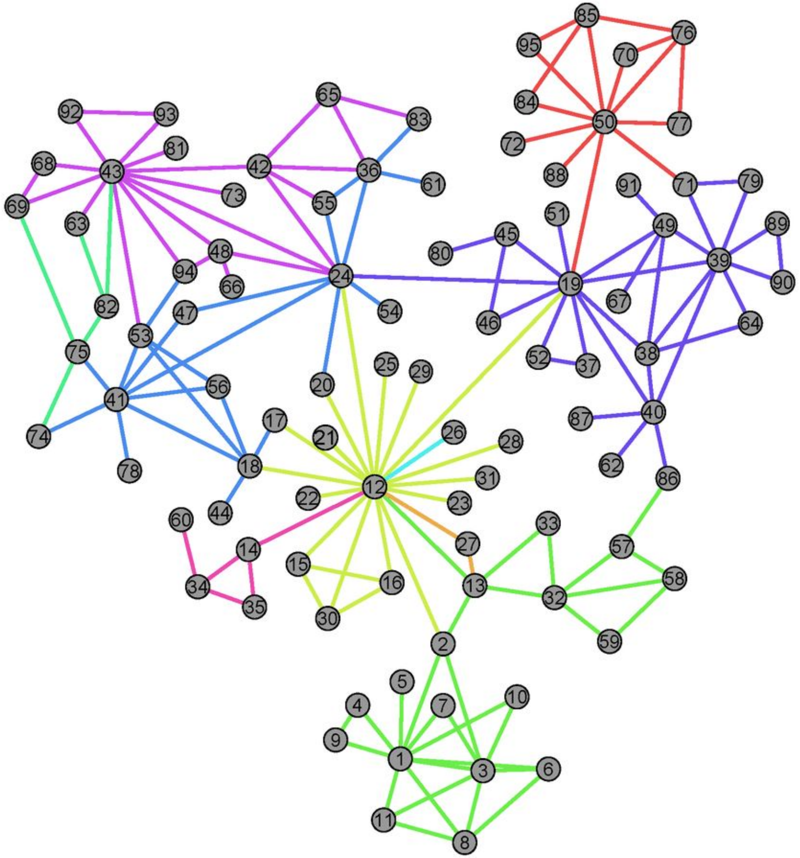
$\boxed{p_{\phi,\psi}(z=j|w)}$ : ***Softmax*** of node embedding over community embeddings

$\boxed{p_{\psi,\varphi}(c|z=j)}$ : ***Softmax*** of community embedding over node embeddings

$\underline{q_{\phi,\psi}(z=j|w,c)}$ : ***Softmax*** of $\phi_w \odot \phi_c$ over community embeddings

# Infer overlapping communities

$$\mathcal{F}(w) = \{\arg\max_{k} q(z = k | w, c)\}_{c \in N(w)}.$$



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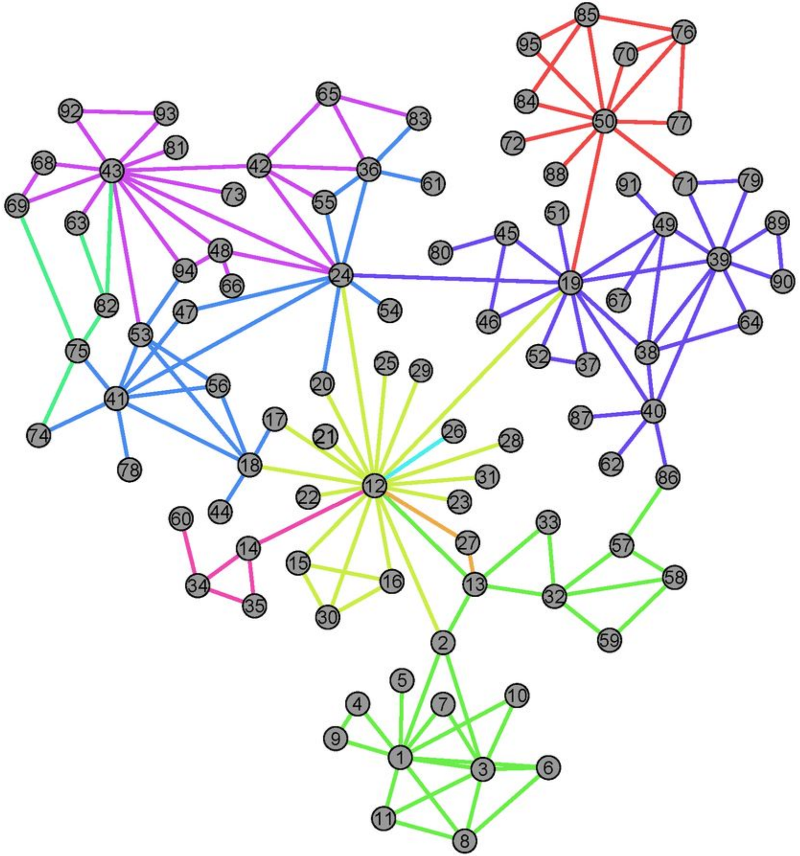


# vGraph - Community-smoothness Regularization

$$\mathcal{L}_{reg} = \lambda \sum_{(w,c)\in\mathcal{E}} \underline{\alpha_{w,c}} \cdot \underline{d(p(z|c), p(z|w))}$$

$$\Downarrow$$

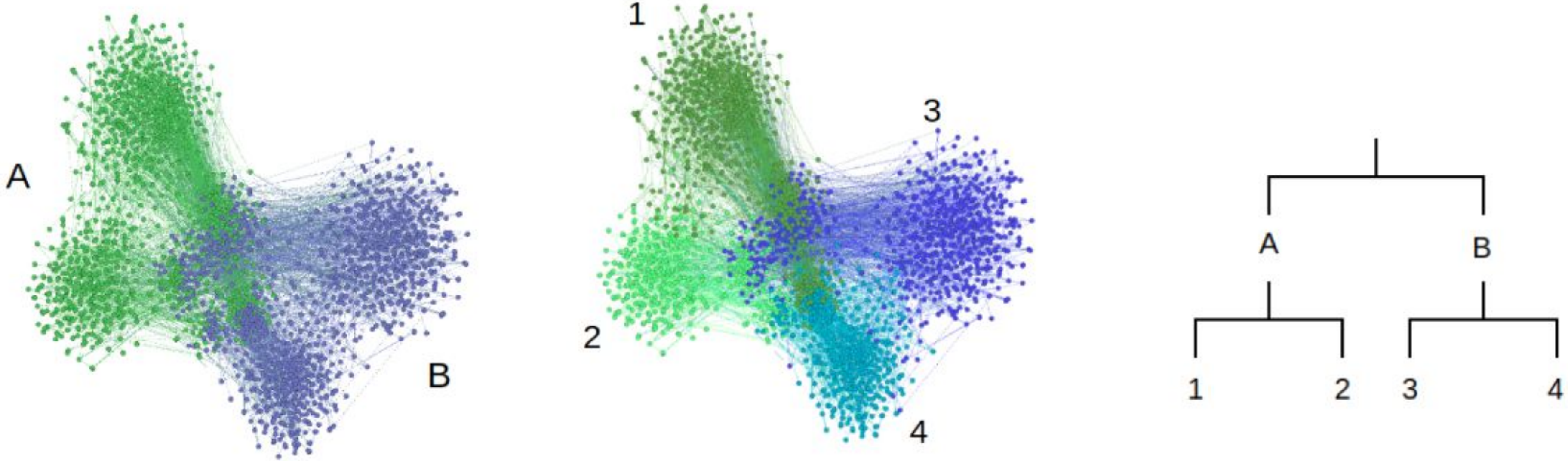
$$\alpha_{w,c} = \frac{|N(w) \cap N(c)|}{|N(w) \cup N(c)|},$$

# vGraph - hierarchical extension

$$p(c|w) = \sum_z p(c|z)p(z|w).$$

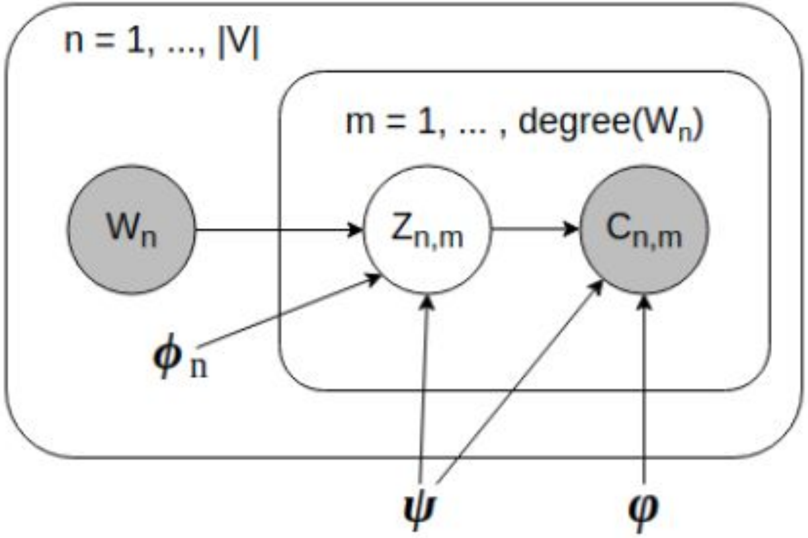
$$\Downarrow$$

$$p_{\boldsymbol{\phi},\boldsymbol{\varphi},\boldsymbol{\psi}}(c|w) = \sum_{\vec{z}} p_{\boldsymbol{\phi},\boldsymbol{\psi}}(c|\vec{z})p_{\boldsymbol{\phi},\boldsymbol{\psi}}(\vec{z}|w).$$

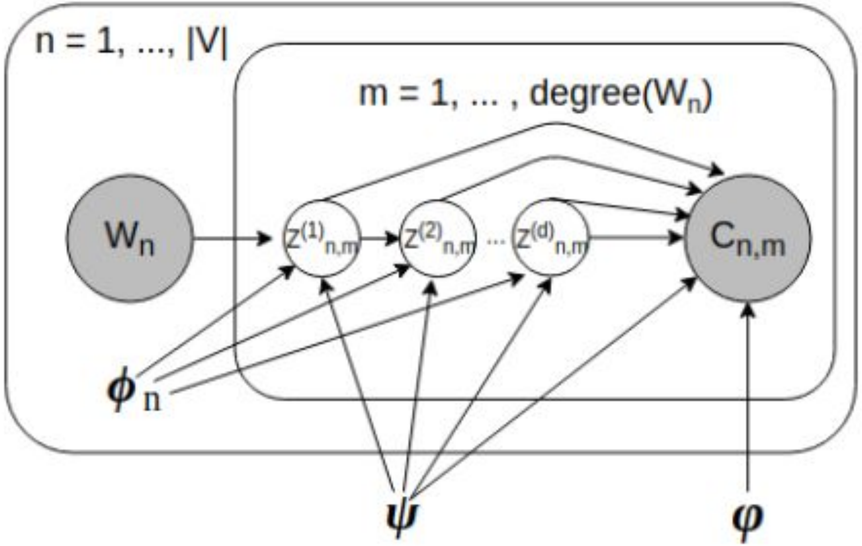



(a) (b) (c)

# vGraph - hierarchical extension




(a) vGraph




(b) Hierarchical vGraph

# Agenda

- Motivation
- vGraph
  - Community-smoothness Regularization
  - Hierarchical vGraph
- Experiments
  - Non-overlapping community detection & Node classification
  - Overlapping community detection
  - Visualization

# Evaluation of community detection

- Normalized Mutual Information
- Modularity (w/o ground truth)
- F1-score
- Jaccard Index

# Non-overlapping community detection & Node classification

Table 3: Evaluation (in terms of NMI and Modularity) on networks with non-overlapping ground-truth communities.

| | NMI | | | | | | Modularity | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dataset | MF | deepwalk | LINE | node2vec | ComE | vGraph | MF | deepwalk | LINE | node2vec | ComE | vGraph |
| cornell | 0.0632 | 0.0789 | 0.0697 | 0.0712 | 0.0732 | **0.0803** | 0.4220 | 0.4055 | 0.2372 | 0.4573 | 0.5748 | **0.5792** |
| texas | 0.0562 | 0.0684 | **0.1289** | 0.0655 | 0.0772 | 0.0809 | 0.2835 | 0.3443 | 0.1921 | 0.3926 | **0.4856** | 0.4636 |
| washington | 0.0599 | 0.0752 | **0.0910** | 0.0538 | 0.0504 | 0.0649 | 0.3679 | 0.1841 | 0.1655 | 0.4311 | 0.4862 | **0.5169** |
| wisconsin | 0.0530 | 0.0759 | 0.0680 | 0.0749 | 0.0689 | **0.0852** | 0.3892 | 0.3384 | 0.1651 | 0.5338 | 0.5500 | **0.5706** |
| cora | 0.2673 | 0.3387 | 0.2202 | 0.3157 | **0.3660** | 0.3445 | 0.6711 | 0.6398 | 0.4832 | 0.5392 | 0.7010 | **0.7358** |
| citeseer | 0.0552 | 0.1190 | 0.0340 | 0.1592 | **0.2499** | 0.1030 | 0.6963 | 0.6819 | 0.4014 | 0.4657 | 0.7324 | **0.7711** |

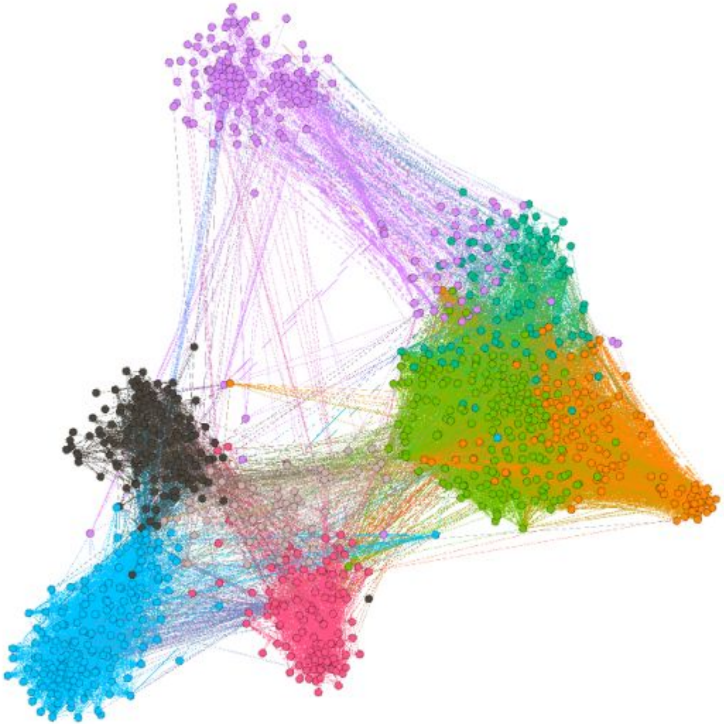
Table 4: Results of node classification on 6 datasets.

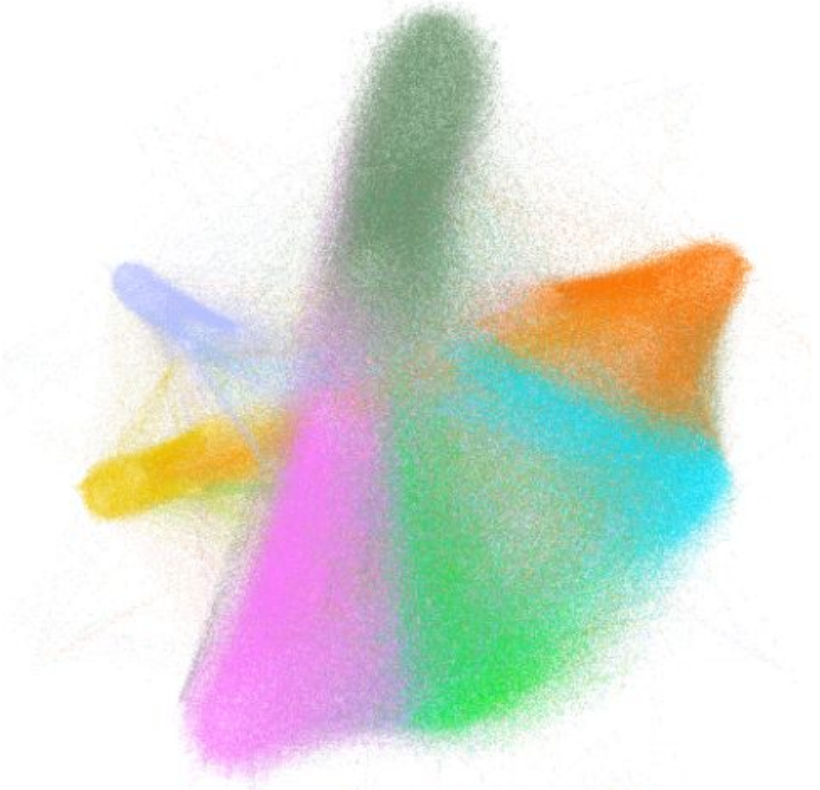
| | Macro-F1 | | | | | | Micro-F1 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Datasets | MF | DeepWalk | LINE | Node2Vec | ComE | vGraph | MF | DeepWalk | LINE | Node2Vec | ComE | vGraph |
| Cornell | 13.05 | 22.69 | 21.78 | 20.70 | 19.86 | **29.76** | 15.25 | 33.05 | 23.73 | 24.58 | 25.42 | **37.29** |
| Texas | 8.74 | 21.32 | 16.33 | 14.95 | 15.46 | **26.00** | 14.03 | 40.35 | 27.19 | 25.44 | 33.33 | **47.37** |
| Washington | 15.88 | 18.45 | 13.99 | 21.23 | 15.80 | **30.36** | 15.94 | 34.06 | 25.36 | 28.99 | 33.33 | **34.78** |
| Wisconsin | 14.77 | 23.44 | 19.06 | 18.47 | 14.63 | **29.91** | 18.75 | **38.75** | 28.12 | 25.00 | 32.50 | 35.00 |
| Cora | 11.29 | 13.21 | 11.86 | 10.52 | 12.88 | **16.23** | 12.79 | 22.32 | 14.59 | 27.74 | **28.04** | 24.35 |
| Citeseer | 14.59 | 16.17 | 15.99 | 16.68 | 12.88 | **17.88** | 15.79 | 19.01 | 16.80 | **20.82** | 19.42 | 20.42 |

# Overlapping community detection

| | F1-score | | | | | | Jaccard | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dataset | Bigclam | CESNA | Circles | SVI | vGraph | vGraph+ | Bigclam | CESNA | Circles | SVI | vGraph | vGraph+ |
| facebook0 | **0.2948** | 0.2806 | 0.2860 | 0.2810 | 0.2440 | 0.2606 | **0.1846** | 0.1725 | 0.1862 | 0.1760 | 0.1458 | 0.1594 |
| facebook107 | **0.3928** | 0.3733 | 0.2467 | 0.2689 | 0.2817 | 0.3178 | **0.2752** | 0.2695 | 0.1547 | 0.1719 | 0.1827 | 0.2170 |
| facebook1684 | 0.5041 | **0.5121** | 0.2894 | 0.3591 | 0.4232 | 0.4379 | 0.3801 | **0.3871** | 0.1871 | 0.2467 | 0.2917 | 0.3272 |
| facebook1912 | 0.3493 | 0.3474 | 0.2617 | 0.2804 | 0.2579 | **0.3750** | 0.2412 | 0.2394 | 0.1672 | 0.2010 | 0.1855 | **0.2796** |
| facebook3437 | 0.1986 | 0.2009 | 0.1009 | 0.1544 | 0.2087 | **0.2267** | 0.1148 | 0.1165 | 0.0545 | 0.0902 | 0.1201 | **0.1328** |
| facebook348 | 0.4964 | 0.5375 | 0.5175 | 0.4607 | **0.5539** | 0.5314 | 0.3586 | 0.4001 | 0.3927 | 0.3360 | **0.4099** | 0.4050 |
| facebook3980 | 0.3274 | 0.3574 | 0.3203 | NA | **0.4450** | 0.4150 | 0.2426 | 0.2645 | 0.2097 | NA | **0.3376** | 0.2933 |
| facebook414 | 0.5886 | 0.6007 | 0.4843 | 0.3893 | 0.6471 | **0.6693** | 0.4713 | 0.4732 | 0.3418 | 0.2931 | 0.5184 | **0.5587** |
| facebook686 | 0.3825 | 0.3900 | 0.5036 | 0.4639 | 0.4775 | **0.5379** | 0.2504 | 0.2534 | 0.3615 | 0.3394 | 0.3272 | **0.3856** |
| facebook698 | 0.5423 | 0.5865 | 0.3515 | 0.4031 | 0.5396 | **0.5950** | 0.4192 | 0.4588 | 0.2255 | 0.3002 | 0.4356 | **0.4771** |
| Youtube | 0.4370 | 0.3840 | 0.3600 | 0.4140 | 0.5070 | **0.5220** | 0.2929 | 0.2416 | 0.2207 | 0.2867 | 0.3434 | **0.3480** |
| Amazon | 0.4640 | 0.4680 | **0.5330** | 0.4730 | **0.5330** | 0.5320 | 0.3505 | 0.3502 | 0.3671 | 0.3643 | 0.3689 | **0.3693** |
| Dblp | 0.2360 | 0.3590 | NA | NA | 0.3930 | **0.3990** | 0.1384 | 0.2226 | NA | NA | 0.2501 | **0.2505** |
| Coauthor-CS | 0.3830 | 0.4200 | NA | 0.4070 | 0.4980 | **0.5020** | 0.2409 | 0.2682 | NA | 0.2972 | **0.3517** | 0.3432 |

# Visualization





# Conclusion

- Motivation
  - community information is important for node
    - **model nodes as mixture of communities**
  - community detection can use node embedding as features
    - **Model communities as multinomial distribution over nodes**
- vGraph: Generative model with Variational Inference & (stochastic) gradient descent
  - ~~Bayesian inference~~
  - ~~MCMC~~
- Efficient & Scalable → ***O(*** d * |E| * K *)*
  - d: embedding dimension
  - K: number of communities

# Thanks!

Arvix Full paper: https://arxiv.org/pdf/1906.07159.pdf

## Reference (figures):


https://www.google.com/url?sa=i&source=images&cd=&ved=2ahUKEwie7-PqsLbkAhVMHTQIHdHEDW4QjRx6BAgBEAQ&url=https%3A%2F%2Fbigdata.oden.utexas.edu%2Fproject%2Fgraph-clustering%2F&psig=AOvVaw2w8OQzTar0QlHqKYxNQc_8&ust=1567659483561269

https://www.semanticscholar.org/paper/Representation-Learning-on-Graphs%3A-Methods-and-Hamilton-Ying/ecf6c42d84351f34e1625a6a2e4cc6526da45c74/figure/4

https://www.google.com/url?sa=i&source=images&cd=&ved=2ahUKEwj4-pKOwLbkAhX0KDQIHci-DEEQjRx6BAgBEAQ&url=https%3A%2F%2Fwww.researchgate.net%2Ffigure%2FVisualization-of-edge-clustering-using-a-subset-of-DBLP-with-95-instances-Edges-are_fig2_313422448&psig=AOvVaw0j2v8wHOpfuIUCPz-EOGxN&ust=1567663587564454